SECTION 12
BEAM AND GIRDER BRIDGES

Alfred Hedefine, P.E.
Former President, Parsons Brinckerhoff Quade & Douglas Inc., New York, NY

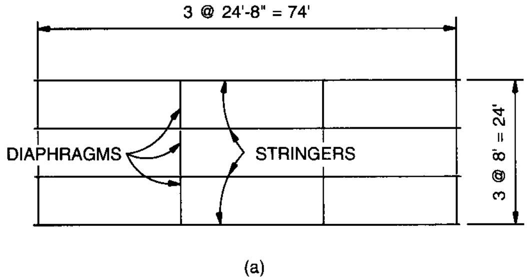
John Swindlehurst, P.E.
Former Senior Professional Associate, Parsons Brinckerhoff Quade & Douglas Inc., Newark, N.J.

Mahir Sen, P.E.
Professional Associate, Parsons Brinckerhoff-FG, Inc., Princeton, N.J.

Steel beam and girder bridges are often the most economical type of framing. Contemporary capabilities for extending beam construction to longer and longer spans safely and economically can be traced to the introduction of steel and the availability, in the early part of the twentieth century, of standardized rolled beams. By the late thirties, after wide-flange shapes became generally available, highway stringer bridges were erected with simply supported, wide-flange beams on spans up to about 110 ft. Riveted plate girders were used for highway-bridge spans up to about 150 ft. In the fifties, girder spans were extended to 300 ft by taking advantage of welding, continuity, and composite construction. And in the sixties, spans two and three times as long became economically feasible with the use of high-strength steels and box girders, or orthotropic-plate construction, or stayed girders. Thus, now, engineers, as a matter of common practice, design girder bridges for medium and long spans as well as for short spans.

12.1 CHARACTERISTICS OF BEAM BRIDGES

Rolled wide-flange shapes generally are the most economical type of construction for short-span bridges. The beams usually are used as stringers, set, at regular intervals, parallel to the direction of traffic, between piers or abutments (Fig. 12.1). A concrete deck, cast on the top flange, provides lateral support against buckling. Diaphragms between the beams offer additional bracing and also distribute loads laterally to the beams before the concrete deck has cured.



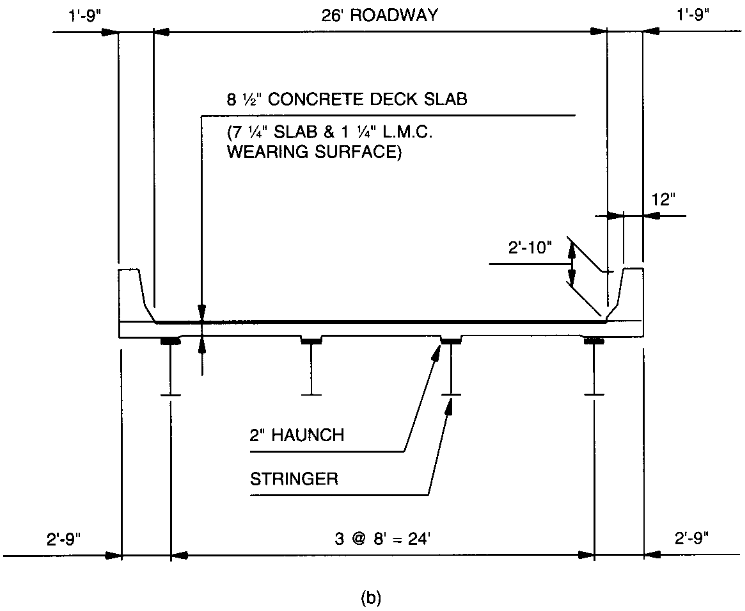


FIGURE 12.1 Two-lane highway bridge with rolled-beam stringers. (*a*) Framing plan. (*b*) Typical cross section.

Spacing. For railroad bridges, two stringers generally carry each track. They may, however, be more widely spaced than the rails, for stability reasons. If a bridge contains only two stringers, the distance between their centers should be at least 6 ft 6 in. When more stringers are used, they should be placed to distribute the track load uniformly to all beams.

For highway bridges, one factor to be considered in selection of stringer spacing is the minimum thickness of concrete deck permitted. For the deck to serve at maximum efficiency, its span between stringers should be at least that requiring the minimum thickness. But when stringer spacing requires greater than minimum thickness, the dead load is increased, cutting into the savings from use of fewer stringers. For example, if the minimum thickness of concrete slab is about 8 in, the stringer spacing requiring this thickness is about 8 ft for 4,000-psi concrete. Thus, a 29-ft 6-in-wide bridge, with 26-ft roadway, could be carried on four girders with this spacing. The outer stringers then would be located 1 ft from the curb into the roadway, and the outer portion of the deck, with parapet, would cantilever 2 ft 9 in beyond the stringers.

If an outer stringer is placed under the roadway, the distance from the center of the stringer to the curb preferably should not exceed about 1 ft.

Stringer spacing usually lies in the range 6 to 15 ft. The smaller spacing generally is desirable near the upper limits of rolled-beam spans.

The larger spacing is economical for the longer spans where deep, fabricated, plate girders are utilized. Wider spacing of girders has resulted in development of long-span stay-in-place forms. This improvement in concrete-deck forming has made steel girders with a concrete deck more competitive.

Regarding deck construction, while conventional cast-in-place concrete decks are commonplace, precast-concrete deck slab bridges are often used and may prove practical and economical if stage construction and maintenance of traffic are required. Additionally, use of lightweight concrete, a durable and economical product, may be considered if dead weight is a problem.

Other types of deck are available such as steel orthotropic plates (Arts. 12.14 and 12.15). Also, steel grating decks may be utilized, whether unfilled, half-filled, or fully filled with concrete. The latter two deck-grating construction methods make it possible to provide composite action with the steel girder.

Short-Span Stringers. For spans up to about 40 ft, noncomposite construction, where beams act independently of the concrete slab, and stringers of AASHTO M270 (ASTM A709), Grade 36 steel often are economical. If a bridge contains more than two such spans in succession, making the stringers continuous could improve the economy of the structure. Savings result primarily from reduction in number of bearings and expansion joints, as well as associated future maintenance costs. A three-span continuous beam, for example, requires four bearings, whereas three simple spans need six bearings.

For such short spans, with relatively low weight of structural steel, fabrication should be kept to a minimum. Each fabrication item becomes a relatively large percentage of material cost. Thus, cover plates should be avoided. Also, diaphragms and their connections to the stringers should be kept simple. For example. they may be light channels field bolted or welded to plates welded to the beam webs (Fig. 12.2).

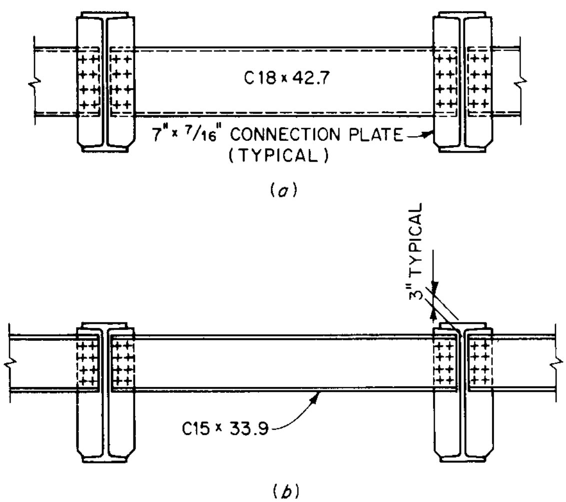


FIGURE 12.2 Diaphragms for rolled-beam stringers. (*a*) Intermediate diaphragm. (*b*) End diaphragm.

For spans 40 ft and less, each beam reaction should be transferred to a bearing plate through a thin sole plate welded to the beam flange. The bearing may be a flat steel plate or an elastomeric pad. At interior supports of continuous beams, sole plates should be wider than the flange. Then, holes needed for anchor bolts can be placed in the parts of the plates extending beyond the flange. This not only reduces fabrication costs by avoiding holes in the stringers but also permits use of lighter stringers, because the full cross section is available for moment resistance.

At each expansion joint, the concrete slab should be thickened to form a transverse beam, to protect the end of the deck. Continuous reinforcement is required for this beam. For the purpose, slotted holes should be provided in the ends of the steel beams to permit the reinforcement to pass through.

Live Loads. Although AASHTO "Standard Specifications for Highway Bridges" specify for design H15-44, HS15-44, H20-44, and HS20-44 truck and lane loadings (Art. 11.4), many state departments of transportation are utilizing larger live loadings. The most common is HS20-44 plus 25% (HS25). An alternative military loading of two axles 4 ft apart, each axle weighing 24 kips, is usually also required and should be used if it causes higher stresses. Some states prefer 30 kip axles instead of 24 kips.

Dead Loads. Superstructure design for bridges with a one-course deck slab should include a 25-psf additional dead load to provide for a future 2-in-thick overlay wearing surface. Bridges with a two-course deck slab generally do not include this additional dead load. The assumption is that during repaving of the adjoining roadway, the 1¼-in wearing course (possibly latex modified concrete) will be removed and replaced only if necessary.

If metal stay-in-place forms are permitted for deck construction, consideration should be given to providing for an additional 8 to 12 psf to be included for the weight of the permanent steel form plus approximately 5 psf for the additional thickness of deck concrete required. The specific additional dead load should be determined for the form to be utilized. The additional dead load is considered secondary and may be included in the superimposed dead load supported by composite construction, when shoring is used.

Long-Span Stringers. Composite construction with rolled beams (Art. 11.16) may become economical when simple spans exceed about 40 ft, or the end span of a continuous stringer exceeds 50 ft, or the interior span of a continuous stringer exceeds 65 ft. W36 rolled wide-flange beams of Grade 36 steel designed for composite action with the concrete slab are economical for spans up to about 85 ft, though such beams can be used for longer spans. When spans exceed 85 ft, consideration should be given to rolled beams made of high-strength steels, W40 rolled wide-flange beams, or to plate-girder stringers. In addition to greater economy than with noncomposite construction, composite construction offers smaller deflections or permits use of shallower stringers, and the safety factor is larger.

For long-span, simply supported, composite, rolled beams, costs often can be cut by using a smaller rolled section than required for maximum moment and welding a cover plate to the bottom flange in the region of maximum moment (partial-length cover plate). For the purpose, one plate of constant width and thickness should be used. It also is desirable to use cover plates on continuous beams. The cover plate thickness should generally be limited to about 1 in and be either 2 in narrower or 2 in maximum wider than the flange. Longitudinal fillet welds attach the plate to the flange. Cover plates may be terminated and end-welded within the span at a developed length beyond the theoretical cutoff point. American Association of State Highway and Transportation Officials (AASHTO) specifications provide for a Category E′ allowable fatigue-stress range that must be utilized in the design of girders at this point.

Problems with fatigue cracking of the end weld and flange plate of older girders has caused designers to avoid terminating the cover plate within the span. Some state departments of transportation specify that cover plates be full length or terminated within 2 ft of the end bearings. The end attachments may be either special end welds or bolted connections.

Similarly, for continuous, noncomposite, rolled beams, costs often can be cut by welding cover plates to flanges in the regions of negative moment. Savings, however, usually will not be achieved by addition of a cover plate to the bottom flange in positive-moment areas. For composite construction, though, partial-length cover plates in both negative-moment and positive-moment regions can save money. In this case, the bottom cover plate is effective because the tensile forces applied to it are balanced by compressive forces acting on the concrete slab serving as a top cover plate.

For continuous stringers, composite construction can be used throughout or only in positive-moment areas. Costs of either procedure are likely to be nearly equal.

Design of composite stringers usually is based on the assumption that the forms for the concrete deck are supported on the stringers. Thus, these beams have to carry the weight of the uncured concrete. Alternatively, they can be shored, so that the concrete weight is transmitted directly to the ground. The shores are removed after the concrete has attained sufficient strength to participate in composite action. In that case, the full dead load may be assumed applied to the composite section. Hence, a slightly smaller section can be used for the stringers than with unshored erection. The savings in steel, however, may be more than offset by the additional cost of shoring, especially when provision has to be made for traffic below the span.

Diaphragms for long-span rolled beams, as for short-span, should be of minimum permitted size. Also, connections should be kept simple (Fig. 12.2). At span ends, diaphragms should be capable of supporting the concrete edge beam provided to protect the end of the concrete slab. Consideration should also be given to designing the end diaphragms for jacking forces for future bearing replacements.

For simply supported, long-span stringers, one end usually is fixed, whereas arrangements are made for expansion at the other end. Bearings may be built up of steel or they may be elastomeric pads. A single-thickness pad may be adequate for spans under 85 ft. For longer spans, laminated pads will be needed. Expansion joints in the deck may be made economically with extruded or preformed plastics.

Cambering of rolled-beam stringers is expensive. It often can be avoided by use of different slab-haunch depths over the beams.

12.2 EXAMPLE-ALLOWABLE-STRESS DESIGN OF COMPOSITE, ROLLED-BEAM STRINGER BRIDGE

To illustrate the design procedure, a two-lane highway bridge with simply supported, composite, rolled-beam stringers will be designed. As indicated in the framing plan in Fig. 12.1*a*, the stringers span 74 ft center to center (c to c) of bearings. The typical cross section in Fig. 12.1*b* shows a 26-ft-wide roadway flanked by 1-ft 9-in parapets. Structural steel to be used is Grade 36. Loading is HS25. Appropriate design criteria given in Sec. 11 will be used for this structure. Concrete to be used for the deck is Class A, with 28-day compressive strength $f'_c = 4{,}000$ psi and allowable compressive strength $f_c = 1{,}400$ psi. Modulus of elasticity $E_c = 33w^{1.5}\sqrt{f'_c} = 33(145)^{1.5}\sqrt{4{,}000} = 3{,}644{,}000$ psi, say 3,600,000 psi.

Assume that the deck will be supported on four rolled-beam stringers, spaced 8 ft c to c, as shown in Fig. 12.1.

Concrete Slab. The slab is designed to span transversely between stringers, as in noncomposite design. The effective span S is the distance between flange edges plus half the flange width, ft. In this case, if the flange width is assumed as 1 ft, $S = 8 - 1 + \frac{1}{2} = 7.5$ ft. For computation of dead load, assume a 9-in-thick slab, weight 112 lb/ft^2 plus 5 lb/ft^2 for the additional thickness of deck concrete in the stay-in-place forms. The 9-in-thick slab consists of a 7¾-in base slab plus a 1¼-in latex-modified concrete (LMC) wearing course. Total dead load then is 117 lb/ft^2. With a factor of 0.8 applied to account for continuity of the slab over the stringers, the maximum dead-load bending moment is

$$M_D = \frac{w_D S^2}{10} = \frac{117(7.5)^2}{10} = 660 \text{ ft-lb per ft}$$

From Table 11.27, the maximum live-load moment, with reinforcement perpendicular to traffic, plus a 25% increase for conversion to HS25 loading, equals

$$M_L = 1.25 \times 400(S + 2) = 500(7.5 + 2) = 4{,}750 \text{ ft-lb/ft}$$

Allowance for impact is 30% of this, or 1,425 ft-lb/ft. The total maximum moment then is

$$M = 660 + 4{,}750 + 1{,}425 = 6{,}835 \text{ ft-lb/ft}$$

For balanced design of the concrete slab, the depth $k_b d_b$ of the compression zone is determined from

$$k_b = \frac{1}{1 + f_s/nf_c} = \frac{1}{1 + 24{,}000/8(1{,}400)} = 0.318$$

where d_b = effective depth of slab, in, for balanced design
f_s = allowable tensile stress for reinforcement, psi = 24,000 psi
n = modular ratio = E_s/E_c = 8
E_s = modulus of elasticity of the reinforcement, psi = 29,000,000 psi
E_c = modulus of elasticity of the concrete, psi = 3,600,000 psi

For determination of the moment arm $j_b d_b$ of the tensile and compressive forces on the cross section,

$$j_b = 1 - k_b/3 = 1 - 0.318/3 = 0.894$$

Then the required depth for balanced design, with width of slab b taken as 1 ft, is

$$d_b = \sqrt{2M/f_c bjk} = 5.86 \text{ in}$$

For the assumed dimensions of the concrete slab, the depth from the top of slab to the bottom reinforcement is

$$d = 9 - 0.5 - 1 - 0.38 = 7.12 \text{ in}$$

The depth from bottom of slab to top reinforcement is

$$d = 7.75 + 1.25 - 2.75 - 0.38 = 5.88 \text{ in}$$

Since $d > d_b$, this will be an underreinforced section. Use d = 5.88 in. Then, the maximum compressive stress on a slab of the assumed dimensions is

$$f_c = \frac{M}{(kd)(jd)b/2} = \frac{6{,}835 \times 12}{1.87 \times 5.26 \times {}^{12}\!/_{2}} = 1{,}390 < 1{,}400 \text{ psi}$$

Hence, a 9-in-thick concrete slab is satisfactory.

Required reinforcement area transverse to traffic is

$$A_s = \frac{12M}{f_s jd} = \frac{12 \times 6{,}835}{24{,}000 \times 5.26} = 0.65 \text{ in}^2/\text{ft}$$

Use No. 6 bars at 8-in intervals. These supply 0.66 in^2/ft. For distribution steel parallel to traffic, use No. 5 bars at 9 in, providing an area about two-thirds of 0.65 in^2/ft.

Stringer Design Procedure. A composite stringer bridge may be considered to consist of a set of T beams set side by side. Each T beam comprises a steel stringer and a portion of the concrete slab (Art. 11.16). The usual design procedure requires that a section be assumed for the steel stringer. The concrete is transformed into an equivalent area of steel. This is done for a short-duration load by dividing the effective area of the concrete flange by the ratio n of the modulus of elasticity of steel to the modulus of elasticity of the concrete, and for a long-duration load, under which the concrete may creep, by dividing by $3n$. Then, the properties of the transformed section are computed. Next, bending stresses are checked at top and bottom of the steel section and top of concrete slab. After that, cover-plate lengths are determined, web shear is investigated, and shear connectors are provided to bond the concrete slab to the steel section. Finally, other design details are taken care of, as in non-composite design.

Fabrication costs often will be lower if all the stringers are identical. The outer stringers, however, carry different loads from those on interior stringers. Sometimes girder spacing can be adjusted to equalize the loads. If not, and the load difference is large, it may be necessary to provide different designs for inner and outer stringers. Exterior stringers, however, should have at least the same load capacity as interior stringers. Since the design procedure is the same in either case, only a typical interior stringer will be designed in this example.

Loads, Moments, and Shears. Assume that the stringers will not be shored during casting of the concrete slab. Hence, the dead load on each stringer includes the weight of an 8-ft-wide strip of concrete slab as well as the weights of steel shape, cover plate, and framing details. This dead load will be referred to as *DL.*

DEAD LOAD CARRIED BY STEEL BEAM, KIPS PER FT:

Slab: $0.150 \times 8 \times 7.75 \times 1/12$	= 0.775
Haunch—12 × 1 in: $0.150 \times 1 \times 1/12$	= 0.013
Stay-in-place forms: 0.013×7	= 0.091
Rolled beam and details—assume	0.296
DL per stringer	1.175

Maximum moment occurs at the center of the 74-ft span:

$$M_{DL} = 1.175(74)^2/8 = 804 \text{ ft-kips}$$

Maximum shear occurs at the supports and equals

$$V_{DL} = 1.175 \times 74/2 = 43.5 \text{ kips}$$

The safety-shaped parapets will be placed after the concrete has cured. Their weights may be equally distributed to all stringers. No allowance will be made for a future wearing surface, but provision will be made for the weight of the 1¼-in LMC wearing course. The total superimposed dead load will be designated SDL.

DEAD LOAD CARRIED BY COMPOSITE SECTION, KIPS PER FT

Two parapets: 1.060/4	0.265
LMC wearing course:	0.125

$0.150 \times 8 \times 1.25/12$	
SDL per stringer:	0.390

Maximum moment occurs at midspan and equals

$$M_{SDL} = 0.390(74)^2/8 = 267 \text{ ft-kips}$$

Maximum shear occurs at the supports and equals

$$V_{SDL} = 0.390 \times 74/2 = 14.4 \text{ kips}$$

The HS25 live load imposed may be a truck load or a lane load. For maximum effect with the truck load, the two 40-kip axle loads, with variable spacing *V*, should be placed 14 ft apart, the minimum permitted (Fig. 12.3*a*). Then the distance of the center of gravity of the three axle loads from the center load is found by taking moments about the center load.

$$a = \frac{40 \times 14 - 10 \times 14}{40 + 40 + 10} = 4.67 \text{ ft}$$

Maximum moment occurs under the center axle load when its distance from mid-span is the same as the distance of the center of gravity of the loads from midspan, or $4.67/2 = 2.33$ ft. Thus, the center load should be placed $\frac{74}{2} - 2.33 = 34.67$ ft from a support (Fig. 12.3*a*). Then, the maximum moment due to the 90-kip truck load is

$$M_T = \frac{90(\frac{74}{2} + 2.33)^2}{74} - 40 \times 14 = 1{,}321 \text{ ft-kips}$$

This loading governs, because the maximum moment due to lane loading (Fig. 12.3*b*) is smaller:

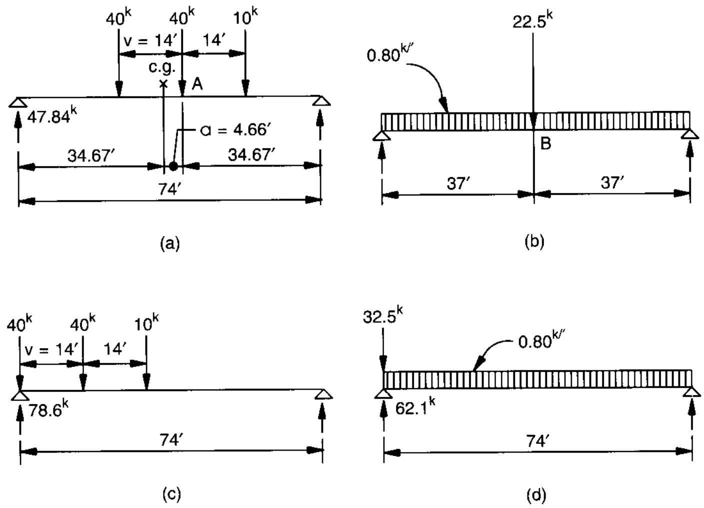


FIGURE 12.3 Positions of load for maximum stress in a simply supported stringer. (*a*) Maximum moment in the span with truck loads. (*b*) Maximum moment in the span with lane loading. (*c*) Maximum shear in the span with truck loads. (*d*) Maximum shear in the span with lane loading.

$$M_L = 0.80(74)^2/8 + 22.5 \times {}^{74}\!/_4 = 964 < 1{,}321 \text{ ft-kips}$$

The distribution of the live load to a stringer may be obtained from Table 11.14, for a bridge with two traffic lanes.

$$\frac{S}{5.5} = \frac{8}{5.5} = 1.454 \text{ wheels} = 0.727 \text{ axle}$$

Hence, the maximum live-load moment is

$$M_{LL} = 0.727 \times 1{,}321 = 960 \text{ ft-kips}$$

While this moment does not occur at midspan as do the maximum dead-load moments, stresses due to M_{LL} may be combined with those from M_{DL} and M_{SDL} to produce the maximum stress, for all practical purposes.

For maximum shear with the truck load, the outer 40-kip load should be placed at the support (Fig. 12.3*c*). Then, the shear is

$$V_T = \frac{90(74 - 14 + 4.66)}{74} = 78.6 \text{ kips}$$

This loading governs, because the shear due to lane loading (Fig. 12.3*d*) is smaller:

$$V_L = 32.5 + 0.80 \times {}^{74}\!/_2 = 62.1 < 78.6 \text{ kips}$$

Since the stringer receives 0.727 axle loads, the maximum shear on the stringer is

$$V_{LL} = 0.727 \times 78.6 = 57.1 \text{ kips}$$

Impact is the following fraction of live-load stress:

$$I = \frac{50}{L + 125} = \frac{50}{74 + 125} = 0.251$$

Hence, the maximum moment due to impact is

$$M_I = 0.251 \times 960 = 241 \text{ ft-kips}$$

and the maximum shear due to impact is

$$V_I = 0.251 \times 57.1 = 14.3 \text{ kips}$$

MIDSPAN BENDING MOMENTS, FT-KIPS:

M_{DL}	M_{SDL}	$M_{LL} + M_I$
804	267	1,201

END SHEAR, KIPS:

V_{DL}	V_{SDL}	$V_{LL} + V_I$	Total V
43.5	14.4	71.4	129.3

Properties of Composite Section. The 9-in-thick roadway slab includes an allowance of 0.5 in for a wearing surface. Hence, the effective thickness of the concrete slab for composite action is 8.5 in.

The effective width of the slab as part of the top flange of the T beam is the smaller of the following:

¼ span = ¼ × 74 = 222 in
Stringer spacing, c to c = 8 × 12 = 96 in
12 × slab thickness = 12 × 8.5 = 102 in

Hence, the effective width is 96 in (Fig. 12.4).

To complete the T beam, a trial steel section must be selected. As a guide in doing this, formulas for estimated required flange area given in I. C. Hacker, "A Simplified Design of Composite Bridge Structures," *Journal of the Structural Division, ASCE, Proceedings Paper* 1432, November, 1957, may be used. To start, assume the rolled beam will be a 36-in-deep wide-flange shape, and take the allowable bending stress F_b as 20 ksi. The required bottom-flange area, in^2, then may be estimated from

$$A_{sb} = \frac{12}{F_b}\left(\frac{M_{DL}}{d_{cg}} + \frac{M_{SDL} + M_{LL} + M_I}{d_{cg} + t}\right) \tag{12.1a}$$

where d_{cg} = distance, in, between center of gravity of flanges of steel shape and t = thickness, in, of concrete slab. With d_{cg} assumed as 36 in, the estimated required bottom-flange area is

$$A_{sb} = \frac{12}{20}\left(\frac{804}{36} + \frac{267 + 1201}{36 + 8.5}\right) = 33.2 \text{ in}^2$$

The ratio $R = A_{st}/A_{sb}$, where A_{st} is the area, in^2, of the top flange of the steel beam, may be estimated to be

$$R = 50/(190 - L) = 50/(190 - 74) = 0.43 \tag{12.1b}$$

Then, the estimated required area of the top flange is

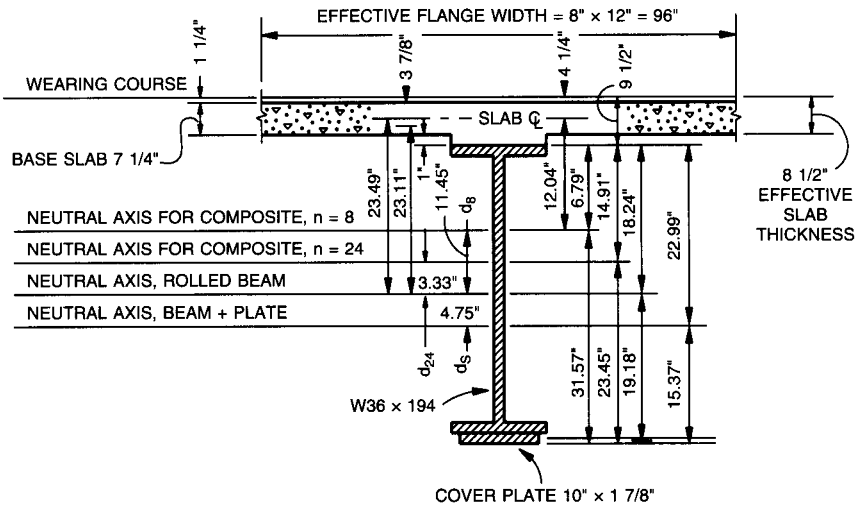


FIGURE 12.4 Cross section of composite stringer at midspan.

$$A_{st} = RA_{sb} = 0.43 \times 33.2 = 14.3 \text{ in}^2$$

A W36 × 194 provides a flange with width 12.117 in, thickness 1.26 in, and area

$$A_{st} = 12.117 \times 1.26 = 15.27 > 14.3 \text{ in}^2\text{—OK}$$

With this shape, a bottom cover plate with an area of at least $33.2 - 15.27 = 17.9$ in^2. Maximum thickness permitted for a cover plate on a rolled beam is 1.5 times the flange thickness. In this case, therefore, plate thickness should not exceed $1.5 \times 1.26 = 1.89$ in. These requirements are met by a 10 × 1⅞-in plate, with an area of 18.75 in^2.

The trial section chosen consequently is a W36 × 194 with a partial-length cover plate 10 × 1⅞ in on the bottom flange (Fig. 12.4). Its neutral axis can be located by taking moments about the neutral axis of the rolled beam. This computation and that for the section moduli S_{st} and S_{sb} of the steel section are conveniently tabulated in Table 12.1.

In computation of the properties of the composite section, the concrete slab, ignoring the haunch area, is transformed into an equivalent steel area. For the purpose, for this bridge, the concrete area is divided by the modular ratio $n = 8$ for short-time loading, such as live loads and impact. For long-time loading, such as superimposed dead loads, the divisor is $3n = 24$, to account for the effects of creep. The computations of neutral-axis location and section moduli for the composite section are tabulated in Table 12.2. To locate the neutral axis, moments are taken about the neutral axis of the rolled beam.

Stresses in Composite Section. Since the stringers will not be shored when the concrete is cast and cured, the stresses in the steel section for load *DL* are determined with the section moduli of the steel section alone (Table 12.1). Stresses for load *SDL* are computed with section moduli of the composite section when $n = 24$ from Table 12.2*a*. And stresses in the steel for live loads and impact are calculated with section moduli of the composite section when $n = 8$ from Table 12.2*b* (Table 12.3*a*).

Stresses in the concrete are determined with the section moduli of the composite section with $n = 24$ for *SDL* from Table 12.2*a* and $n = 8$ for $LL + I$ from Table 12.2*b* (Table 12.3*b*).

TABLE 12.1 Steel Section for Maximum Moment

Material	A	d	Ad	Ad^2	I_o	I
W36 × 194	57.00				12,100	12,100
Cover plate 10 × 1⅞	18.75	−19.18	−359.6	6,898		6,698
	75.75		−359.6			18,998
$d_s = -359.6/75.75 = -4.75$ in					$-4.75 \times 359.6 =$	−1,708
					$I_{NA} =$	17,290

Distance from the neutral axis of the steel section to:

$$\text{Top of steel} = 18.24 + 4.75 = 22.99 \text{ in}$$

$$\text{Bottom of steel} = 18.24 - 4.75 + 1.88 = 15.37 \text{ in}$$

Section moduli	
Top of steel	Bottom of steel
$S_{st} = 17{,}290/22.99 = 752 \text{ in}^3$	$S_{sb} = 17{,}290/15.37 = 1{,}125 \text{ in}^3$

TABLE 12.2 Composite Section for Maximum Moment

(*a*) For superimposed dead loads, $n = 24$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	75.75		−360			18,998
Concrete 96 × 7.75/24*	31.00	23.11	716	16,556	155	16,711
	106.75		356			35,709
$d_{24} = 356/106.75 = 3.33$ in					$-3.33 \times 356 =$	−1,185
					$I_{NA} =$	34,534

Distance from the neutral axis of the composite section to:

Top of steel = 18.24 − 3.33 = 14.91 in

Bottom of steel = 18.24 + 3.33 + 1.88 = 23.45 in

Top of concrete = 14.91 + 1 + 7.75 = 23.66 in

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 34{,}534/14.91 = 2{,}316$ in^3	$S_{sb} = 34{,}534/23.45 = 1{,}473$ in^3	$S_c = 34{,}534/23.66 = 1{,}460$ in^3

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	75.75		−360			18,998
Concrete 96 × 8.5/8	102.00	23.49	2,396	56,280	615	56,895
	177.75		2,036			75,893
$d_8 = 2{,}036/177.75 = 11.45$ in					$-11.45 \times 2{,}036 =$	−23,312
					$I_{NA} =$	52,581

Distance from the neutral axis of the composite section to:

Top of steel = 18.24 − 11.45 = 6.79 in

Bottom of steel = 18.24 + 11.45 + 1.88 = 31.57 in

Top of concrete = 6.79 + 1 + 8.5 = 16.29 in

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 52{,}580/6.79 = 7{,}744$ in^3	$S_{sb} = 52{,}580/31.57 = 1{,}666$ in^3	$S_c = 52{,}580/16.29 = 3{,}228$ in^3

*Depth of the top slab is taken as 7.75 in, inasmuch as the 1¼-in wearing course is included in the superimposed load.

TABLE 12.3 Stresses in the Composite Section, ksi, at Section of Maximum Moment

(*a*) Steel stresses

Top of steel (compression)		Bottom of steel (tension)	
DL: $f_b = 804 \times 12/752$	$= 12.83$	$f_b = 804 \times 12/1{,}125 =$	8.58
SDL: $f_b = 267 \times 12/2{,}316 =$	1.38	$f_b = 267 \times 12/1{,}473 =$	2.18
LL + I: $f_b = 1{,}201 \times 12/7{,}744 =$	1.86	$f_b = 1{,}201 \times 12/1{,}666 =$	8.66
Total:	$16.07 < 20$		$19.42 < 20$

(*b*) Stresses at top of concrete

$$SDL\text{: } f_c = 267 \times 12/(1{,}460 \times 24) = 0.09$$
$$LL + I\text{: } f_c = 1{,}201 \times 12/(3{,}228 \times 8) = \underline{0.56}$$
$$0.65 < 1.6$$

Since the bending stresses in steel and concrete are less than the allowable, the assumed steel section is satisfactory. Use the W36 × 194 with 10 × 1⅞-in bottom cover plate. Total weight of steel will be about 0.274 kip per ft, including 0.016 kip per ft for diaphragms, whereas 0.297 kip per ft was assumed in the dead-load calculations.

Maximum Shear Stress. Though shear rarely is critical in wide-flange shapes adequate in bending, the maximum shear in the web should be checked. The total shear at the support has been calculated to be 129.3 kips. The web of the steel beam is about 36 in deep and the thickness is 0.770 in. Thus, the web area is

$$36 \times 0.770 = 27.7 \text{ in}^2$$

and the average shear stress is

$$f_v = \frac{129.3}{27.7} = 4.7 < 12 \text{ ksi}$$

This indicates that the beam has ample shear capacity.

End bearing stiffeners are not required for a rolled beam if the web shear does not exceed 75% of the allowable shear for girder webs, 12 ksi. The ratio of actual to allowable shears is

$$\frac{f_v}{F_v} = \frac{4.7}{12} = 0.39 < 0.75$$

Hence, bearing stiffeners are not required.

Cover-Plate Cutoff. Bending moments decrease almost parabolically with distance from midspan, to zero at the supports. At some point on either side of the center, therefore, the cover plate is not needed for carrying bending moment. For locating this cutoff point, the properties of the composite section without the cover plate are needed, with $n = 24$ and $n = 8$ (Fig. 12.5). The computations are tabulated in Table 12.4.

The length L_{cp}, ft, required for the cover plate may be estimated by assuming that the curve of maximum moments is a parabola. Approximately,

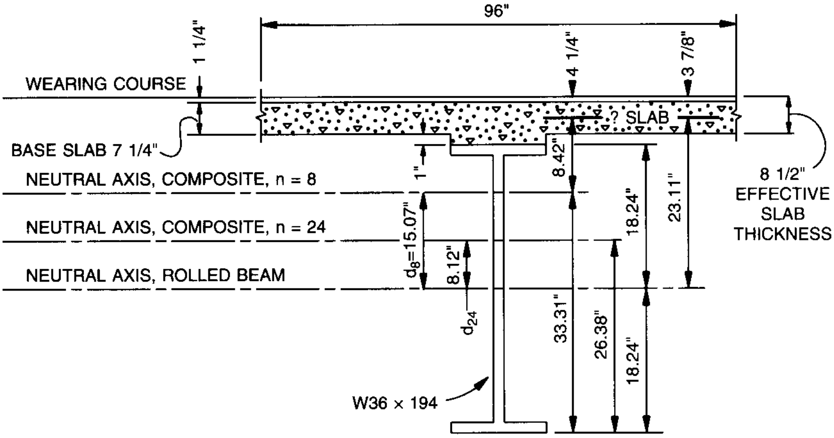


FIGURE 12.5 Cross section of composite stringer near supports.

TABLE 12.4 Composite Section Near Supports

(*a*) For dead loads, $n = 24$

Material	A	d	Ad	Ad^2	I_o	I
W36 × 194	57.0				12,100	12,100
Concrete 96 × 7.75/24	31.0	23.11	716	16,556	155	16,711
	88.0		716			28,811
d_{24} = 716/88.0 = 8.14 in					−8.14 × 716 =	−5,826
Half-beam depth = 18.24					I_{NA} =	22,985
26.38 in						

S_{sb} = 22,985/26.38 = 871 in^3

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
W36 × 194	57.0				12,100	12,100
Concrete 96 × 8.5/8	102.0	23.49	2,396	56,282	615	56,900
	159.0		2,396			69,000
d_8 = 2,396/159 = 15.07 in					−15.07 × 2,396 =	−36,110
Half-beam depth = 18.24					I_{NA} =	32,890
33.31 in						

S_{sb} = 32,890/33.31 = 987 in^3

$$L_{cp} = L\sqrt{1 - \frac{S'_{sb}}{S_{sb}}} \tag{12.2}$$

where L = span, ft
S'_{sb} = section modulus with respect to bottom of steel shape with lighter flange (without cover plate), in^3
S_{sb} = section modulus with respect to bottom of steel shape with heavier flange (with cover plate), in^3

For the W36 × 194, $S'_{sb} = 665$. Hence,

$$L_{cp} = 74\sqrt{1 - \frac{665}{1{,}125}} = 48 \text{ ft}$$

If the cover plate is welded along its ends, the terminal distance that the plate must be extended beyond its theoretical cutoff point is about 1.5 times the plate width. For the 10-in plate, therefore, the terminal distance is 1.5 × 10 = 15 in. Use 1.5 ft. Thus, L_{cp} must be increased by 2 × 1.5, to 51 ft.

Assume a 51-ft-long cover plate. It would then terminate 11.5 ft from each support (Fig. 12.6). The theoretical cutoff point is therefore 11.5 + 1.5 = 13.0 ft from each support. The stresses at that point should be checked to ensure that allowable bending stresses in the composite section without the cover plate are not exceeded. Table 12.5*a* presents the calculations for maximum flexural tensile stress at the theoretical cutoff points, 13-ft from the supports, and Table 12.5*b*, calculations for stresses at the actual terminations of the cover plate, 11.5 ft from the supports. The composite section without the cover plate is adequate at the theoretical cutoff point. But fatigue stresses in the beam should be checked at the actual termination of the plate, 11.5 ft from each support.

From Table 12.5*b*, the stress range equals the stress due to live load plus impact, 8.23 ksi. On the assumption that the bridge is a redundant-load-path structure, for base metal adjacent to a fillet weld (Category E′) subjected to 500,000 loading cycles, the allowable fatigue stress range permitted by AASHTO standard specifications is $F_{sr} = 9.2$ ksi > 8.23. The cover plate is satisfactory. (Because of past experience with fatigue cracking at termination welds for cover plates, however, the usual practice, when a cover plate is specified, is to extend it the full length of the beam.)

Cover-Plate Weld. The fillet weld connecting the cover plate to the bottom flange must be capable of resisting the shear at the bottom of the flange. The shear is a maximum at the end of the cover plate, 11.5 ft from the supports. The position of the truck load to produce maximum shear there is the same as that for maximum movement at those points (Fig. 12.8). Maximum shears and resulting shear stresses are given in Table 12.6.

The shear stress at the section is computed from

$$v = \frac{VQ}{I} \tag{12.3}$$

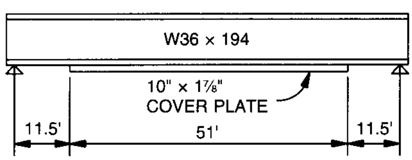


FIGURE 12.6 Elevation view of stringer.

TABLE 12.5 Stresses in Composite Steel Beam without Cover Plate

(*a*) At theoretical cutoff point, 13 ft from supports		
	Bending moments, ft-kips	
M_{DL}	M_{SDL}	$M_{LL} + M_I$
466	155	744 (Fig. 12.7)
	Stresses at bottom of steel (tension), ksi	
DL: $f_b = 466 \times 12/665 =$	8.41 (S_{sb} for W36 × 194)	
SDL: $f_b = 155 \times 12/871 =$	2.14 (S_{sb} from Table 12.4*a*)	
LL + *I*: $f_b = 744 \times 12/987 =$	9.04 (S_{sb} from Table 12.4*b*)	
Total:	19.59 < 20	

(*b*) At cover-plate terminal, 11.5 ft from support		
	Bending moments, ft-kips	
M_{DL}	M_{SDL}	$M_{LL} + M_I$
422	140	677 (Fig. 12.8)
	Stresses at bottom of steel (tension), ksi	
DL: $f_b = 422 \times 12/665 =$	7.62 (S_{sb} for W36 × 194)	
SDL: $f_b = 140 \times 12/871 =$	1.93 (S_{sb} from Table 12.4*a*)	
LL + *I*: $f_b = 677 \times 12/987 =$	8.23 (S_{sb} from Table 12.4*b*)	
Total:	17.78	

where v = horizontal shear stress, kips per in
V = vertical shear on cross section, kips
Q = statical moment about neutral axis of area of cross section on one side of axis and not included between neutral axis and horizontal line through given point, in^3
I = moment of inertia, in^4, of cross section about neutral axis

AASHTO specifications permit a stress $F_v = 0.27F_u = 15.7$ ksi in fillet welds when the base metal is Grade 36 steel. The minimum size of fillet weld permitted with the 1⅞-in-thick cover plate is 5⁄16 in. If a 5⁄16-in weld is used on opposite sides of the plate, the two welds would be allowed to resist a shear stress of

$$v_a = 2 \times 0.313 \times 0.707 \times 15.7 = 6.9 > 1.23 \text{ kips per in}$$

Therefore, use 5⁄16-in welds.

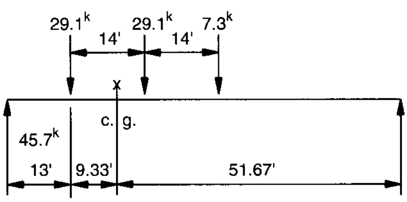


FIGURE 12.7 Position of truck load for maximum moment 13 ft from the support.

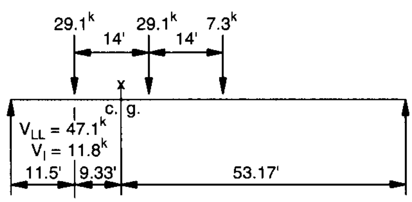


FIGURE 12.8 Position of truck load for maximum moment 11.5 ft from the support.

TABLE 12.6 Shear Stress 11.5 ft from Support

	Shear, kips		
	V_{DL}	V_{SDL}	$V_{LL} + V_I$
	30.0	9.9	58.9
	Shear stress, kips per in		
DL: v = 30.0 × 18.75 × 14.43/17,290 =	0.47 (*I* from Table 12.1)		
SDL: v = 9.9 × 18.75 × 22.51/34,530 =	0.12 (*I* from Table 12.2*a*)		
LL + *I*: v = 58.9 × 18.75 × 30.63/52,580 =	0.64 (*I* from Table 12.2*b*)		
Total:	1.23		

Shear Connectors. To ensure composite action of concrete deck and steel stringer, shear connectors welded to the top flange of the stringer must be embedded in the concrete (Art. 11.16). For this structure, ¾-in dia. welded studs are selected. They are to be installed in groups of three at specified locations to resist the horizontal shear at the top of the steel stringer (Fig. 12.9). With height $h = 6$ in, they satisfy the requirement $h/d \geq 4$, where d = stud diameter, in.

With $f'_c = 4000$ psi for the concrete, the ultimate strength of a ¾-in-dia. welded stud is

$$S_u = 0.4d^2\sqrt{f'_c E_c} = 0.4(0.75)^2\sqrt{4{,}000 \times 3{,}600{,}000} = 27 \text{ kips}$$

This value is needed for determining the number of shear connectors required to develop the strength of the steel stringer or the concrete slab, whichever is smaller. At midspan, the strength of the rolled beam and cover plate, with area $A_s = 75.75$ in² from Table 12.1, is

$$P_1 = A_s F_y = 75.75 \times 36 = 2{,}727 \text{ kips}$$

The compressive strength of the concrete slab is

$$P_2 = 0.85 f'_c bt = 0.85 \times 4.0 \times 96 \times 8.5 = 2{,}774 > 2{,}727 \text{ kips}$$

Steel strength governs. Hence, the number of studs provided between midspan and each support must be at least

$$N_1 = \frac{P_1}{\phi S_u} = \frac{2{,}727}{0.85 \times 27} = 119$$

With the studs placed in groups of three, therefore, there should be at least 40 groups on each half of the stringer.

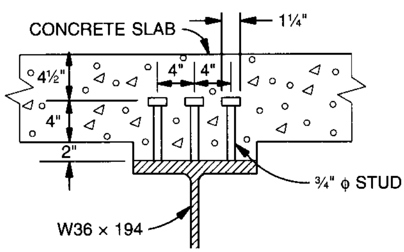


FIGURE 12.9 Welded studs on beam flange.

Between the end of the cover plate and the support, the strength of the rolled beam alone, with $A_s = 57.0$, is

$$P_1 = A_s F_y = 57.0 \times 36 = 2{,}052 < 2{,}727 \text{ kips}$$

Steel strength still governs.

Pitch is determined by fatigue requirements. The allowable load range, kips per stud, may be computed from

$$Z_r = ad^2 \tag{12.4}$$

With $\alpha = 10.6$ for 500,000 cycles of load (AASHTO specifications),

$$Z_r = 10.6(0.75)^2 = 5.97 \text{ kips per stud}$$

At the supports, the shear range $V_r = 71.4$ kips, the shear produced by live load plus impact. Consequently, with $n = 8$ for the concrete, and the transformed concrete area equal to 102 in^2 and $I = 32{,}980$ in^4 from Table 12.4*b*, the range of horizontal shear stress is

$$S_r = \frac{V_r Q}{I} = \frac{71.4 \times 102.0 \times 8.42}{32{,}980} = 1.859 \text{ kips per in}$$

Hence, the pitch required for stud groups near the supports is

$$p = \frac{3Z_r}{S_r} = \frac{3 \times 5.97}{1.859} = 9.63 \text{ in}$$

At 5 ft from the supports, the shear range $V_r = 66.1$ kips, produced by live load plus impact. Since the cross section is the same as at the support, the pitch required for the studs is

$$p = \frac{9.63 \times 71.4}{66.1} = 10.40 \text{ in}$$

At 25 ft from the supports, $V_r = 46.1$ kips (Fig. 12.10). With $I = 52{,}580$ in^4 from Table 12.2*b*, the range of horizontal shear stress is

$$S_r = \frac{V_r Q}{I} = \frac{46.1 \times 102.0 \times 12.04}{52{,}580} = 1.077 \text{ kips per in}$$

Hence, the pitch required is

$$p = \frac{3 \times 5.97}{1.077} = 16.6 \text{ in}$$

The shear-connector spacing selected to meet the preceding requirements is shown in Fig. 12.11.

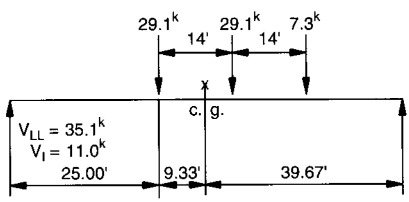


FIGURE 12.10 Position of loads for maximum shear 25 ft from the support.

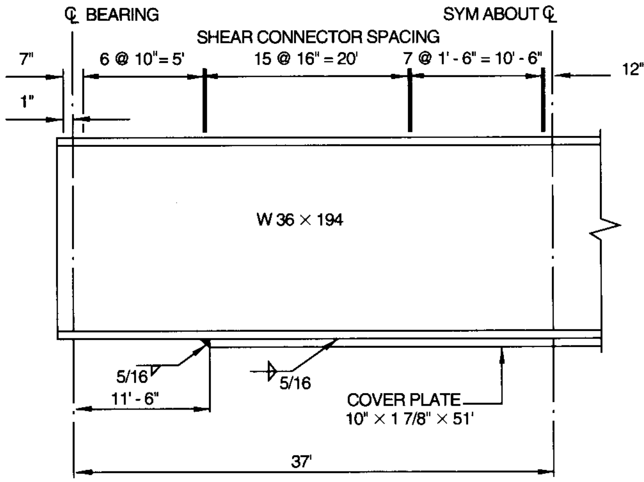


FIGURE 12.11 Shear connector spacing along the top flange of a stringer.

Deflections. Dead-load deflections may be needed so that concrete for the deck may be finished to specified elevations. Cambering of rolled beams to offset dead-load deflections usually is undesirable because of the cost. The beams may, however, be delivered from the mill with a slight mill camber. If so, advantage should be taken of this, by fabricating and erecting the stringers with the camber upward.

The dead-load deflection has two components, one corresponding to *DL* and one to *SDL*. For computation for *DL*, the moment of inertia *I* of the steel section alone should be used. For *SDL, I* should apply to the composite section with $n = 24$ (Table 12.2*a*). Both components can be computed from

$$\delta = \frac{22.5wL^4}{EI} \tag{12.5}$$

where w = uniform load, kips per ft
L = span, ft
E = modulus of elasticity of steel, ksi
I = moment of inertia of section about neutral axis

For *DL,* $w = 1.175$ kips per ft, and for *SDL,* $w = 0.390$ kip per ft.

Dead-Load Deflection

$$DL\text{: } \delta = 22.5 \times 1.175(74)^4/(29{,}000 \times 17{,}290) = 1.60 \text{ in}$$

$$SDL\text{: } \delta = 22.5 \times 0.390(74)^4/(29{,}000 \times 34{,}530) = \underline{0.27}$$

$$\text{Total: } \qquad 1.87 \text{ in}$$

Maximum live-load deflection should be checked and compared with $12L/800$. If desired, this deflection can be calculated accurately by the methods given in Sec. 3, including the effects of changes in moments of inertia. Or the midspan deflection of a simply supported stringer under AASHTO HS truck loading may be obtained with acceptable accuracy from the approximate formula:

$$\delta = \frac{324}{EI} P_T(L^3 - 555L + 4780) \tag{12.6}$$

where P_T = weight, kips, of one front truck wheel multiplied by the live-load distribution factor, plus impact, kips. In this case, $P_T = 10 \times 0.727 + 0.251 \times 10 \times 0.727 = 9.1$ kips. From Table 11.2*b*, for $n = 8$, $I = 52{,}580$. Hence,

$$\delta = \frac{324 \times 9.1}{29{,}000 \times 52{,}580}(74^3 - 555 \times 74 + 4{,}780) = 0.70 \text{ in}$$

And the deflection-span ratio is

$$\frac{0.70}{74 \times 12} = \frac{1}{1{,}200} < \frac{1}{800}$$

Thus, the live-load deflection is acceptable.

12.3 CHARACTERISTICS OF PLATE-GIRDER STRINGER BRIDGES

For simple or continuous spans exceeding about 85 ft, plate girders may be the most economical type of construction. Used as stringers instead of rolled beams, they may be economical even for long spans (350 ft or more). Design of such bridges closely resembles that for bridges with rolled-beam stringers (Arts. 12.1 and 12.2). Important exceptions are noted in this and following articles.

The decision whether to use plate girders often hinges on local fabrication costs and limitations imposed on the depth of the bridge. For shorter spans, unrestricted depth favors plate girders over rolled beams. For long spans, unrestricted depth favors deck trusses or arches. But even then, cable-supported girders may be competitive in cost. Stringent depth restrictions, however, favor through trusses or arches.

Composite construction significantly improves the economy and performance of plate girders and should be used wherever feasible. (See also Art. 12.1.) Advantage also should be taken of continuity wherever possible, for the same reasons.

Spacing. For stringer bridges with spans up to about 175 ft, two lanes may be economically carried on four girders. Where there are more than two lanes, five or more girders should be used at spacings of 7 ft or more. With increase in span, economy improves with wider girder spacing, because of the increase in load-carrying capacity with increase in depth for the same total girder area.

For stringer bridges with spans exceeding 175 ft, girders should be spaced about 14 ft apart. Consequently, this type of construction is more advantageous where roadway widths exceed about 40 ft. For two-lane bridges in this span range, box girders may be less costly.

Steel Grades. In spans under about 100 ft, Grade 36 steel often will be more economical than higher-strength steels. For longer spans, however, designers should consider use of stronger steels, because some offer maintenance benefits as well as a favorable strength-cost ratio. But in small quantities, these steels may be expensive or unavailable. So where only a few girders are required, it may be uneconomical to use a high-strength steel for a light flange plate extending only part of the length of a girder.

In spans between 100 and 175 ft, hybrid girders, with stronger steels in the flanges than in the web (Art. 11.19), often will be more economical than girders completely of Grade 36 steel. For longer spans, economy usually is improved by making the web of higher-strength steels than Grade 36. In such cases, the cost of a thin web with stiffeners should be compared with that of a thicker web with fewer stiffeners and thus lower fabrication costs. Though high-strength steels may be used in flanges and web, other components, such as stiffeners, bracing, and connection details, should be of Grade 36 steel, because size is not determined by strength.

Haunches. In continuous spans, bending moments over interior supports are considerably larger than maximum positive bending moments. Hence, theoretically, it is advantageous to make continuous girders deeper at interior supports than at midspan. This usually is done by providing a haunch, usually a deepening of the girders along a pleasing curve in the vicinity of those supports.

For spans under about 175 ft, however, girders with straight soffits may be less costly than with haunches. The expense of fabricating the haunches may more than offset savings in steel obtained with greater depth. With long spans, the cost of haunching may be further increased by the necessity of providing horizontal splices, which may not be needed with straight soffits. So before specifying a haunch, designers should make cost estimates to determine whether its use will reduce costs.

Web. In spans up to about 100 ft, designers may have the option of specifying a web with stiffeners or a thicker web without stiffeners. For example, a $\frac{5}{16}$-in-thick stiffened plate or a $\frac{7}{16}$-in-thick unstiffened plate often will satisfy shear and buckling requirements in that span range. A girder with the thinner web, however, may cost more than with the thicker web, because fabrication costs may more than offset savings in steel. But if the unstiffened plate had to be thicker than $\frac{7}{16}$ in, the girder with stiffeners probably would cost less.

For spans over 100 ft, transverse stiffeners are necessary. Longitudinal stiffeners, with the thinner webs they permit, may be economical for Grade 36 as well as for high-strength steels.

Flanges. In composite construction, plate girders offer greater flexibility than rolled beams, and thus can yield considerable savings in steel. Flange sizes of plate girders, for example, can be more closely adjusted to variations in bending stress along the span. Also, the grade of steel used in the flanges can be changed to improve economy. Furthermore, changes may be made where stresses theoretically permit a weaker flange, whereas with cover-plated rolled beams, the cover plate must be extended beyond the theoretical cutoff location.

Adjoining flange plates are spliced with a groove weld. It is capable of developing the full strength of the weaker plate when a gradual transition is provided between groove-welded material of different width or thickness. AASHTO specifies transition details that must be followed.

Designers should avoid making an excessive number of changes in sizes and grades of flange material. Although steel weight may be reduced to a minimum in that manner, fabrication costs may more than offset the savings in steel.

For simply supported, composite girders in spans under 100 ft, it may be uneconomical to make changes in the top flange. For spans between 100 and 175 ft, a single reduction in thickness of the top flange on either side of midspan may be economical. Over 175 ft, a reduction in width as well as thickness may prove worthwhile. More frequent changes are economically justified in the bottom flange, however, because it is more sensitive to stress changes along the span. In simply supported spans up to about 175 ft, the bottom flange may consist of three plates of two sizes—a center plate extending over about the middle 60% of the span and two thinner plates extending to the supports. (See Art. 11.17).

Note that even though high-strength steels may be specified for the bottom flange of a composite girder, the steel in the top flange need not be of higher strength than that in the web. In a continuous girder, however, if the section is not composite in negative-moment regions, the section should be symmetrical about the neutral axis.

In continuous spans, sizes of top and bottom flanges may be changed economically once or twice in a negative-moment region, depending on whether only thickness need be changed or both width and thickness have to be decreased. Some designers prefer to decrease thickness first and then narrow the flange at another location. But a constant-width flange should be used between flange splices. In positive-moment regions, the flanges may be treated in the same way as flanges of simply supported spans.

Welding of stiffeners or other attachments to a tension flange usually should be avoided. Transverse stiffeners used as cross-frame connections, should be connected to both girder flanges (Art. 11.12.6). The flange stress should not exceed the allowable fatigue stress for base metal adjacent to or connected by fillet welds. Stiffeners, however, should be welded to the compression flange. Though not required for structural reasons, these welded connections increase lateral rigidity of a girder, which is a desirable property for transportation and erection.

Bracing. Intermediate cross frames usually are placed in all bays and at intervals as close to 25 ft as practical, but no farther apart than 25 ft. Consisting of minimum-size angles, these frames provide a horizontal angle near the bottom flange and V bracing (Fig. 12.12) or X bracing. The angles usually are field-bolted to connection plates welded to each girder web. Eliminating gusset plates and bolting directly to stiffeners is often economical.

Cross frames also are required at supports. Those at interior supports of continuous girders usually are about the same as the intermediate cross frames. At end supports, however, provision must be made to support the end of the concrete deck. For the purpose, a horizontal channel of minimum weight, consistent with concrete edge-beam requirements, often is used near the top flange, with V or X bracing, and a horizontal angle near the bottom flange.

Lateral bracing in a horizontal plane near the bottom flange is sometimes required. The need for such bracing must be investigated, based on a wind pressure of 50 psf. (Spans with nonrigid decks may also require a top lateral system.) This bracing usually consists of crossing diagonal angles and the bottom angles of the cross frames.

Bearings. Laminated elastomeric pads may be used economically as bearings for girder spans up to about 175 ft. Welded steel rockers or rollers are an alternative for all spans but may not meet seismic requirements. Seismic attenuation bearings, pot bearings, or spherical bearings with teflon guided surfaces for expansion are other alternatives.

Camber. Plate girders should be cambered to compensate for dead-load deflections. When the roadway is on a grade, the camber should be adjusted so that the girder flanges will parallel the profile grade line. For the purpose, designers should calculate dead-load deflec-

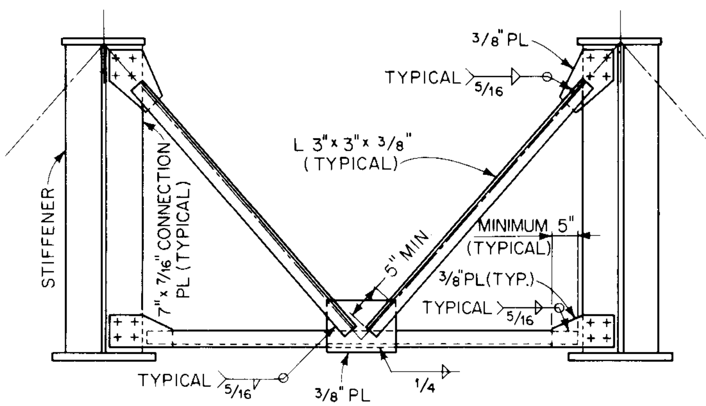


FIGURE 12.12 Intermediate cross frame for a stringer bridge.

tions at sufficient points along each span to indicate to the fabricator the desired shape for the unloaded stringer.

12.4 EXAMPLE—ALLOWABLE-STRESS DESIGN OF COMPOSITE, PLATE-GIRDER BRIDGE

To illustrate the design procedure, a two-lane highway bridge with simply supported, composite, plate-girder stringers will be designed. As indicated in the framing plan in Fig. 12.13*a*, the stringers span 100 ft c to c of bearings. The typical cross section in Fig. 12.13*b* shows a 26-ft roadway flanked by 1-ft 9-in-wide barrier curbs. Structural steel to be used is Grade 36. Loading is HS25. Appropriate design criteria given in Sec. 11 will be used for this

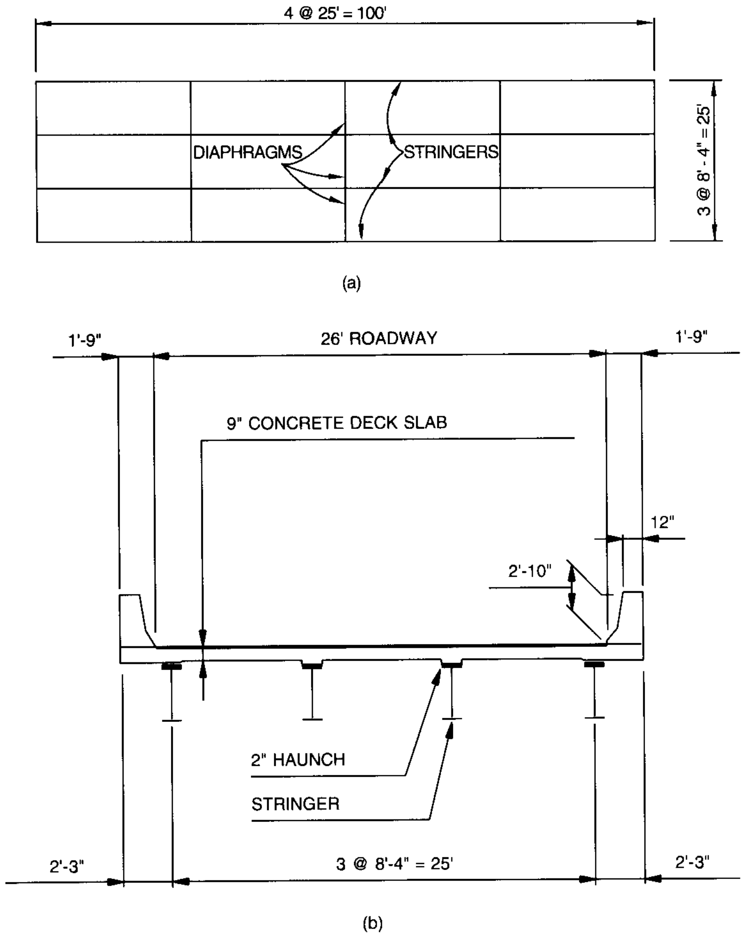


FIGURE 12.13 Two-lane highway bridge with plate-girder stringers. (*a*) Framing plan. (*b*) Typical cross section.

structure. Concrete to be used for the deck is class A, with 28-day strength $f'_c = 4{,}000$ psi and allowable compressive stress $0.4f'_c = 1600$ psi. Modulus of elasticity $E_c = 3{,}600{,}000$ psi.

Assume that the deck will be supported on four plate-girder stringers, spaced 8 ft 4 in c to c, as indicated in Fig. 12.13.

Concrete Slab. The slab is designed, to span transversely between stringers, in the same way as for rolled-beam stringers (Art. 12.2). A 9-in-thick one-course, concrete slab will be used with the plate-girder stringers.

Stringer Design Procedure. The general design procedure outlined in Art. 12.2 for rolled beams also holds for plate girders. In this example, too, only a typical interior stringer will be designed.

Loads, Moments, and Shears. Assume that the girders will not be shored during casting of the concrete slab. Hence, the dead load on each steel stringer includes the weight of an 8.33-ft-wide strip of slab as well as the weights of steel girder and framing details. This dead load will be referred to as *DL*.

DEAD LOAD CARRIED BY STEEL BEAM, KIPS PER FT

Slab: $0.150 \times 8.33 \times 9/12$	= 0.938
Haunch—16 × 2 in: $0.150 \times 1.33 \times 0.167$	= 0.034
Steel stringer and framing details—assume:	0.327
Stay-in-place forms and additional concrete in forms:	0.091
DL per stringer:	1.390

Maximum moment occurs at the center of the 100-ft span and equals

$$M_{DL} = \frac{1.39(100)^2}{8} = 1{,}738 \text{ ft-kips}$$

Maximum shear occurs at the supports and equals

$$V_{DL} = \frac{1.39 \times 100}{2} = 69.5 \text{ kips}$$

Barrier curbs will be placed after the concrete slab has cured. Their weights may be equally distributed to all stringers. In addition, provision will be made for a future wearing surface, weight 25 psf. The total superimposed dead load will be designated *SDL*.

DEAD LOAD CARRIED BY COMPOSITE SECTION, KIPS PER FT

Two barrier curbs: $2 \times 0.530/4$	= 0.265
Future wearing surface: 0.025×8.33	= 0.208
SDL per stringer:	0.473

Maximum moment occurs at midspan and equals

$$M_{SDL} = \frac{0.473(100)^2}{8} = 592 \text{ ft-kips}$$

Maximum shear occurs at supports and equals

$$V_{SDL} = \frac{0.473 \times 100}{2} = 23.7 \text{ kips}$$

The HS25 live load imposed may be a truck load or lane load. But for this span, the truck load shown in Fig. 12.14*a* governs. The center of gravity of the three axles lies between the two heavier loads and is 4.66 ft from the center load. Maximum moment occurs under the center-axle load when its distance from midspan is the same as the distance of the center of gravity of the loads from midspan, or $4.66/2 = 2.33$ ft. Thus, the center load should be placed $100/2 - 2.33 = 47.67$ ft from a support (Fig. 12.14*a*). Then, the maximum moment is

$$M_T = \frac{90(100/2 + 2.33)^2}{100} - 40 \times 14 = 1{,}905 \text{ ft-kips}$$

The distribution of the live load to a stringer may be obtained from Table 11.14, for a bridge with two traffic lanes.

$$\frac{S}{5.5} = \frac{8.33}{5.5} = 1.516 \text{ wheels} = 0.758 \text{ axle}$$

Hence, the maximum live-load movement is

$$M_{LL} = 0.758 \times 1{,}905 = 1{,}444 \text{ ft-kips}$$

While this moment does not occur at midspan as do the maximum dead-load moments, stresses due to M_{LL} may be combined with those from M_{DL} and M_{SDL} to produce the maximum stress, for all practical purposes.

For maximum shear with the truck load, the outer-40-kip load should be placed at the support (Fig. 12.14*b*). Then, the shear is

$$V_T = \frac{90(100 - 14 + 4.66)}{100} = 81.6 \text{ kips}$$

Since the stringer receives 0.758 axle load, the maximum shear on the stringer is

$$V_{LL} = 0.758 \times 81.6 = 61.9 \text{ kips}$$

Impact is taken as the following fraction of live-load stress:

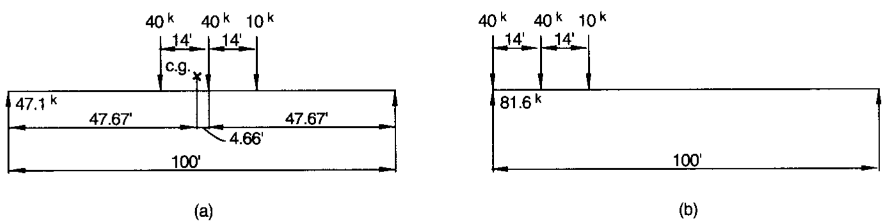


FIGURE 12.14 Positions of loads on a plate girder for maximum stress. (*a*) For maximum moment in the span. (*b*) For maximum shear in the span.

$$I = \frac{50}{L + 125} = \frac{50}{100 + 125} = 0.222$$

Hence, the maximum moment due to impact is

$$M_I = 0.222 \times 1{,}444 = 321 \text{ ft-kips}$$

and the maximum shear due to impact is

$$V_I = 0.222 \times 61.9 = 13.8 \text{ kips}$$

MIDSPAN BENDING MOMENTS, FT-KIPS

M_{DL}	M_{SDL}	$M_{LL} + M_I$
1,738	592	1,765

END SHEAR, KIPS

V_{DL}	V_{SDL}	$V_{LL} + V_I$	Total V
69.5	23.7	75.7	168.9

Properties of Composite Section. The 9-in thick roadway slab includes an allowance of 0.5 in for a wearing surface. Hence, the effective thickness of the concrete slab for composite action is 8.5 in.

The effective width of the slab as part of the top flange of the T beam is the smaller of the following:

¼ span = ¼ × 100 × 12 = 300 in
Stringer spacing, c to c = 8.33 × 12 = 100 in
12 × slab thickness = 12 × 8.5 = 102 in

Hence, the effective width is 100 in (Fig. 12.15).

To complete the T beam, a trial section must be selected for the plate girder. As a guide in doing this, formulas for estimating required flange area given in J. C. Hacker, "A Simplified Design of Composite Bridge Structures," *Journal of the Structural Division, ASCE, Proceedings Paper* 1432, November, 1957, may be used. To start, assume that the girder web will be 60 in deep. This satisfies the requirements that the depth-span ratio for girder plus slab exceed 1⁄25 and for girder alone, 1⁄30. With stiffeners, the web thickness is required to be at least 1⁄165 of the depth, or 0.364 in.

Use a web plate 60 × 7⁄16 in. With a cross-sectional area $A_w = 26.25$ in², the web will be subjected to maximum shearing stress considerably below the 12 ksi permitted.

$$f_v = \frac{168.9}{26.25} = 6.4 \text{ ksi} < 12$$

The required bottom-flange area may be estimated from Eq. (12.1*a*) with allowable bending stress $F_b = 20$ ksi and distance between centers of gravity of steel flanges taken as $d_{cg} = 63$ in.

$$A_{sb} = \frac{12}{20}\left(\frac{1{,}738}{63} + \frac{2{,}357}{63 + 9}\right) = 36.2 \text{ in}^2$$

Try a 20 × 1¾-in bottom flange, area = 35 in².

The ratio of flange areas $R = A_{st}/A_{sb}$ may be estimated from Eq. (12.1*b*) as

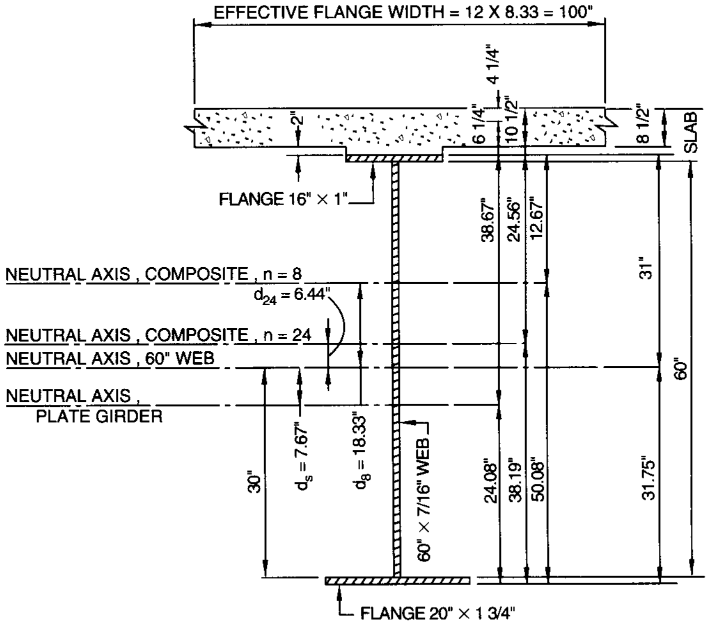


FIGURE 12.15 Cross section of composite plate girder at midspan.

$$R = \frac{50}{190 - L} = \frac{50}{190 - 100} = 0.55$$

Then, the estimated required area of the steel top flange is

$$A_{st} = RA_{sb} = 0.55 \times 35 = 19.3 \text{ in}^2$$

If the flange will be fully stressed in compression, the maximum permissible width-thickness ratio for the flange plate is 23 for Grade 36 steel. On the assumption of a 16-in-wide plate, the minimum thickness permitted is 16/23, or about ¾ in. Try a 16 × 1-in top flange, area = 16 in^2.

The trial section is shown in Fig. 12.15. Its neutral axis can be located by taking moments of web and flange areas about middepth of the web. This computation and that for the section moduli S_{st} and S_{sb} of the plate girder alone are conveniently tabulated in Table 12.7.

In computation of the properties of the composite section, the concrete slab, ignoring the haunch area, is transformed into an equivalent steel area. For the purpose, for this bridge, the concrete area is divided by the modular ratio $n = 8$ for short-time loading, such as live loads and impact. For long-time loading, such as superimposed dead loads, the divisor is $3n = 24$, to account for the effects of creep. The computations of neutral-axis location and section moduli for the composite section are tabulated in Table 12.8. To locate the neutral axis, moments are taken about middepth of the girder web.

Stresses in Composite Section. Since the girders will not be shored when the concrete is cast and cured, the stresses in the steel section for load *DL* are determined with the section moduli of the steel section alone (Table 12.7). Stresses for load *SDL* are computed with section moduli of the composite section when $n = 24$ from Table 12.8*a*, and stresses in the

TABLE 12.7 Steel Section for Maximum Moment

Material	A	d	Ad	Ad^2	I_o	I
Top flange 16 × 1	16.0	30.50	488	14,880		14,880
Web 60 × 7⁄16	26.3				7,880	7,880
Bottom flange 20 × 1¾	35.0	−30.88	−1,081	33,380		33,380
	77.3		−593			56,140
$d_s = -593/77.3 = -7.67$ in					$-7.67 \times 593 =$	−4,550
					$I_{NA} =$	51,590

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 30 + 1 + 7.67 = 38.67 \text{ in}$$

$$\text{Bottom of steel} = 30 + 1.75 - 7.67 = 24.08 \text{ in}$$

Section moduli	
Top of steel	Bottom of steel
$S_{st} = 51{,}590/38.67 = 1{,}334 \text{ in}^3$	$S_{sb} = 51{,}590/24.08 = 2{,}142 \text{ in}^3$

steel for live loads and impact are calculated with section moduli of the composite section when $n = 8$ from Table 12.8*b* (Table 12.9*a*).

The width-thickness ratio of the compression flange now can be checked by the general formula applicable for any stress level:

$$\frac{b}{t} = \frac{103}{\sqrt{f_b}} = \frac{103}{\sqrt{19.24}} = 23.5 < \frac{24}{1}$$

Hence, the trial steel section is satisfactory.

Stresses in the concrete are determined with the section modulus of the composite section with $n = 24$ for *SDL* from Table 12.8*a* and $n = 8$ for $LL + I$ from Table 12.8*b* (Table 12.9*b*). Therefore, the composite section is satisfactory. Use the section shown in Fig. 12.15 in the region of maximum moment.

Changes in Flange Sizes. One change in size of each flange will be made on both sides of midspan. (Though steel weight can be cut by reducing the area of the bottom flange in two steps, say from 1¾ to 1¼ in and then to ¾ in, higher fabrication costs probably would offset the savings in steel costs.) Each flange will maintain its width throughout the span, but thickness will be reduced at locations to be determined.

For the top flange near the supports, assume a plate 16 × ¾ in, area = 12 in². Its width-thickness ratio is 16/¾ = 21.3 < 23 and therefore is satisfactory. The cross section of the girder after the reduction in size of the top flange is shown in Fig. 12.16. The neutral axes of the steel section and of the composite section are located by taking moments of the areas about middepth of the girder web. These computations and those for the section moduli are given in Tables 12.10 and 12.11.

The location of the transition from the thicker plate to the thinner one can be estimated from Eq. (12.7), which gives the approximate length L_p, ft, of the thicker plate on the assumption of a parabolic bending-moment diagram.

TABLE 12.8 Composite Section for Maximum Moment

(*a*) For dead loads, $n = 24$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	77.3		−593			56,140
Concrete $100 \times 8.5/24$	35.4	37.25	1,319	49,120	210	49,330
	112.7		726			105,470
$d_{24} = 726/112.7 = 6.44$ in					$-6.44 \times 726 =$	−4,680
					$I_{NA} =$	100,790

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 31.00 - 6.44 = 24.56 \text{ in}$$

$$\text{Bottom of steel} = 31.75 + 6.44 = 38.19 \text{ in}$$

$$\text{Top of concrete} = 24.56 + 2 + 8.5 = 35.06 \text{ in}$$

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 100{,}790/24.56 = 4{,}104 \text{ in}^3$	$S_{sb} = 100{,}790/38.19 = 2{,}639 \text{ in}^3$	$S_c = 100{,}790/35.06 = 2{,}875 \text{ in}^3$

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	77.3		−593			56,140
Concrete $100 \times 8.5/8$	106.3	37.25	3,958	147,500	640	148,140
	183.6		3,365			204,280
$d_{10} = 3{,}365/183.6 = 18.33$ in					$-18.33 \times 3{,}365 =$	−61,680
					$I_{NA} =$	142,600

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 31.00 - 18.33 = 12.67 \text{ in}$$

$$\text{Bottom of steel} = 31.75 + 18.33 = 50.08 \text{ in}$$

$$\text{Top of concrete} = 12.67 + 2 + 8.5 = 23.17 \text{ in}$$

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 142{,}600/12.67 = 11{,}254 \text{ in}^3$	$S_{sb} = 142{,}600/50.08 = 2{,}847 \text{ in}^3$	$S_c = 142{,}600/23.17 = 6{,}154 \text{ in}^3$

TABLE 12.9 Stresses, ksi, in Composite Plate Girder at Section of Maximum Moment

(*a*) Steel stresses	
Top of steel (compression)	Bottom of steel (tension)
DL: $f_b = 1{,}738 \times 12/1{,}334 = 15.63$	$f_b = 1{,}738 \times 12/2{,}142 = 9.74$
SDL: $f_b = 592 \times 12/4{,}104 = 1.73$	$f_b = 592 \times 12/2{,}639 = 2.69$
LL + *I*: $f_b = 1{,}765 \times 12/11{,}254 = \underline{1.88}$	$f_b = 1{,}765 \times 12/2{,}847 = \underline{7.44}$
Total: $19.24 < 20$	$19.87 < 20$

(*b*) Stresses at top of concrete
SDL: $f_c = 592 \times 12/(2{,}875 \times 8) = 0.31$
LL + *I*: $f_c = 1{,}765 \times 12/(6{,}154 \times 8) = \underline{0.43}$
$0.74 < 1.6$

$$L_p = L\sqrt{1 - \frac{S'_{st}}{S_{st}}} \tag{12.7}$$

where L = span, ft
S'_{st} = section modulus with respect to top of steel girder, with lighter flange, in^3

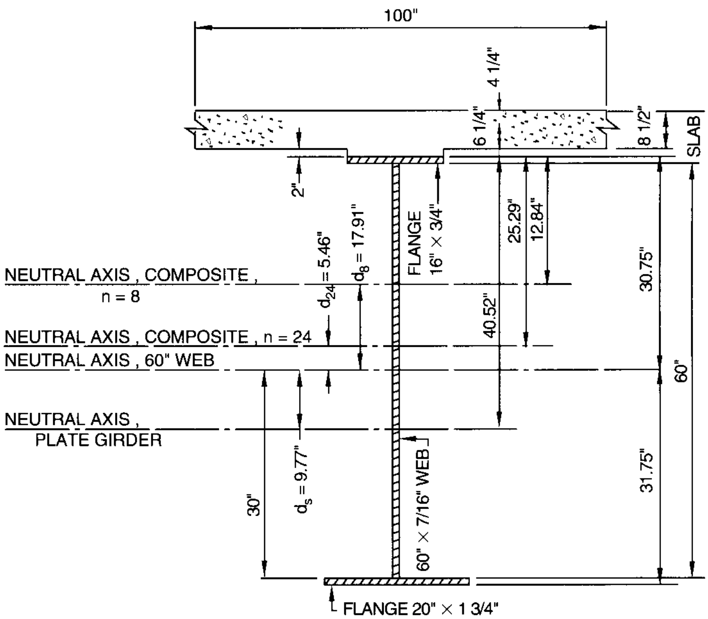


FIGURE 12.16 Cross section of composite plate girder about 30 ft from the supports.

TABLE 12.10 Steel Section about 30 ft from Supports

Material	A	d	Ad	Ad^2	I_o	I
Top flange 16 × ¾	12.0	30.38	365	11,070		11,070
Web 60 × 7⁄16	26.3				7,880	7,880
Bottom flange 20 × 1¾	35.0	−30.88	−1,081	33,380		33,380
	73.3		−716			52,330
$d_s = -716/73.3 = -9.77$ in					$-9.77 \times 716 =$	−7,000
					$I_{NA} =$	45,330

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 30 + 0.75 + 9.77 = 40.52 \text{ in}$$

Section modulus, top of steel

$$S_{st} = 45{,}330/40.52 = 1{,}119 \text{ in}^3$$

S_{st} = section modulus with respect to top of steel girder, with heavier flange, in^3

From Table 12.10, $S'_{st} = 1119$, and from Table 12.7, $S_{st} = 1334$. Hence.

$$L_p = 100 \sqrt{1 - \frac{1{,}119}{1{,}334}} = 40.1 \text{ ft}$$

Assume a 40-ft length for the 1-in top flange. It would then terminate 30 ft from the supports. Stresses should be checked at that location to ensure that they are within the allowable (Table 12.12).

Fatigue seldom governs at flange transitions in simply supported spans, but it should be checked for the final design.

For the bottom flange near the supports, assume a plate 20 × ⅞ in, area = 17.5 in^2. The cross section of the girder after this reduction in size of the bottom flange is shown in Fig. 12.18. The neutral axes of the steel section and of the composite section are located by taking moments of the areas about middepth of the girder web. These computations and those for the section moduli are given in Tables 12.13 and 12.14.

The approximate location of the transition from the thicker bottom plate to the thinner one can be determined from Eq. (12.2), by setting the length L_p, ft, of the thicker plate equal to L_{cp}. From Table 12.13, $S'_{sb} = 1244$, and from Table 12.7, $S_{sb} = 2142$. Hence,

$$L_p = 100 \sqrt{1 - \frac{1{,}244}{2{,}142}} = 64.7 \text{ ft}$$

Assume a 66-ft length for the 1½-in bottom flange. It will then terminate 17 ft from the supports. Stresses are checked at that point to ensure that they are within the allowable (Table 12.15). Since the stresses are within the allowable, the bottom flange can be reduced in thickness to ⅞ in 17 ft from the supports.

Flange-to-Web Welds. Fillet welds placed on opposite sides of the girder web to connect it to each flange must resist the horizontal shear between flange and web. The minimum size of weld permissible for the thickest plate at the connection usually determines the size of

TABLE 12.11 Composite Section about 30 ft from Supports

(*a*) For dead loads, $n = 24$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	73.3		−716			52,330
Concrete 100 × 8½/24	35.4	37.0	1,310	48,470	210	48,680
	108.7		594			101,010
$d_{24} = 594/108.7 = 5.46$ in					$-5.46 \times 594 =$	−3,240
					$I_{NA} =$	97,770

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 30.75 - 5.46 = 25.29 \text{ in}$$

Section modulus, top of steel

$$S_{st} = 97{,}770/25.29 = 3{,}866 \text{ in}^3$$

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	73.3		−716			52,330
Concrete 100 × 8½/8	106.3	37.0	3,933	145,520	640	146,160
	179.6		3,217			198,490
$d_8 = 3217/179.6 = 17.91$ in					$-17.91 \times 3{,}217 =$	−57,620
					$I_{NA} =$	140,870

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 30.75 - 17.91 = 12.84 \text{ in}$$

Section modulus, top of steel

$$S_{st} = 140{,}870/12.84 = 10{,}971 \text{ in}^3$$

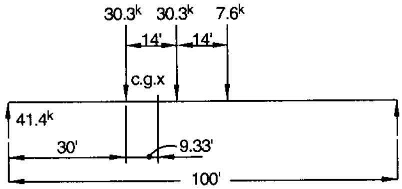


FIGURE 12.17 Positions of loads for maximum moment 30 ft from a support.

TABLE 12.12 Stresses in Composite Plate Girder 30 ft from Supports

Bending moments, ft-kips		
M_{DL}	M_{SDL}	$M_{LL} + M_I$
1,460	498	1,518 (Fig. 12.17)

Stresses at top of steel (compression), ksi		
DL: 1,460 × 12/1,119	=	15.66 (S_{st} from Table 12.10)
SDL: 498 × 12/3,866	=	1.55 (S_{st} from Table 12.11*a*)
LL + *I*: 1,518 × 12/10,971	=	1.66 (S_{st} from Table 12.11*b*)
Total:		18.87 < 20

weld. In some cases, however, the size of weld may be governed by the maximum shear. In this example, shear does not govern, but the calculations are presented to illustrate the procedure.

The maximum shears, which occur at the supports, have been calculated previously but are included in Table 12.16*b*. Moments of inertia may be obtained from Table 12.13 for *DL* and from Table 12.14*b* for *SDL* and *LL* + *I*. Computations for the static moments *Q* of the flange areas are presented in Table 12.16*a*. Then, shear, kips per in, on the two fillet welds can be computed from $v = VQ/I$ (Table 12.16*c*). The allowable stress on the weld is the

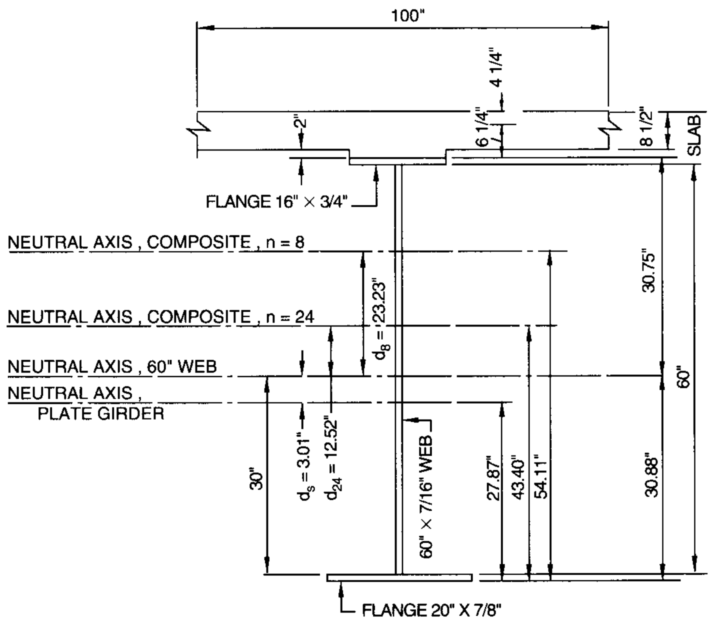


FIGURE 12.18 Cross section of composite plate girder near the supports.

TABLE 12.13 Steel Section Near Supports

Material	A	d	Ad	Ad^2	I_o	I
Top flange 16 × ¾	12.0	30.38	365	11,070		11,070
Web 60 × 7⁄16	26.3				7,880	7,880
Bottom flange 20 × ⅞	17.5	−30.44	−533	16,220		16,220
	55.8		−168			35,170
$d_s = -168/55.8 = 3.01$ in					$-3.01 \times 168 =$	−510
					$I_{NA} =$	34,660

Distance from neutral axis of steel section to:

$$\text{Bottom of steel} = 30 + 0.88 - 3.01 = 27.87 \text{ in}$$

Section modulus, top of steel

$$S_{sb} = 34{,}660/27.87 = 1{,}244 \text{ in}^3$$

smaller of the allowable shear stress, 12.4 ksi, for static loads and the allowable fatigue stress for 500,000 cycles of load. Fatigue does not govern in this example. Hence, the allowable load per weld is 12.4 × 0.707 = 8.76 kips per in. With a weld on each side of the web, the shear per weld is 2,445/2 = 1.223. So the weld size required to resist the shear is 1.223/8.76 = 0.14 in. (See *Flange-to-Web Welds* in Art. 12.9.4.)

The minimum sizes of welds permitted with the flange material are all larger than this. Use two ¼-in fillets with the 16 × ¾-in top flange, and two 5⁄16-in fillets with the other flange plates (Fig. 12.20).

Stiffeners. See Arts. 12.9.4 to 12.9.6.

Bearings. See Arts. 12.9.8 and 12.9.9.

Shear Connectors. See Art. 12.2.

Deflections. See Art. 12.2.

Load-Factor Design. See Art. 12.5.

12.5 EXAMPLE—LOAD-FACTOR DESIGN OF COMPOSITE PLATE-GIRDER BRIDGE

The ‘‘Standard Specifications for Highway Bridges’’ of the American Association for State Highway and Transportation Officials (AASHTO) allow load-factor design as an alternative method to allowable-stress design for design of simple and continuous beam and girder structures of moderate length and it is widely used for highway bridges.

Load-factor design (LFD) is a method of proportioning structural members for multiples of the design loads. The moments, shears, and other forces are determined by assuming elastic behavior of the structure. To ensure serviceability and durability, consideration is given to control of permanent deformations under overloads, to fatigue characteristics under service loadings, and to control of live-load deflections under service loadings. To illustrate

TABLE 12.14 Composite Section Near Supports

(*a*) For dead loads, $n = 34$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	55.8		−168			35,170
Concrete 100 × 8.5/24	35.4	37.0	1,310	48,470	210	48,680
	91.2		1,142			83,850
$d_{24} = 1{,}142/91.2 = 12.52$ in					$-12.52 \times 1{,}142 =$	−14,300
					$I_{NA} =$	69,550

Distance from neutral axis of steel section to:

Bottom of stteel = 30 + 0.88 + 12.52 = 43.40 in

Section modulus, bottom of steel

$$S_{sb} = 69{,}550/43.40 = 1{,}602 \text{ in}^3$$

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	55.8		−168			35,170
Concrete 100 × 8.5/8	106.3	37.0	3,933	145,520	640	146,160
	162.1		3,765			181,330
$d_8 = 3{,}765/162.1 = 23.23$ in					$-23.23 \times 3{,}765 =$	−87,460
					$I_{NA} =$	93,870

Distance from neutral axis of steel section to:

Bottom of steel = 30 + 0.88 + 23.23 = 54.11 in

Section modulus, bottom of steel

$$S_{sb} = 93{,}870/54.11 = 1{,}735 \text{ in}^3$$

load-factor design, a simply supported, composite, plate-girder stringer of the two-lane highway bridge in Art. 12.4 will be designed. The framing plan and the typical cross section are the same as for that bridge (Fig. 12.13). Structural steel is Grade 36, with yield strength $f_y = 36$ ksi and concrete for the deck slab is Class A, with 28-day strength $f'_c = 4{,}000$ psi. Loading is HS25.

12.5.1 Stringer Design Procedure

In the usual design procedure, the concrete deck slab is designed to span between the girders. A section is assumed for the steel stringer and classified as either symmetrical or unsymmetrical, compact or noncompact, braced or unbraced, and transversely or longitudinally stiffened. Section properties of a steel girder alone, and composite section properties of the steel girder and concrete slab are then determined, in a similar way as for allowable-stress design, for long- and short-duration loads. Next, flange local buckling is checked for the composite section. Fatigue stress checks are made for the most common connections found

TABLE 12.15 Stresses in Composite Plate Girder 17 ft from Supports

Bending moments, ft-kips		
M_{DL}	M_{SDL}	$M_{LL} + M_I$
981	335	1,044 (Fig. 12.19)

Stresses at top of steel (compression), ksi

DL: 981 × 12/1,244 = 9.46 (S_{st} from Table 12.13)
SDL: 335 × 12/1,602 = 2.51 (S_{st} from Table 12.14*a*)
$LL + I$: 1,044 × 12/1,735 = 7.22 (S_{st} from Table 12.14*b*)
Total: 19.19 < 20

TABLE 12.16 Shear Stresses in Composite Plate Girder, ksi, at Supports

(*a*) Static moment Q, in^3, of flange

Top flange: Q_t = 12.0 × 33.39 = 401
Bottom flange: Q_b = 17.5 × 27.43 = 480

Composite section, $n = 8$

Steel top flange: Q_{st} = 12.0 × 7.15 = 86
Concrete slab: Q_c = 106.3 × 13.77 = 1,464
Total: Q_t = 1,550
Steel bottom flange: Q_b = 17.5 × 53.67 = 939

(*b*) Maximum shears, kips, at supports

V_{DL}	V_{SDL}	$V_{LL} + V_I$
69.5	23.7	75.7

(*c*) Shear stresses, kips per in

Top-flange welds	Bottom-flange welds
DL: 69.5 × 401/34,660 = 0.804	DL: 69.5 × 480/34,660 = 0.962
SDL: 23.7 × 1,550/93,870 = 0.391	SDL: 23.7 × 939/93,870 = 0.237
$LL + I$: 75.7 × 1,550/93,870 = 1.250	$LL + I$: 75.7 × 939/93,870 = 0.757
v = 2.445	v = 1.956

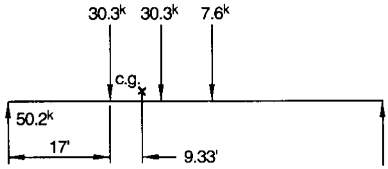


FIGURE 12.19 Positions of loads for maximum moment 17 ft from a support.

in a welded plate girder, such as those for transverse stiffeners, flange plate splices, gusset plates for lateral bracing, and flanges to webs.

The trial section is checked for compactness. The allowable stresses may have to be reduced if the section is noncompact and unbraced. Next, bending strength and shear capacity of the section are checked, and the section is adjusted as necessary. Then, transverse and longitudinal stiffeners are designed, if required. In addition, for a complete design, flange-web welds and shear connectors (fatigue to be included), bearing stiffeners (as concentrically loaded columns), lateral bracing (for wind loading) are designed and a deflection check is made.

12.5.2 Concrete Slab

The slab is designed to span transversely between stringers in the same way as for the allowable-stress method (Art. 12.4). A 9-in-thick, one-course concrete slab is used, as in Art. 12.4. The effective span *S*, the distance, ft, between flange edges plus half the flange width, is, for an assumed flange width of 16 in (1.33 ft),

$$S = 8.33 - 1.33 + 1.33/2 = 7.67 \text{ ft}$$

For computation of dead load,

Weight of concrete slab: $0.150 \times 9/12$	= 0.113
3/8-in extra concrete in stay-in-place forms: $0.150(3/8)/12$	= 0.005
Future wearing surface	= 0.025
Total dead load w_D:	0.143 kips per ft

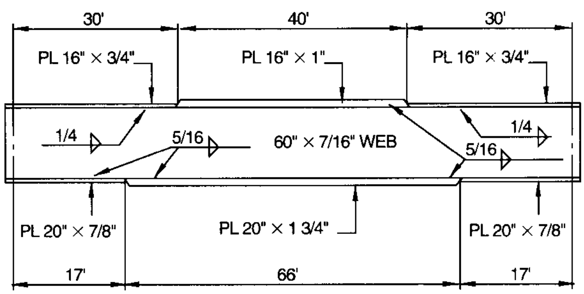


FIGURE 12.20 Plate girder with splices in top and bottom flanges.

With a factor of 0.8 applied to account for continuity of slab over more than three stringers, the maximum dead-load bending moment is

$$M_D = \frac{w_D S^2}{10} = \frac{0.143(7.67)^2}{10} = 0.84 \text{ ft-kips per ft}$$

Maximum live-load moment, with reinforcement perpendicular to traffic, using a factor of 0.8 applied to account for continuity, equals

$$M_L = \frac{(S + 2)}{32} P(0.8) \tag{12.8}$$

where P is the load on one rear wheel of a truck. Since $P = 16 \times 1.25 = 20$ kips for an HS25 truck,

$$M_L = \frac{(7.67 + 2)}{32} 20 \times 0.8 = 4.84 \text{ ft-kips per ft}$$

Allowance for impact is 30% of this, or 1.45 ft-kips per ft. The total live load moment then is

$$M_L = 4.84 + 1.45 = 6.29 \text{ ft-kips per ft}$$

The factored total moment for AASHTO Group I loading on a straight bridge is

$$M_T = 1.3[DL + 1.67(LL + I) = 1.3(0.84 + 1.67 \times 6.29)$$

$$= 14.75 \text{ ft-kips per ft}$$

For a strip of slab $b = 12$ in wide, the effective depth d of the steel reinforcement is determined based on the assumption that No. 6 bars with 2.5 in of concrete cover will be used:

$$d = 9 - 2.5 - (6/8)/2 = 6.13 \text{ in}$$

For determination of the moment capacity of the concrete slab, the depth of the equivalent rectangular compressive-stress block is given by

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{12.9}$$

where A_s = the area, in^2, of the reinforcing steel.

For $f'_c = 4$ ksi and the yield strength of the reinforcing steel $F_y = 60$ ksi,

$$a = \frac{60A_s}{0.85 \times 4 \times 12} = 1.47A_s$$

Design moment strength ϕM_n is given by

$$\phi M_n = \phi A_s f_y (d - a/2) \tag{12.10}$$

where the strength reduction factor $\phi = 0.90$ for flexure. If the nominal moment capacity ϕM_n is equated to the total factored moment M_T, the required area of reinforcement steel A_s can be obtained with Eq. (12.10) by solving a quadratic equation:

$$14.75 \times 12 = 0.9 \times 60A_s(6.13 - 1.47A_s/2)$$

from which $A_s = 0.58$ in^2 per ft. Number 6 bars at 9-in intervals supply 0.59 in^2 per ft and will be specified. The provided area should be checked to ensure that its ratio ρ to the concrete area does not exceed 75% of the balanced reinforcement ratio ρ_b.

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{87}{87 + f_y}\right) \tag{12.11}$$

where the factor $\beta_1 = 0.85$ for $f'_c = 4$-ksi concrete.

$$\rho_b = \frac{0.85 \times 0.85 \times 4}{60}\left(\frac{87}{87 + 60}\right) = 0.0285$$

For the provided steel area,

$$\rho = 0.59/(12 \times 6.13) = 0.008 < (0.75\rho_b = 0.0214)\text{—OK}$$

AASHTO standard specifications state that, at any section of a flexural member where tension reinforcement is required by analysis, the reinforcement provided shall be adequate to develop a moment at least 1.2 times the cracking moment M_u calculated on the basis of modulus of rupture f_r for normal-weight concrete.

$$f_r = 7.5\sqrt{f'_c} \tag{12.12}$$

For $f'_c = 4$ ksi, $f_r = 7.5\sqrt{4000} = 474$ psi. The cracking moment is obtained from

$$M_u = f_r S \tag{12.13}$$

where the section modulus $S = bh^2/6 = 12 \times 8.5^2/6 = 144.5$ in^3. (One-half inch is deducted from the section for the wearing course.)

$$M_u = 474 \times 144.5/12{,}000 = 5.71 \text{ ft-kips per ft}$$

From Eq. (12.9), the depth of the equivalent rectangular stress block is

$$a = \frac{60 \times 0.59}{0.85 \times 4 \times 12} = 0.87 \text{ in}$$

Substitution of the preceding values in Eq. (12.10) yields the moment capacity

$$\phi M_n = 0.90 \times 0.59 \times 60(6.13 - 0.87/2)/12$$

$$= 15.12 > (1.2M_u = 6.85 \text{ ft-kips per ft})$$

Therefore, the minimum reinforcement requirement is satisfied.

For a complete slab design, serviceability requirements in the AASHTO standard specifications for fatigue and distribution of reinforcement in flexural members also need to be satisfied. Only a typical interior stringer will be designed in this example.

12.5.3 Loads, Moment and Shears

As in Art. 12.4, it is assumed that the girders will not be shored during casting of the concrete slab. The factored moments and shears will be obtained from the combination of dead load (DL) plus live load and impact ($LI + I$).

For the AASHTO Group I loading combination, the factored moment is

$$M_f = \gamma[\beta_D M_{DL} + 1.67(M_L + M_I)] \tag{12.14}$$

and the factored shear is

$$V_f = \gamma[B_D V_{DL} + 1.67(V_L + V_I)] \tag{12.15}$$

where γ is the load factor (γ = 1.30 for moment and 1.41 for shear) and β is 1.0. Thus, the unfactored loads M_{DL}, M_{SDL} and V_{DL} in Art. 11.4 are still valid. Then,

$$M_{fT} = 1.30[1.0 \times 1{,}738 + 1.0 \times 592 + 1.67 \times (1{,}444 + 321)] = 6{,}862$$

$$V_{fT} = 1.41[1.0 \times 69.5 + 1.0 \times 23.7 + 1.67 \times (61.9 + 13.8)] = 309.7$$

FACTORED BENDING MOMENTS AT MIDSPAN, FT-KIPS

M_{fDL}	M_{fSDL}	$M_{fLL} + M_{fI}$	M_{fT}
2,260	770	3,832	6,862

FACTORED END SHEAR, KIPS

V_{fDL}	V_{fSDL}	$V_{fLL} + V_{fI}$	V_{fT}
98.0	33.4	178.3	309.7

12.5.4 Trial Girder Section

A trial section with a web plate 60 × 7⁄16 in is assumed as in Art. 12.4. Bottom flange area can be estimated from

$$A_{sb} = \frac{12(M_{DL} + M_{LL} + M_I)}{F_y d} \tag{12.16}$$

For the preceding bending moments,

$$A_{sb} = \frac{12(2260 + 770 + 3832)}{36 \times 60} = 38.1 \text{ in}^2$$

Since the part of the web below the neutral axis will also carry some force, a bottom flange 20 × 1½ in (A_{sb} = 30.0 in^2) will be tried first. For the top flange plate, $A_{sb}/2$ = 15.0 in^2, a top flange of 16 × 1 in will be tried.

The concrete section for an interior stringer, not including the concrete haunch, is 8 ft 4 in wide (c to c of stringers) and 8½ in deep (½ in of slab is deducted from the concrete depth for the wearing course). The concrete area A_c = 8.33 × 12 × 8.50 = 850 in^2. Thus, this is an unsymmetrical composite section.

Check for Local Buckling. The trial section is assumed to be braced and noncompact. The width-thickness ratio b'/t of the projecting compression-flange element may not exceed

$$\frac{b'}{t} = \frac{69.6}{\sqrt{F_y}} \tag{12.17}$$

where b' is the width of the projecting element, t, the flange thickness, and F_y, the specified yield stress, ksi. For flange width b = 16 in and F_y = 36 ksi, the thickness should be at least

$$t = \frac{\sqrt{36}}{69.6} \times \frac{16}{2} = 0.69 \text{ in}$$

The 1-in-thick top flange is satisfactory.

Properties of Trial Section. The trial section is shown in Fig. 12.21. The computations for the location of the neutral axis and for the section moduli S_{st} and S_{sb} of the trial plate-girder section are tabulated in Table 12.17.

For unsymmetrical girders with transverse stiffeners but without longitudinal stiffeners, the minimum thickness of the web is obtained from:

$$\frac{D_c}{t_w} \leq \frac{577}{\sqrt{F_y}} \qquad D_c > D/2 \tag{12.18}$$

where D_c is the clear distance, in, between the neutral axis and the compression flange; D, the web depth, in; and t_w, the web thickness, in. For the trial section, $D_c = 36.02 > (D/2 = 30)$. Hence, from Eq. (12.18), the web thickness should be at least

$$t_w = \frac{D_c\sqrt{F_y}}{577} = \frac{36.02\sqrt{36}}{577} = 0.38 \text{ in, or } 3/8 \text{ in}$$

Since the assumed 7⁄16-in web thickness exceeds 3⁄8 in, the requirement for minimum web thickness without longitudinal stiffeners is met.

The computations for the location of the neutral axis and for the section moduli are given in Table 12.18 for the composite section, with $n = 8$ for short-time loading, such as live load and impact, and $n = 3 \times 8 = 24$ for long-time loading, such as superimposed dead loads. To locate the neutral axis, moments are taken about middepth of the girder web. Depth of the concrete haunch atop the girder is assumed to be 2 in. In addition, since the girder is composite, for prevention of flange buckling, the width-thickness ratio of the projecting element of the compression flange may not exceed

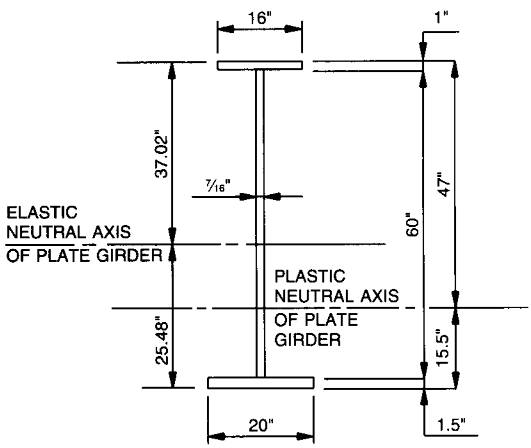


FIGURE 12.21 Cross section assumed for plate girder for load-factor design.

TABLE 12.17 Steel Section for Maximum Factored Moment

Material	A	d	Ad	Ad^2	I_o	I
Top flange 16 × 1	16.0	30.50	488	14,880		14,880
Web 60 × 7⁄16	26.3				7,880	7,880
Bottom flange 20 × 1½	30.0	−30.75	−923	28,370		28,370
	72.3		−435			51,130
$d_s = -435/72.3 = -6.02$ in					$-6.02 \times 435 =$	−2,620
					$I_{NA} =$	48,510

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 30 + 1 + 6.02 = 37.02 \text{ in}$$

$$\text{Bottom of steel} = 30 + 1.50 - 6.02 = 25.48 \text{ in}$$

Section moduli	
Top of steel	Bottom of steel
$S_{st} = 48{,}510/37.02$ $= 1{,}310 \text{ in}^3$	$S_{sb} = 48{,}510/25.48$ $= 1{,}904 \text{ in}^3$

$$\frac{b'}{t} = \frac{69.6}{\sqrt{1.3 f_{dl1}}} \tag{12.19}$$

where f_{dl1} is the compression stress ksi, in the top flange due to noncomposite dead load.

$$f_{dl1} = \frac{2{,}260 \times 12}{1{,}310} = 20.70 \text{ ksi}$$

From Eq. (12.19), for a flange width of 16 in, the flange thickness t_1 should be at least

$$t_1 = \frac{\sqrt{1.3 \times 20.7}}{69.6} \times \frac{16}{2} = 0.6 \text{ in}$$

The 1-in-thick top flange is satisfactory.

12.5.5 Fatigue Stresses

In the next step of the design procedure, fatigue stresses will be investigated. The four-stringer system is considered to have multiple load paths. A single fracture in a member cannot lead to collapse of the bridge. Hence, the structure is not fracture-critical.

Determination of the allowable stress range F_{sr} for fatigue is based on the stress category for the connection under consideration, the type of load path (redundant or nonredundant), and the stress cycle.

The bridge is located on a major highway (Case II) with an average daily truck traffic in one direction (ADTT) less than 2,500. The plate girders incorporate the four connection types tabulated in Table 12.19 with corresponding stress types and categories. For main (longitudinal) load-carrying members, the number of stress cycles of the maximum stress

TABLE 12.18 Composite Section for Maximum Factored Moment

(*a*) For superimposed dead loads, $n = 24$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	72.3		−435			51,130
Concrete $100 \times 8.5/24$	$\underline{35.4}$	37.25	$\underline{1319}$	48.120	210	$\underline{49,330}$
	107.7		884			100,460
$d_{24} = 884/107.7 = 8.21$ in					$-8.21 \times 884 =$	$\underline{-7,260}$
					$I_{NA} =$	93,200

Distance from neutral axis of composite section to:

Top of steel = 31.00 − 8.21 = 22.79 in

Bottom of steel = 31.50 + 8.21 = 39.71 in

Top of concrete = 22.79 + 2+ 8.5 = 33.29 in

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 93,200/22.79$ $= 4,089$ in^3	$S_{sb} = 93,200/39.71$ $= 2,347$ in^3	$S_c = 93,200/33.29$ $= 2,800$ in^3

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	72.3		−435			51,130
Concrete $100 \times 8.5/8$	$\underline{106.3}$	37.25	$\underline{3,958}$	147,440	640	$\underline{148,080}$
	178.6		3,523			199,210
$d_8 = 3523/178.6 = 19.73$ in					$-19.73 \times 3,523 =$	$\underline{-69,510}$
					$I_{NA} =$	129,700

Distance from neutral axis of composite section to:

Top of steel = 31.00 − 19.73 = 11.27 in

Bottom of steel = 31.00 + 19.73 = 50.73 in

Top of concrete = 11.27 + 2 + 8.5 = 21.77 in

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 129,700/11.27$ $= 11,510$ in^3	$S_{sb} = 129,700/50.73$ $= 2,557$ in^3	$S_c = 129,700/21.77$ $= 5,958$ in^3

TABLE 12.19 Categories and Allowable Fatigue Stress Ranges for Connections*

Connection type	Stress type	Category	Allowable stress range F_{sr}, ksi: 500,000 cycles	100,000 cycles
Toe of transverse stiffener	Tension or reversal	C	21	35.5
Groove weld at flanges	Tension or reversal	B	29	49
Gusset plate for lateral bracing	Tension or reversal	B		
Flange-to-web weld	Shear	F	12	15

*See AASHTO Specifications for full requirements that apply.

range for Case II, with ADTT < 2,500, the AASHTO standard specifications specify 500,000 loading cycles for truck loading and 100,000 for lane loading. Table 12.20 also lists for the four types of connections the allowable stress ranges F_{sr} for the redundant-load-path structure based on the connection stress category and the number of stress cycles. The fatigue stress in the bottom flange is checked for unfactored HS25 loading on the composite section. For live-load moment plus impact, $M = 1{,}765$ ft-kips, and the corresponding stress is

$$f_b = \frac{1{,}765 \times 12}{2{,}557} = 8.3 \text{ ksi}$$

Since the plate girder under consideration is simply supported, the minimum live-load moment would be zero and the live-load stress range becomes $f_{sr} = 8.3 \text{ ksi} < F_{sr} = 21$ ksi. The section is OK for fatigue.

12.5.6 Check for Compactness

The allowable stresses may have to be reduced if the section is noncompact and unbraced. Composite beams in positive bending qualify as compact when the web depth-thickness ratio D/t_w of the steel section meets the following requirement:

TABLE 12.20 Stresses, ksi, in Composite Girder at Section of Maximum Moment

Top of steel (compression)		Bottom of steel (tension)	
DL: $f_b = 2{,}260 \times 12/1{,}310 =$	20.70	*DL*: $f_b = 2{,}260 \times 12/1{,}904 =$	14.24
SDL: $f_b = 770 \times 12/4{,}089 =$	2.26	*SDL*: $f_b = 770 \times 12/2{,}347 =$	3.94
LL + I: $f_b = 3{,}832 \times 12/11{,}510 =$	<u>3.99</u>	*LL + I*: $f_b = 3{,}832 \times 12/2{,}557 =$	<u>17.98</u>
Total:	26.95 < 36	Total:	36.16 ≈ 36

Top of Concrete

$$SDL\text{: } f_c = 770 \times 12/(2{,}800 \times 8) = 0.41$$

$$LL + I\text{: } f_c = 3{,}832 \times 12/(5{,}958 \times 8) = \underline{0.96}$$

$$1.37 < 4.0$$

$$\frac{D}{t_w} \leq \frac{608}{\sqrt{F_y}} \tag{12.20}$$

where t_w is the web thickness, in, and D is the clear distance, in, between the flanges. For composite beams used in simple spans, D may be replaced by $2D_{cp}$, the distance, in, from the compression flange to the neutral axis in plastic bending. The compression depth of the composite section in plastic bending, including the slab, may not exceed

$$d_c = \frac{d + t_s}{7.5} \tag{12.21}$$

where d is the depth of the steel girder, in, and t_s is the thickness, in, of slab.

$$d_c = \frac{d + t_s}{7.5} = \frac{62.5 + 8.5}{7.5} = 9.47 \text{ in}$$

Therefore, the maximum allowed $D_{cp} = 9.47 - 8.5 = 0.97$ in. From Eq. (12.20), with D replaced by $2D_{cp} = 2 \times 0.97 = 1.94$,

$$t_w = \frac{\sqrt{36}}{608} \times 1.94 = 0.02 < 7/16 \text{ in}$$

The section meets the requirement for compactness.

Check of Unbraced Length of Top Flange. For live loads, the top flange of the girder is continuously supported by the concrete deck slab. But it is necessary to check the unbraced length L_b of the top flange for dead loads on the noncomposite section. For compact sections, spacing of lateral bracing of the compression flange may not exceed

$$\frac{L_b}{r_y} = \frac{[3.6 - 2.2(M_1/M_u)]10^3}{F_y} \tag{12.22}$$

where r_y is the radius of gyration with respect to the y-y axis, in, M_1 is the smaller moment at the end of the unbraced length of the member, M_u is the maximum bending strength $= F_yZ$, and Z is the plastic section modulus, in^3. For the 16×1-in top flange,

$$r_y = \sqrt{\frac{I}{A}} = \sqrt{\frac{1 \times 16^3/12}{1 \times 16}} = 4.62 \text{ in}$$

To determine the plastic modulus Z, the location of the axis that divides the section into two equal areas has to be found. The total area A of the girder is 72.25 in^2 (Table 12.17), and $A/2 = 36.13$ in^2. If $\bar{y}$ is the distance from top of steel to the axis, then $\bar{y} - 1$ is the web length from the axis to the flange. Since the web thickness is $7/16$ in and the flange area $A_f = 16$ in^2, $16 + (\bar{y} - 1)(7/16) = 36.83$, and $\bar{y} = 47.0$ in (Fig. 12.21). Z is computed by taking moments about the axis:

$$\begin{aligned} Z &= (16 \times 46.5 + 20.13 \times 23) + (30 \times 14.75 + 6.13 \times 7) \\ &= 1{,}692 \text{ in}^3 \end{aligned}$$

The bending strength then is

$$M_u = F_yZ = 36 \times 1{,}692/12 = 5076 \text{ ft-kips}$$

From Eq. (12.22) with $M_1 = M_{DL} = 1{,}738$ ft-kips, the maximum allowable unbraced length is

$$L_b = \frac{4.62[3.6 - 2.2(1{,}738/5076)]10^3}{36} = 365 \text{ in}$$

Since the spacing of bracing cross frames is 25 ft = 300 in and L_b is larger, the section may be treated as braced and compact.

12.5.7 Bending Strength of Girder

The flexural stresses in the composite section of the interior girder are checked for all the factored loads to ensure that the maximum stresses do not exceed $F_y = 36$ ksi. The computations in Table 12.20 indicate that the composite section is OK.

12.5.8 Shear Capacity of Girder

For girders with transverse stiffeners, shear capacity V_u, ksi, is given by

$$V_u = V_p\left[C + \frac{0.87(1 - C)}{\sqrt{1 + (d_o/D)^2}}\right] \qquad \frac{d_o}{D} \leq 3 \qquad \frac{d_o}{D} \leq \frac{67{,}600}{(D/t_w)^2} \tag{12.23}$$

where V_p = shear yielding strength of web, ksi = $0.58Dt_wF_y$
d_o = spacing, in, of intermediate stiffeners
D = clear distance, in, between flanges
t_w = web thickness, in
C = web buckling coefficient (Art. 11.12.4)

Stiffeners are usually equally spaced between cross frames. Spacing ranges up to the maximum of 1.5D for the first stiffener. The cross frames in the example are spaced about 25 ft apart. For a first trial, $d_o = 25/2 = 12.50$ ft = 150 in.

$$\frac{d_o}{D} = \frac{150}{60} = 2.5 < 3\text{—OK}$$

$$\frac{67{,}000}{[60/(7/16)]^2} = 3.6 > \frac{d_o}{D}\text{—OK}$$

The plastic shear force is

$$V_p = 0.58 \times 36 \times 60 \times 7/16 = 548 \text{ kips}$$

The coefficient C, the buckling shear stress divided by the shear yield stress, is computed from

$$C = \frac{45{,}000k}{(D/t_w)^2F_y} \qquad \frac{D}{t_w} > 237\sqrt{k/F_y} \tag{12.24}$$

where $k = 5\left[1 + \frac{1}{(d_o/D)^2}\right] = 5\left[1 + \frac{1}{(2.5/60)^2}\right] = 5.8$ and $D/t_w = 60/(7/16) = 147$.

$$C = \frac{45{,}000 \times 5.8}{137^2 \times 36} = 0.39$$

From Eq. (12.23), the shear capacity of the girder is

$$V_u = 548\left[0.39 + \frac{0.87(1 - 0.39)}{\sqrt{1 + (2.5)^2}}\right] = 322 \text{ kips}$$

The total factored end shear $V_{max} = 309.7$ kips < 322 kips. Thus, the section is adequate for shear.

12.5.9 Transverse Stiffener Design

For girders not meeting the shear capacity requirement, $V_u = CV_p$, transverse stiffeners are required. The trial section in this example meets this requirement. Girders designed to meet the shear requirement without transverse stiffeners normally have thicker webs but usually cost less than girders designed with thinner webs and transverse stiffeners, because of high welding costs for attaching stiffeners to webs. Another added advantage of a design without transverse stiffeners is elimination of the fatigue-prone welds between webs and stiffeners.

12.5.10 Shear Connectors

The horizontal shears at the interface of the concrete slab and steel girder are resisted by shear connectors throughout the simply supported span to develop composite action. Shear connectors are mechanical devices, such as welded studs or channels, placed in transverse rows across the top flange of the girder and embedded in the slab. The shear connectors for the girder will be designed for ultimate strength and the number of connectors provided for that purpose will be checked for fatigue.

For ultimate strength, the number N_1 of shear connectors required between a section of maximum positive moment and an adjacent end support should be at least

$$N_1 = P/\phi S_u \tag{12.25}$$

where S_u = ultimate strength of a shear connector, kips
ϕ = reduction factor = 0.85
P = force, kips in the concrete slab taken as the smaller of P_1 and P_2
$P_1 = A_s F_y$
$P_2 = 0.85 f'_c b t_s$
A_s = area, in^2, of the steel section
b = effective width, in slab for composite action
t_s = slab thickness, in
$P_1 = 72.3 \times 36 = 2{,}603$ kips
$P_2 = 0.85 \times 4 \times 100 \times 8.5 = 2{,}890 > 2{,}603$ kips

Hence, P for Eq. (12.25) = 2,603 kips.

For shear connectors, welded studs ⅞ in in diameter and 6 in long will be used. According to AASHTO standard specifications, the ultimate strength S_u, kips, of welded studs for $h/d > 4$, where h is stud height, in, and d = stud diameter, in, may be determined from

$$S_u = 0.4 d^2 \sqrt{f'_c E_c} \tag{12.26}$$

where E_c = modulus of elasticity of the concrete, ksi = $1{,}800\sqrt{f'_c} = 3{,}600$ ksi. For the ⅞-in-dia. welded studs,

$$S_u = 0.4(7/8)^2\sqrt{4 \times 3{,}600} = 36.75 \text{ kips}$$

from which the number of studs required is

$$N_1 = \frac{2{,}603}{0.85 \times 36.75} = 83$$

With the studs placed in groups of three, there should be at least 28 groups on each half of the girder.

Pitch is determined by fatigue requirements. The allowable load range, kips per stud, is given by Eq. (12.4), with $\alpha = 10.6$ for 500,000 cycles of loading. Hence, the allowable load range is

$$Z_r = 10.6(7/8)^2 = 8.12 \text{ kips}$$

At the supports, the shear range $V_r = 75.7$ kips, the shear produced by live load plus impact service loads. Consequently, with $n = 8$ for the concrete, the transformed concrete area equal to 106.3 in^2, and $I = 129{,}700$ in^4 from Table 12.18*b*, the range of horizontal shear is

$$S_r = \frac{V_r O}{I} = \frac{75.7 \times 106.3 \times 17.52}{129{,}700} = 1.087 \text{ kips per in}$$

The pitch required for stud groups near a support is

$$p = \frac{3Z_r}{S_r} = \frac{3 \times 8.12}{1.087} = 22.41 \text{ in}$$

The average pitch required for ultimate strength for 28 groups between midspan and a support is $\frac{1}{2} \times 100 \times 12/28 = 21$ in. Use three 7/8-in-dia. by 6-in-long studs per row, spaced at 18 in.

See also Arts. 12.9.4 to 12.9.6.

12.6 CHARACTERISTICS OF CURVED GIRDER BRIDGES

Past practice in design of new highways often located bridges first, then aligned the roadway with them. Current practice, in contrast, usually fits bridges into the desired highway alignment. Since curved crossings are sometimes unavoidable, and curved ramps at interchanges often must span other highways, railroads, or structures, bridges in those cases must be curved. Plate or box girders usually are the most suitable type of framing for such bridges.

Though the deck may be curved in accordance with the highway alignment, the girders may be straight or curved between skewed supports. Straight girders require less steel and have lower fabrication costs. But curved girders offer better appearance, and often the overall cost of a bridge with such girders may not be greater than that of a structure with straight members. Curved girders may reduce the number of foundations required because longer spans may be used; deck design and construction is simpler, because girder spacing and deck overhangs may be kept constant throughout the span; and cost savings may accrue from use of continuous girders, which may not be feasible with straight, skewed girders. Consequently, curved girders are generally used in curved bridges.

Curved girders introduce a new dimension in bridge design. The practice used for straight stringers of distributing loads to an individual stringer, as indicated in a standard specification, and then analyzing and designing the stringer by itself, cannot be used for curved bridges. For these structures, the entire superstructure must be designed as a unit. Diaphragms or cross frames as well as the stringers serve as main load-carrying members, because of the torsion induced by the curvature.

Analyses of such grids are very complicated, because they are statically indeterminate to a high degree. Computer programs, however, have been developed for performing the analyses. In addition, experience with rigorous analyses indicates that under certain conditions approximate methods give sufficiently accurate results.

The approximate methods described in this article are suitable for manual computations. They appear to be applicable to concentric, circular stringers where the arc between supports subtends an angle not much larger than about 0.5 radian, or about 30°. Also, where the spans are continuous, the methods may be used if the sum of the central angles subtended by each span does not exceed 90°. Accuracy of these methods, however, also seems to depend on the flexural rigidity of the deck in the radial direction and of the diaphragms.

The limitation of central angle indicates that the maximum span, along the arc, for a radius of curvature of 300 ft is about $300 \times 0.5 = 150$ ft for the approximate analysis. If the curved span is 200 ft, the approximate method should not be used unless the radius is at least $200/0.5 = 400$ ft.

Each simply supported or continuous girder should have at least one torsionally fixed support.

For box girders, in addition, accuracy depends on the ratio of bending stiffness to torsional rigidity EI/GK, where E is the modulus of elasticity. G the shearing modulus, I the moment of inertia for longitudinal bending, and K the torsional constant for the radial cross section. (For a hollow, rectangular tube,

$$K = \frac{4A^2}{\Sigma(l/t)} \tag{12.27}$$

where A = area, in^2, enclosed within the mean perimeter of tube
l = length of a side, in
t = thickness of that side, in

For inclusion in the summation in the denominator, a concrete slab in composite construction should be transformed into an equivalent steel plate by dividing the concrete cross-sectional area by the modular ratio n.) If the central angle of a curved span is about 0.5 radian, the approximate method should give satisfactory results if the weighted average of EI/GK in the span does not exceed 2.5.

A curved-girder bridge may have open framing, closed framing, or a combination of the two types. In open framing, curved plate girders are assisted in resisting torsion only by cross frames, diaphragms, or floorbeams at intervals along the span. In closed framing, the curved members may be box girders or plate girders assisted in resisting torsion by horizontal lateral bracing as well as by cross frames, diaphragms, or floorbeams.

12.6.1 Approximate Analysis of Open Framing

The approximate method for open framing derives from a rigorous method based on consistent deformations. Various components of the structure when distorted by loads must retain geometric compatibility with each other and simultaneously stay in equilibrium. The equations developed for these conditions can be satisfied only by a unique set of internal forces. In the rigorous method, a large number of such equations must be solved simultaneously. In the approximate method, considerable simplification is achieved by neglecting the stiffness of the plate girders in St. Venant (pure) torsion.

In the following, girders between the bridge centerline and the center of curvature are called **inner girders.** The rest are called **outer girders.**

The method will be described for a bridge with concentric circular stringers, equally spaced. Thus, for the four girders shown in Fig. 12.22*a*, if the distance from outer girder G_1

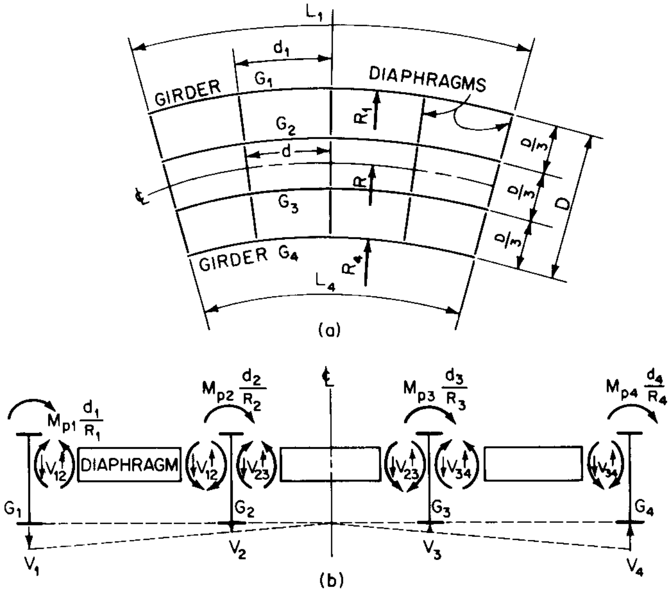


FIGURE 12.22 Curved-girder highway bridge. (*a*) Framing plan. (*b*) Cross section through the bridge at a diaphragm.

to inner girder G_4 is D, the girder spacing is $D/3$. The radius of the bridge centerline is R and of any girder G_n, R_n. Diaphragms are equally spaced at distance d apart along the centerline and placed radially between the girders.

Initially, the girders are assumed to be straight, and the span of each girder is taken as its developed length between supports. Preliminary moments M_p and shears V_p are computed as for straight girders.

These values must be corrected for the effects of curvature. The primary effect is a torque acting on every radial cross section of each girder. The torque per unit length at any section of a girder G_n is given approximately by

$$T_n = \frac{M_{pn}}{R_n} \tag{12.28}$$

where M_{pn} = preliminary bending moment at section and R_n = radius of G_n. If the diaphragm spacing along G_n is d_n and M_{pn} is taken as the preliminary moment at a diaphragm, then the total torque between diaphragms is

$$M_{tn} = \frac{M_{pn} d_n}{R_n} = \frac{M_{pn} d}{R} \tag{12.29}$$

This torque must be resisted by end moments in the diaphragms (Fig. 12.22*b*).

For equilibrium, the end moments on a diaphragm must be balanced by end shears forming an oppositely directed couple. For example, the diaphragm between G_2 and G_3 in Fig. 12.22*b* is subjected to end shears V_{23}. Also, the diaphragm between G_1 and G_2 is subjected to shears V_{12}. Consequently, G_2 is acted on by a net downward force V_2, called a V load, at the diaphragm

$$V_2 = V_{12} + V_{23} \tag{12.30}$$

where upward forces are taken as positive and downward forces as negative.

The V loads applied by the diaphragms are treated as additional loads on the girders.

For a bridge with two girders, the V load on the inner girder equals that on the outer girder, at a specific diaphragm, but is oppositely directed. Determined by equilibrium conditions at the diaphragm, this V load may be computed from

$$V = \frac{M_{p1} + M_{p2}}{K} \tag{12.31}$$

where $K = RD/d$ and D = girder spacing in two-girder bridges and distance between inner and outer girders in bridges with more than two girders.

For a bridge with more than two girders, the method assumes that the V load on a girder at a diaphragm is proportional to the distance of the girder from the centerline of the bridge. Then, equilibrium conditions require that the V load on the outer girder of a multigirder bridge be computed from

$$V = \frac{\Sigma M_{pn}}{CK} \tag{12.32}$$

where C = constant given in Table 12.21. The numerator in Eq. (12.13) consists of the sum of the preliminary moments in the girders at the line of diaphragms. Thus, for the four-girder bridge in Fig. 12.22*b*, the V load on G_1 and G_4 equals

$$-V_1 = V_4 = \frac{M_{p1} + M_{p2} + M_{p3} + M_{p4}}{1.11K} \tag{12.33}$$

By proportion, $-V_2 = V_3 = V_4/3$.

The bending moment produced by the V loads at any section of a girder G_n must be added to the preliminary moment at that section to produce the final bending moment M_n there. Thus,

$$M_n = M_{pn} + M_{vn} \tag{12.34}$$

where M_{vn} = bending moment produced by V loads. Similarly, the shear due to the V loads must be added to the preliminary shears to yield the final shears. Stresses are computed in the same way as for straight girders.

Between diaphragms, the girder flanges resist the torsion. At any section, the stresses in the top and bottom flanges of a girder provide a couple equal to the torque but oppositely directed. The forces comprising this couple induce lateral bending in the flanges. If q_n is the force per unit length of flange in girder G_n resisting torque,

$$q_n = \frac{M_n}{R_n h_n} \tag{12.35}$$

where h_n = distance between centroids of flanges. Each flange may be considered to act

TABLE 12.21 Values of C for Eq. (12.32)

No. of girders	2	3	4	5	6	7	8	9	10
C	1.00	1.00	1.11	1.25	1.40	1.56	1.72	1.88	2.04

under this loading as a continuous beam spanning between diaphragms. The maximum negative moment for design purposes may be taken as

$$M_{Ln} = -\frac{0.1M_n d_n^2}{R_n h_n} \tag{12.36}$$

The stress due to lateral bending should be added to that due to M_n to obtain the maximum stress in each flange. Where provision is made for composite action, however, the lateral bending stress in that flange may be neglected.

For preliminary design purposes, a rough approximation of the effects of curvature may be obtained by use of

$$p = 5.25\,\frac{(1 + r)mL_c^2}{CRD} \tag{12.37}$$

where p = percent increase in moment in outer girder due to curvature

$r = \dfrac{\text{loading on inner girder}}{\text{loading on outer girder}}\left(\dfrac{R'}{R}\right)^2$

R' = radius of curvature of inner girder

R = radius of curvature of outer girder

m = number of girders

L_c = developed length of outer girder between supports when simply supported or between inflection points when continuous

C = constant given by Table 12.21

D = distance between inner and outer girders

12.6.2 Approximate Analysis of Closed Framing

Analysis of bridges with box girders or similar boxlike framing must take into account the torsional stiffness of these members. The method to be described is based on the following assumptions:

Girder cross sections are symmetrical about the vertical axis. Supports are radial. Curvature may vary so long as it does not change direction within a span. Diaphragms prevent distortion of the cross sections. Secondary stresses due to torsional warping are negligible.

Differential equations for determining the internal forces acting on a curved girder can be obtained from the equilibrium conditions for a differential segment (Fig. 12.23). Because upward and downward vertical forces must balance, the shear V is related to the loading w by

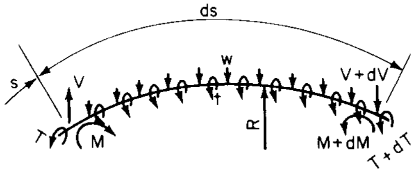


FIGURE 12.23 Forces acting on a differential length *ds* of a girder curved to radius *R*. The distributed vertical load *w* and torque *t* cause vertical end shears *V*, end moments *M*, and end torques *t*.

$$\frac{dV}{ds} = -w \tag{12.38}$$

Thus, as for a straight beam, the change in shear between any two sections of the girder equals the area of the load diagram between those sections. Because the sum of the moments about a radial plane must equal zero, bending moments M, torques T, and shears V are related by

$$\frac{dM}{ds} = -\frac{T}{R} + V \tag{12.39}$$

where R = radius of curvature of girder. In the approximate method, with the limitations on central angle subtended by the span and on the ratio of bending stiffness to torsional stiffness, the T/R term can be ignored. Thus, the equation becomes

$$\frac{dM}{ds} = V \tag{12.40}$$

As for straight beams, the change in bending moments between any two sections of the girder equals the area of the shear diagram between those sections.

Hence, bending moments in a curved girder of the closed-framing type may be computed approximately by treating it as a straight beam with span equal to the developed length of the curve.

A third equation is obtained by taking moments about a tangential plane:

$$\frac{dT}{ds} = \frac{M}{R} - t \tag{12.41}$$

where t = applied torque. With the bending moments throughout the girder known, the torque at any section can be found from Eq. (12.41). [For a more rigorous solution, Eqs. (12.38), (12.39), and (12.41) may be solved simultaneously. This can be done by differentiating Eq. (12.39), solving for dT/ds, substituting the result in Eq. (12.41), and then solving the resulting second-order differential equation.]

Equation (12.41) indicates that the change in torque between any two sections of the girder equals the area of the $M/R - t$ diagram between those sections. Consequently, the torque on a curved girder of the closed-framing type can be determined by a method similar to the conjugate-beam method for determining deflections. In the approximate method, however, the moments M determined from Eq. (12.40) are used instead of those from the more complex rigorous solution.

Thus, first the bending-moment diagram (Fig. 12.24*b*) is obtained for the vertical loading on the developed length of the girder (Fig. 12.24*a*). Then, all ordinates are divided by the radius R. Next, the applied-torque diagram (Fig. 12.24*d*) is plotted for the twisting moments applied by the loading to the girder (Fig. 12.24*c*). The ordinates of this diagram are subtracted from the corresponding ordinates of the M/R diagram. The resulting $M/R - t$ diagram then is used as a loading diagram on the developed length of the girder (Fig. 12.24*e*). The resulting shears (Fig. 12.24*f*) equal the torques T in the curved girder. Note that positive $M/R - t$ is equivalent to an upward load on the conjugate beam.

The conjugate beam shown in Fig. 12.24*c* is simply supported. This requires that the angle of twist at the supports be zero. Hence, for this case, the curved girder is torsionally fixed at the supports. This condition is attained with a line of diaphragms at each support and a bearing under each web capable of resisting uplift, a common practice. Sometimes, interior supports of a continuous box girder are not fixed against torsion, for example, where a single bearing is placed under a diaphragm. In such cases, the span of the conjugate beam

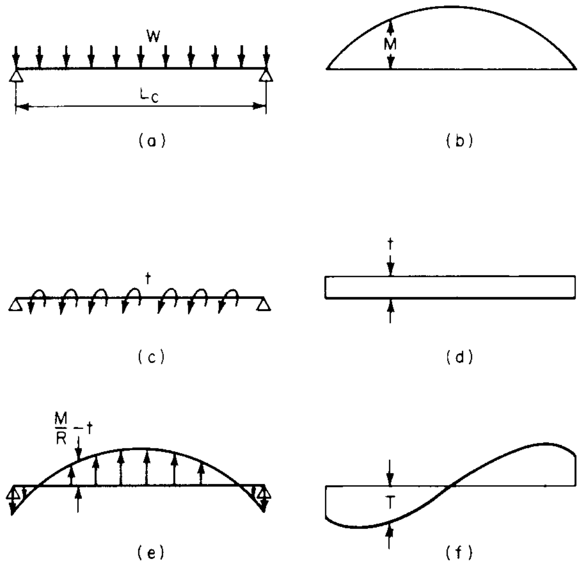


FIGURE 12.24 Loading diagrams for a curved box girder. (*a*) Uniform load *w* on the developed length. (*b*) Bending moment diagram for the uniform load. (*c*) Applied torque *t*. (*d*) Torsion diagram. (*e*) $M/R - t$ diagram applied as a load to the conjugate beam. (*f*) Shear diagram for the loading in (*e*).

should be taken as the developed length of girder between supports that are fixed against torsion.

12.6.3 Loading

Dead loads may be distributed to curved girders in the same way as for straight girders. For live loads, the designer also may use any method commonly used for straight girders. If the distribution procedure of the AASHTO "Standard Specifications for Highway Bridges" is used, however, a correction factor should be applied. The sum of the AASHTO live-load distribution factors for all the girders in a curved grid will usually exceed the number of wheel loads required for the roadway width. Hence, if these factors were used to compute the live-load moments in the girders at a line of diaphragms, the *V* loads there would be too large, because they are proportional to the sum of those moments. One way to correct the *V* loads determined with the AASHTO factors is to multiply the *V* loads by the ratio of the number of wheel loads required for the roadway width to the number of wheel loads determined by the sum of the AASHTO factors.

Impact may be taken into account, in the same way as for straight girders, as a percent increase in live load.

Centrifugal forces comprise a horizontal, radial loading on curved structures that does not apply to straight bridges. These forces are determined as a percentage of the live load, without impact (Art. 11.4). But the live load is restricted to one standard truck placed for maximum loading in each design lane.

Assumed to act 6 ft above the roadway surface, measured from the roadway centerline, centrifugal forces induce torques and horizontal shears in the superstructure. The shears may be assumed to be resisted by the concrete deck within its plane. The torques, however, must be resisted by the girders. In open-framing systems, the primary effect is on the preliminary bending moments. Resisting couples comprise upward and downward vertical forces in the girders. These forces increase bending moments in the outer girders (those farthest from the center of curvature) and decrease moments in the inner girders. The effect of centrifugal forces on *V* loads, however, is small, because *V* loads are determined by the sum of girder moments at a line of diaphragms and this sum is not significantly changed by centrifugal forces.

12.6.4 Sizing of Girders

Design rules for proportioning straight girders generally are applicable to curved girders, depth-span ratios, for example. But curvature does produce effects that should be considered for maximum economy. For instance, girder flanges in open-framing systems should be made as wide as practical to minimize lateral bending stresses. In some cases, where these stresses become too large, a reduction in spacing of diaphragms or cross frames may be desirable.

If curvature causes large adjustments to the preliminary moments in open framing, deepening of girders farthest from the center of curvature may be advantageous. This may be done without overall increase of the floor system, because of the superelevation of the deck.

Girder webs, in some cases, may have to be thicker than for straight girders with corresponding span, spacing, and loadings, because of the effects of curvature on shear. Reactions, too, may be significantly changed, and the effects on substructure design should be taken into account. For some sharply curved bridges, tie-downs may be required to prevent uplift at supports of girders closest to the center of curvature.

If horizontal lateral bracing is placed in an open-framing system, the effects of curvature should be examined more closely. Connections at frequent intervals can convert the system into the closed-framing type.

12.6.5 Fabrication

Curved plate girders usually are produced in one of two ways. One way is to mechanically bend the web to the desired curvature and then weld to it flange plates that have been flame-cut to the required shape. The procedure differs from fabrication of plate girders in handling procedures, layout for fabrication, and web-to-flange welding methods.

Alternatively, girders may be curved by selectively heating the flanges of members initially fabricated straight. In this method, less steel is required. The heating and cooling induce residual stresses, but research indicates that they do not affect fatigue strength.

Mechanical bending is sometimes used for curving rolled beams.

(L. C. Bell and C. P. Heins, "Analysis of Curved Bridges," *Journal of the Structural Division, ASCE,* vol. 96, no. ST8, pp. 1657–1673, August, 1970.

P. P. Christiano and C. G. Culver, "Horizontally Curved Bridges Subject to Moving Load," *Journal of the Structural Division, ASCE,* vol. 95, no. ST8, pp. 1615–1643, August, 1969.

"Guide Specifications for Horizontally Curved Highway Bridges," and "Standard Specifications for Highway Bridges," American Association of State Highway and Transportation Officials.

R. L. Brockenbrough, "Distribution Factors for Curved I-Girder Bridges," *Journal of Structural Engineering, ASCE,* 1986.

M. A. Grubb, "Horizontally Curved I-Girder Bridge Analysis: V-Load Method," Transportation Research Board, 1984.)

12.7 EXAMPLE—ALLOWABLE-STRESS DESIGN OF CURVED STRINGER BRIDGE

The basic design procedures that apply to bridges with straight stringers apply also to bridges with curved stringers (Arts. 12.1 to 12.5). In determination of stresses, however, the effects of curvature must be taken into account (Art. 12.6).

To illustrate the design procedure, a curved, two-lane highway bridge with simply supported, composite, plate-girder stringers will be designed. As indicated in the framing plan in Fig. 12.25*a*, the stringers are concentric and the supports and diaphragms are placed radially. Outer girder G_1 spans 90 ft and has a radius of curvature R_1 of 300 ft. Spacing of diaphragms along this span is $d_1 = 15$ ft. Distance between inner and outer grids G_1 and G_3 is $D = 22$ ft c to c, and G_2 is midway between them. The typical cross section in Fig. 12.25*b* shows a 22-ft-wide roadway flanked by two 3-ft 3-in-wide safety walks.

Structural steel to be used is Grade 36. Concrete to be used for the deck is Class A, with 28-day strength $f'_c = 3000$ psi and allowable compressive stress $f_c = 1200$ psi. Appropriate design criteria given in Sec. 10 will be used for this structure. The approximate analysis described in Art. 12.6 for open framing will be applied to the design of the girders.

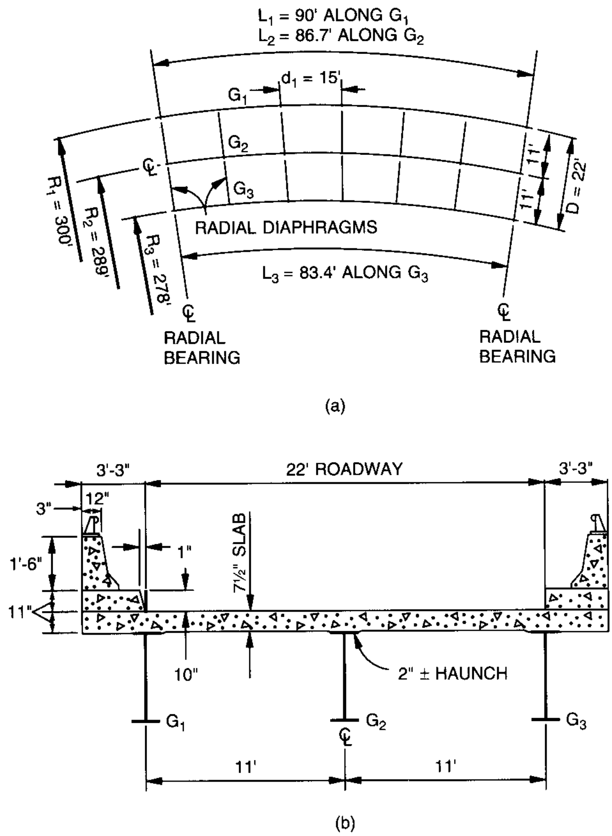


FIGURE 12.25 Two-lane highway bridge with curved stringers. (*a*) Framing plan. (*b*) Typical cross section.

TABLE 12.22 Dead Load Carried by Steel Beams, kips per ft

Stringers G_1 and G_3	
Slab: $0.150(11\frac{1}{2} + 3.25)7.5/12$	$= 0.82$
Haunch and extra concrete: $0.150(3.25 + 0.75)\frac{3}{12}$	$= 0.15$
Steel stringer and framing details—assume:	0.30
DL per stringer:	1.27
Stringer G_2	
Slab: $0.150 \times 11 \times 7.5/12$	$= 1.03$
Haunch—18 × 2 in: $0.150 \times 1.5 \times \frac{2}{12}$	$= 0.04$
Steel stringer and framing details—assume:	0.30
DL per stringer:	1.37

Concrete Slab. The slab is designed, to span transversely between stringers, in the same way as for straight stringers (Art. 12.2). A 7.5-in-thick slab will be used with the curved plate-girder stringers.

Loads, Moments, and Shears for Stringers. Assume that the girders will not be shored during casting of the concrete slab. Hence, the dead load on each steel stringer includes the weight of the concrete slab as well as the weight of stringer and framing details. This dead load will be referred to as *DL* (see Table 12.22).

For design of the grid composed of the stringers and diaphragms not only is the maximum bending moment needed for each stringer but also the bending moment at each line of diaphragms (Table 12.23).

For computation of V loads,

$$K = \frac{RD}{d} = \frac{300 \times 22}{15} = 440$$

From Table 12.21, $C = 1.00$. V loads now can be computed from Eq. (12.32) and are listed in Table 12.24. They act at the diaphragms, downward on G_1, upward on G_3, to resist the torque due to curvature. The V load on G_2, the central girder in a three-girder grid, is assumed to be zero. The reaction due to these loads is

TABLE 12.23 Initial-Dead-Load Preliminary Moments, ft-kips

		Distance from support, ft		
	Span, ft	15, 75	30, 60	45
M_{p1} in G_1	90	714	1,143	1,286
M_{p2} in G_2	86.7	715	1,144	1,287
M_{p3} in G_3	83.4	613	981	1,104
ΣM_{pn}		2,042	3,268	3,677

TABLE 12.24 V Loads from $\Sigma M_{pn}/440$

Distance from support, ft	15, 75	30, 60	45
V loads on G_1 and G_3, kips	4.64	7.43	8.36

$$R_v = 4.64 + 7.43 + \frac{8.36}{2} = 16.25 \text{ kips}$$

The resulting bending moments are given in Table 12.25.

Final moments are the sum of the preliminary bending moments and the moments due to the V loads (Table 12.26).

Maximum shear occurs at the supports. For G_1, the maximum dead-load shear is the sum of the preliminary shear and V-load shear:

$$V_{DL} = \frac{1.27 \times 90}{2} + 16.25 = 73.4 \text{ kips}$$

Parapets, railings, and safety walks will be placed after the concrete slab has cured. Their weights may be equally distributed to all stringers. In addition, provision will be made for a future wearing surface, weight 20 psf. The total superimposed dead load will be designated *SDL.* Table 12.27 lists for the superimposed load on the composite section the dead loads, the preliminary bending moments, and the V loads, Table 12.28 gives the bending moments due to the V loads, and Table 12.29, the final superimposed dead-load moments. For G_1, the maximum shear due to the superimposed dead load is

$$V_{SDL} = \frac{0.607 \times 90}{2} + 7.57 = 34.9 \text{ kips}$$

The HS20-44 live load imposed may be a truck load or lane load. For these girder spans, however, truck loading governs. A standard truck should be placed within each 11-ft lane to produce maximum stresses in the stringers. The extreme left and right positions of the loading are shown in Fig. 12.26. The loads are distributed to the girders on the assumption that the concrete slab is simply supported on them (Table 12.30).

The trucks also should be positioned to produce maximum bending moment in each stringer for design of its central portion. Maximum dead-load moments, however, have been computed at midspan. Since moments are needed at diaphragm locations for computation of V loads and maximum moment occurs near midspan, it is convenient to place the trucks for maximum moment at midspan, where a diaphragm is located, and the error in so doing will be small. Hence, the central 32-kip axle of each truck is placed at midspan, with the other axles, 32 kips and 8 kips, 14 ft on either side.

TABLE 12.25 Initial-Dead-Load V-Load Moments M_{vn}, ft-kips

	Distance from support, ft		
	15, 75	30, 60	45
M_v in G_1	244	418	481
M_{v3} in G_3	−226	−387	−445

TABLE 12.26 Initial-Dead-Load Final Moments, ft-kips

	Distance from support, ft		
	15, 75	30, 60	45
M_1 in G_1	958	1,561	1,767
M_2 in G_2	715	1,144	1,287
M_3 in G_3	387	594	659

The midspan moments M_n for a truck in one lane only are used to produce the maximum midspan moments in each girder for a truck in each of the two lanes. The calculations are given in Tables 12.31 to 12.34.

For maximum live-load moments at other sections of each girder, the trucks should be placed in each lane as for the midspan moments and positioned to produce maximum preliminary moments at the sections. Then, V loads and the moments they cause should be calculated and added to the preliminary moments to yield M_{LL} at each section.

Maximum preliminary shear occurs at a support with a 32-kip axle at the support and the center of gravity of the loading $14 - 4.66 = 9.34$ ft from the support. For girder G_1, a truck should be placed at the left in both lanes 1 and 2 for maximum shear due to curvature. The truck in lane 1 should be located at a support for maximum shear, while the truck in lane 2 should be positioned for maximum midspan moments in G_2 and G_3. The maximum shear in G_1 caused by the truck in lane 2 equals the reaction $R_v = 5.07$, previously computed

TABLE 12.27 Dead Load Carried by Composite Section, kips per ft

Two parapets: $(1/3)2 \times 0.150 \times 1.5(1 + 1.25)/2$	$= 0.169$
Two railings: $(1/3)2 \times 0.015$	$= 0.010$
Two safety walks: $(1/3)2 \times 0.150[3.25(0.917 + 0.833)/2 - 0.833 \times 0.083/2]$	$= 0.281$
Future wearing surface: $(1/3)0.020 \times 22$	$= \underline{0.147}$
SDL per stringer:	$= 0.607$

M_{pn} for superimposed dead load, ft-kips

		Distance from support, ft		
	Span, ft	15, 75	30, 60	45
M_{p1} in G_1	90	342	546	615
M_{p2} in G_2	86.7	317	507	571
M_{p3} in G_3	83.4	293	469	528
ΣM_{pn}		952	1,522	1,714

V Loads from $\Sigma M_{pn}/440$

Distance from support, ft	15, 75	30, 60	45
V load on G_1 and G_3, kips	2.16	3.46	3.90

$$R_v = 2.16 + 3.46 + 3.90/2 = 7.57 \text{ kips}$$

TABLE 12.28 Superimposed Dead Load V-Load Moments M_{vn}, ft-kips

	Distance from support, ft		
	15, 75	30, 60	45
M_{v1} in G_1	114	194	223
M_{v3} in G_3	−104	−180	−205

in determining maximum midspan moments (Table 12.33). The truck in lane 1 produces a maximum preliminary shear in G_1 of

$$V_{p1} = \frac{72(90 - 9.34)}{90} \times 0.55 = 35.5 \text{ kips}$$

This truck also induces the preliminary bending moments given in Table 12.35 in G_1 and G_2 at the diaphragms.

The reaction due to the V loads is

$$R_v = 1.03 \times 5/6 + 1.02 \times 4/6 + 0.76 \times 3/6 + 0.51 \times 2/6 + 0.25 \times 1/6 = 2.12 \text{ kips}$$

Hence, the final shear in G_1 due to the trucks in both lanes is

$$V_{LL} = 35.5 + 2.12 + 5.07 = 42.7 \text{ kips}$$

Impact is taken as the following fraction of live-load stress:

$$I = \frac{50}{L_3 + 125} = \frac{50}{83.4 + 125} = 0.24$$

Thus, the maximum moments due to impact are

$$G_1{:}M_I = 0.24 \times 1{,}046 = 251 \text{ ft-kips}$$

$$G_2{:}M_I = 0.24 \times 1{,}408 = 338 \text{ ft-kips}$$

$$G_3{:}M_I = 0.24 \times 531 = 127 \text{ ft-kips}$$

And the maximum shear in G_1 due to impact is

TABLE 12.29 Superimposed-Dead-Load Final Moments, ft-kips

	Distance from support, ft		
	15, 75	30, 60	45
M_1 in G_1	456	740	838
M_2 in G_2	317	507	571
M_3 in G_3	189	289	323

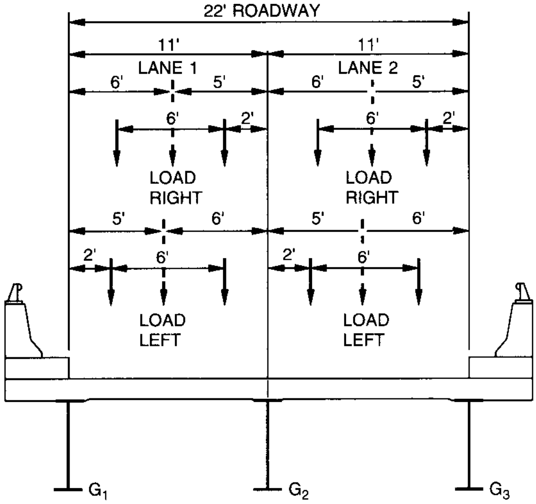


FIGURE 12.26 Position of truck wheel loads in design lanes.

$$V_I = 0.24 \times 42.7 = 10.2 \text{ kips}$$

For centrifugal forces and radial wind forces on live load, both of which induce torques in the superstructure because they are assumed to act 6 ft above the roadway surface, allowable stresses may be increased 25%. Therefore, if the sum of the moments and shears produced by those forces does not exceed 25% of the moments and shears, respectively, without those forces, they may be ignored. For this structure, the effects of the wind and centrifugal forces are small enough to be neglected. But for illustrative purposes, they will be calculated.

Because of the sharp curvature, design speed is taken as 30 mph. Then, the centrifugal forces equal the following percentages of a truck load in lanes 1 and 2:

$$C = \frac{6.68S^2}{R} = \frac{6.68(30)^2}{295} = 20.4\% \qquad C = \frac{6.68(30)^2}{284} = 21.2\%$$

Application of these percentages to the axle load per lane permits use of the results of previous calculations for moments and shears. Thus, the horizontal force per axle is

$$H = 32 \times 0.204 + 32 \times 0.212 = 13.3 \text{ kips}$$

This force is assumed to act 6 ft above the roadway surface or about 8 ft above the centroidal axis of the girders. Thus, it causes a torque

TABLE 12.30 Fraction of Axle Load on Girders

	G_1	G_2	G_3
Load in lane 1 only, at extreme left	0.55	0.45	0
Load in lane 1 only, at extreme right	0.45	0.55	0
Load in lane 2 only, at extreme left	0	0.55	0.45
Load in lane 2 only, at extreme right	0	0.45	0.55

TABLE 12.31 M_{pn} for Maximum Live-Load Moment at Midspan, ft-kips

	Distance from support, ft				
	15	30	45	60	75
	Girder G_1, span 90 ft				
Full axle load	596	1,192	1,340	968	484
0.55 axle load	328	656	737	532	266
0.45 axle load	268	536	603	436	218
	Girder, G_2, span 86.7 ft				
Full axle load	576	1,152	1,281	928	464
0.55 axle load	317	634	704	510	255
0.45 axle load	259	518	577	418	209
	Girder G_3, span 83.4 ft				
Full axle load	556	1,109	1,221	889	444
0.55 axle load	306	610	671	489	244
0.45 axle load	250	499	550	400	200

$$T = 8 \times 13.3 = 106.4 \text{ ft-kips}$$

This is resisted by a couple compromising a downward vertical force on G_1 and an upward vertical force on G_3

$$P = \frac{106.4}{22} = 4.83 \text{ kips}$$

By proportion, the maximum moment M_C in G_1 due to the centrifugal forces can be obtained from the maximum moment M_{p1} previously computed for a truck load in lane 1, 1,340 ft-kips.

$$M_C = \frac{1{,}340 \times 4.83}{32} = 202 \text{ ft-kips}$$

Similarly, the maximum shear in G_1 due to the centrifugal forces is

$$V_C = \frac{35.5 \times 4.83}{32 \times 0.55} = 9.7 \text{ kips}$$

AASHTO specifications require a wind load on the live load of at least 0.1 kip per ft. This would cause a torque of $0.1 \times 8 = 0.8$ ft-kip per ft and a downward vertical force on G_1 of $0.8/22 = 0.0364$ kip per ft. Hence, the maximum shear in G_1 due to wind on live load is

$$V_{WL} = \tfrac{1}{2} \times 0.0364 \times 90 = 1.6 \text{ kips}$$

The maximum moment in G_1 due to this load is

TABLE 12.32 M_{pn} for Truck in One Lane Only, ft-kips and V Load, kips

	Distance from support, ft				
	15	30	45	60	75
At left in lane 1					
M_{p1} for G_1 (0.55 axle load)	328	656	737	532	266
M_{p2} for G_2 (0.45 axle load)	259	518	577	418	209
M_{p3} for G_3 (no load)	0	0	0	0	0
ΣM_{pn}	587	1,174	1,314	950	475
$V = \Sigma M_{pn}/440$	1.33	2.67	2.99	2.16	1.08
At right in lane 1					
M_{p1} for G_1 (0.45 axle load)	268	536	603	436	218
M_{p2} for G_2 (0.55 axle load)	317	634	704	510	255
M_{p3} for G_3 (no load)	0	0	0	0	0
ΣM_{pn}	585	1,170	1,307	946	473
$V = \Sigma M_{pn}/440$	1.33	2.66	2.97	2.15	1.08
At left in lane 2					
M_{p1} for G_1 (no load)	0	0	0	0	0
M_{p2} for G_2 (0.55 axle load)	318	634	704	510	255
M_{p3} for G_3 (0.45 axle load)	250	499	550	400	200
ΣM_{pn}	567	1133	1254	910	455
$V = \Sigma M_{pn}/440$	1.29	2.57	2.85	2.07	1.03
At right in lane 2					
M_{p1} for G_1 (no load)	0	0	0	0	0
M_{p2} for G_2 (0.45 axle load)	259	518	577	418	209
M_{p3} for G_3 (0.55 axle load)	306	610	671	489	244
ΣM_{pn}	565	1,128	1,248	907	453
$V = \Sigma M_{pn}/440$	1.28	2.56	2.84	2.06	1.03

$$M_{WL} = \frac{0.0364(90)^2}{8} = 36 \text{ ft-kips}$$

Combined, centrifugal forces and wind induce in G_1 a maximum shear

$$V_C + V_{WL} = 9.7 + 1.6 = 11.3 \text{ kips}$$

This is less than 25% of the total shear without these forces and may be ignored. Similarly, the combined maximum moment in G_1 is

$$M_C + M_{WL} = 202 + 36 = 238 \text{ ft-kips}$$

This is less than 25% of the total moment without these forces and may be ignored. The maximum moments and shears for design therefore are as given in Tables 12.36 and 12.37.

TABLE 12.33 *V*-Load Reactions, kips, and Final Midspan Moments M_n for Truck in One Lane Only, ft-kips

At left in lane 1
$R_v = 1.33 \times 5/6 + 2.67 \times 4/6 + 2.99 \times 3/6 + 2.16 \times 2/6 + 1.08 \times 1/6 = 5.28$
Midspan $M_{v1} = 5.28 \times 45 - 1.33 \times 30 - 2.67 \times 15 = 158$
Midspan $M_{v2} = 0$
Midspan $M_{v3} = -158 \times 83.4/90 = -146$
Midspan $M_1 = M_{p1} + M_{v1} = 737 + 158 = 895$
Midspan $M_2 = M_{p2} + M_{v2} = 577$
Midspan $M_3 = M_{p3} + M_{v3} = -146$
At right in lane 1
$R_v = 1.33 \times 5/6 + 2.66 \times 4/6 + 2.97 \times 3/6 + 2.15 \times 2/6 + 1.08 \times 1/6 = 5.26$
Midspan $M_{v1} = 5.26 \times 45 - 1.33 \times 30 - 2.66 \times 15 = 157$
Midspan $M_{v2} = 0$
Midspan $M_{v3} = -157 \times 83.4/90 = -145$
Midspan $M_1 = M_{p1} + M_{v1} = 603 + 157 = 760$
Midspan $M_2 = M_{p2} + M_{v2} = 704$
Midspan $M_3 = M_{p3} + M_{v3} = -145$
At left in lane 2
$R_v = 1.29 \times 5/6 + 2.57 \times 4/6 + 2.85 \times 3/6 + 2.07 \times 2/6 + 1.03 \times 1/6 = 5.07$
Midspan $M_{v1} = 5.07 \times 45 - 1.29 \times 30 - 2.57 \times 15 = 151$
Midspan $M_{v2} = 0$
Midspan $M_{v3} = -151 \times 83.4/90 = -140$
Midspan $M_1 = M_{p1} + M_{v1} = 151$
Midspan $M_2 = M_{p2} + M_{v2} = 704$
Midspan $M_3 = M_{p3} + M_{v3} = 550 - 140 = 410$
At right in lane 2
$R_v = 1.28 \times 5/6 + 2.56 \times 4/6 + 2.84 \times 3/6 + 2.06 \times 2/6 + 1.03 \times 1/6 = 5.05$
Midspan $M_{v1} = 5.05 \times 45 - 1.28 \times 30 - 2.56 \times 15 = 150$
Midspan $M_{v2} = 0$
Midspan $M_{v3} = -150 \times 83.4/90 = -140$
Midspan $M_1 = M_{p1} + M_{v1} = 150$
Midspan $M_2 = M_{p2} + M_{v2} = 577$
Midspan $M_3 = M_{p3} + M_{v3} = 671 - 140 = 531$

TABLE 12.34 Midspan Live-Load Moments M_{LL}, ft-kips

Girder	Truck position	M_{LL}
G_1	At left in lanes 1 and 2	895 + 151 = 1,046
G_2	At right in lane 1, left in lane 2	704 + 704 = 1,408
G_3	At right in lane 2	531

TABLE 12.35 M_p for Truck in Lane 1 Placed for Maximum Shear, ft-kips

		Distance from support, ft				
	Span, ft	15	30	45	60	75
M_{p1} for G_1	90	251	246	185	123	62
M_{p2} for G_2	86.7	203	201	151	100	50
ΣM_{pn}		454	447	336	223	112
$V = M_{pn}/440$		1.03	1.02	0.76	0.51	0.25

Properties of Composite Section. Design of the girders follows the procedures indicated for plate-girder stringers in Art. 12.4, except that the lateral bending stress in the flanges due to curvature must be taken into account.

For illustrative purposes, girder G_1 will be designed. The effective width of the concrete slab, governed by its 7-in effective thickness, is 84 in. A trial section for the plate girder is selected with the aid of the Hacker formulas, Eqs. (12.1*a*) and (12.1*b*), with the allowable stress reduced about 20% because of lateral bending.

Assume that the girder web will be 60 in deep. This satisfies the requirements that the depth-span ratio for girder plus slab exceed 1:25 and for girder alone 1:30.
With stiffeners, the web thickness is required to be at least $1/165$ of the depth, or 0.364 in.

Use a web plate 60 × 3/8 in. With a cross-sectional area $A_w = 22.5$ in^2, the web will be subjected to a maximum shearing stress considerably below the 12 ksi permitted.

$$f_v = \frac{161.2}{22.5} = 7.2 < 12 \text{ ksi}$$

The required bottom-flange area may be estimated from Eq. (12.1*a*) with allowable bending stress $F_b = 20 - 4 = 16$ ksi. Distance between centers of gravity of steel flanges is assumed as $d_{cg} = 63$ in.

$$A_{sb} = \frac{12}{16}\left(\frac{1{,}767}{63} + \frac{2{,}135}{63 + 7}\right) = 43.8 \text{ in}^2$$

Try a 22 × 2-in bottom flange, area = 44 in^2.

The ratio of flange areas $R = A_{st}/A_{sb}$ may be estimated from Eq. (12.1*b*) as

$$R = \frac{50}{190 - L} = \frac{50}{190 - 90} = 0.50$$

Then, the estimated required area of the steel top flange is

TABLE 12.36 Midspan Bending Moments, ft-kips

	M_{DL}	M_{SDL}	$M_{LL} + M_I$
Girder G_1	1,767	838	1,297
Girder G_2	1,287	571	1,746
Girder G_3	659	323	658

TABLE 12.37 End Shear, kips, in Girder G_1

V_{DL}	V_{SDL}	$V_{LL} + V_1$	Total V
73.4	34.9	52.9	161.2

$$A_{st} = RA_{sb} = 0.50 \times 43.8 = 21.9 \text{ in}^2$$

Try a 18 × 1½-in top flange, area = 27 in².

The trial section is shown in Fig. 12.27. Its neutral axis can be located by taking moments of web and flange areas about middepth of the web. This computation and that for the section moduli S_{st} and S_{sb} of the plate girder alone are conveniently tabulated in Table 12.38.

In computation of the properties of the composite section, the concrete slab, ignoring the haunch area, is transformed into an equivalent steel area, with $n = 30$ for superimposed dead load and $n = 10$ for live loads. The computations of neutral-axis location and section moduli for the composite section are tabulated in Table 12.39. To locate the neutral axis, moments of the areas are taken about middepth of the girder web.

Stresses in Composite Section. Since the girders will not be shored when the concrete is cast and cured, the stresses in the steel section for load *DL* are determined with the section moduli of the steel section alone (Table 12.38). Stresses for load *SDL* are computed with the section moduli of the composite section when $n = 30$ (Table 12.39*a*). And stresses in the steel for live loads and impact are calculated with section moduli of the composite section when $n = 10$ (Table 12.39*b*). Lateral bending stresses in the bottom flange are superimposed

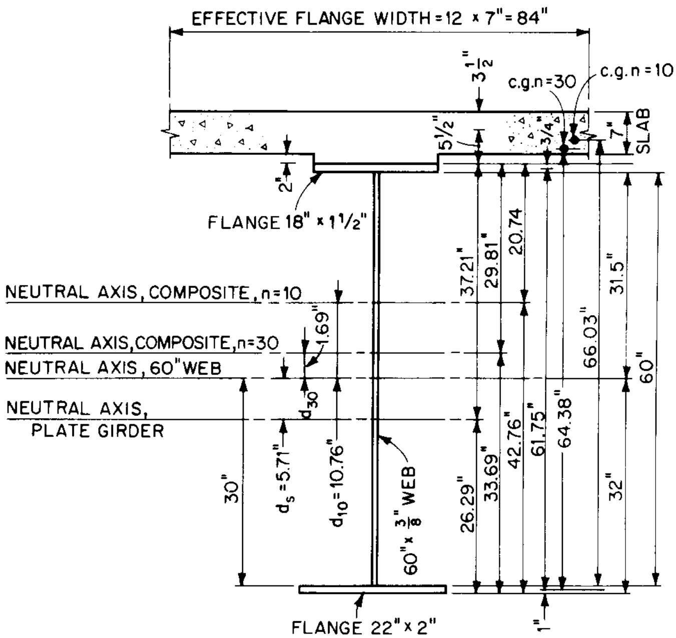


FIGURE 12.27 Cross section of composite plate girder at midspan.

TABLE 12.38 Steel Section for G_1 for Maximum Moment

Material	A	d	Ad	Ad^2	I_o	I
Top flange 18 × 1½	27.0	30.75	830	25,500		25,500
Web 60 × ⅜	22.5				6,750	6,750
Bottom flange 22 × 2	44.0	−31.0	−1,365	42,300		42,300
	93.5		−534			74,550
$d_s = -534/93.5 = -571$ in					$-5.71 \times 534 =$	−3,050
					$I_{NA} =$	71,500

Distance from neutral axis of steel section to:

Top of steel = 30 + 1.50 + 5.71 = 37.21 in

Bottom of steel = 30 + 2.00 − 5.71 = 26.29 in

Section moduli	
Top of steel	Bottom of steel
$S_{st} = 71{,}500/37.21 = 1{,}922 \text{ in}^3$	$S_{sb} = 71{,}500/26.29 = 2{,}720 \text{ in}^3$

on other stresses. Lateral bending moment in the top flange should be computed for load *DL*. For composite beams, lateral bending in the top flange under *SDL* and live load can be ignored.

The moments causing the lateral bending stresses can be computed from Eq. (12.36). For use in this calculation (Table 12.40), the centroid of the compression flange is located by taking the moment of the transformed area of the concrete slab about the centroid of the steel top flange. Thus, for the steel section alone, the distance between flange centroids is

$$h = 60 + \frac{2}{2} + \frac{1.5}{2} = 61.75 \text{ in}$$

For the composite section, $n = 30$

$$h = 61.75 + \frac{6.25 \times 19.6}{27.0 + 19.6} = 61.75 + 2.63 = 64.38 \text{ in}$$

For the composite section, $n = 10$,

$$h = 61.75 + \frac{6.25 \times 58.8}{27.0 + 58.8} = 61.75 + 4.28 = 66.03 \text{ in}$$

The section moduli of the top and bottom flanges about their central vertical axes are

$$S_{ft} = \frac{1.5(18)^2}{6} = 81 \text{ in}^3 \qquad S_{fb} = \frac{2(22)^2}{6} = 161.5 \text{ in}^3$$

Calculations of the steel stresses in G_1 are given in Table 12.41. The trial section is satisfactory.

Stresses in the concrete slab are determined with the section moduli of the composite section with $n = 30$ for *SDL* (Table 12.39*a*) and $n = 10$ for $LL + I$ (Table 12.39*b*). The calculation is given in Table 12.42.

TABLE 12.39 Composite Section for G_1 for Maximum Moment

Material	A	d	Ad	Ad^2	I_o	I
(*a*) For dead loads, $n = 30$						
Steel section	93.5		−534			74,550
Concrete 84 × 7/30	19.6	37.0	725	26,830	80	26,900
	113.1		191			101,450
$d_{s0} = 191/113.1 = 1.669$ in					$-1.69 \times 191 =$	−320
					$I_{NA} =$	101,130

Distance from neutral axis of composite section to:

Top of steel = 31.50 − 1.69 = 29.81 in

Bottom of steel = 32.00 + 1.69 = 33.69 in

Top of concrete = 29.81 + 2 + 7 = 38.81 in

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 101{,}130/29.81 = 3{,}400\ \text{in}^3$	$S_{sb} = 101{,}130/33.69 = 3{,}010\ \text{in}^3$	$S_c = 101{,}130/38.81 = 2{,}615\ \text{in}^3$

(*b*) For live loads, $n = 10$

Materials	A	d	Ad	Ad^2	I_o	I
Steel section	93.5		−534			74,500
Concrete 84 × 7/10	58.8	37.0	2,175	80,510	240	80,750
	152.3		1,641			155,300
$d_{10} = 1{,}641/152.3 = 10.76$					$-10.76 \times 1{,}641 =$	−17,700
					$I_{NA} =$	137,600

Distance from neutral axis of composite section to:

Top of steel = 31.50 − 10.76 = 20.74 in

Bottom of steel = 32.00 + 10.76 = 42.76 in

Top of concrete = 20.74 + 2 + 7 = 29.74 in

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 137{,}600/20.74 = 6{,}630\ \text{in}^3$	$S_{sb} = 137{,}600/42.76 = 3{,}210\ \text{in}^3$	$S_c = 137{,}600/29.74 = 4{,}630\ \text{in}^3$

TABLE 12.40 Maximum Lateral Bending Moments, ft-kips

DL: $M_L = -0.1 \times 12 \times 1{,}767(15)^2/(300 \times 61.75)$	$= -26$
SDL: $M_L = -0.1 \times 12 \times 838(15)^2/(3000 \times 64.38)$	$= -12$
LL + I: $M_L = -0.1 \times 12 \times 1{,}297(15)^2/(300 \times 66.03)$	$= -18$
Total:	-56

Therefore, the composite section for G_1 is satisfactory. Use for G_1 in the region of maximum moment the section shown in Fig. 12.27.

The procedure is the same for design of other sections and for the other stringers. For design of other elements, see Arts. 12.2 and 12.4. Fatigue design is similar to that for straight girders.

12.8 DECK PLATE-GIRDER BRIDGES WITH FLOORBEAMS

For long spans, use of fewer but deeper girders to span the long distance between supports becomes more efficient. With appropriately spaced stringers between the main girders of highway bridges, depth of concrete roadway slab can be kept to the minimum permitted, thus avoiding increase in dead load from the deck. Spans of the longitudinal stringers are kept short by supporting them on transverse floor-beams spanning between the girders. If spacing of the floorbeams is 25 ft or less, additional diaphragms or cross frames between the girders are not required.

This type of construction can be used with deck or through girders. Through girders carry the roadway between them. Their use generally is limited to locations where vertical clearances below the bridge are critical. Deck girders carry the roadway on the top flange. They generally are preferred for highway bridges where vertical clearances are not severely restricted, because the girders, being below the deck, do not obstruct the view from the deck. Structurally, deck girders have the advantage that the concrete deck is available for bracing the top flange of the girders and for composite action. Bracing of the bottom flange is accomplished with horizontal lateral bracing.

The design procedure for through plate girders with floor beams is described in Art. 12.10. Article 12.9 presents an example to indicate the design procedure for a deck girder bridge with floorbeams and stringers. In general, design of the stringers is much like that for a stringer bridge (Art. 12.2), and design of the girders is much like that for the girders of a multigirder bridge (Art. 12.4). In the following example, however, the stringers and girders are not designed for composite action. See also Art. 12.3.

TABLE 12.41 Steel Stresses in G_1, ksi

Top of steel (compression)		Bottom of steel (tension)	
DL: $f_b = 1.767 \times 12/1{,}992$	$= 11.03$	$f_b = 1{,}767 \times 12/2{,}720$	$= 7.79$
SLD: $f_b = 838 \times 12/3{,}400$	$= 2.95$	$f_b = 838 \times 12/3{,}010$	$= 3.34$
LL + I: $f_b = 1{,}297 \times 12/6{,}630$	$= 2.34$	$f_b = 1{,}297 \times 12/3{,}210$	$= 4.85$
L: $f_b = 26 \times 12/81$	$= 3.85$	$f_b = 56 \times 12/161.5$	$= 4.16$
Total:	$20.17 \approx 20$		$20.14 \approx 20$

TABLE 12.42 Stresses in G_1 at Top of Concrete, ksi

SDL:	$f_c = 838 \times 12/(2{,}615 \times 30)$	$= 0.13$
LL + I:	$f_c = 1{,}297 \times 12/(4{,}630 \times 10)$	$= \underline{0.34}$
Total:		$0.47 < 1.2$

12.9 EXAMPLE—ALLOWABLE-STRESS DESIGN OF DECK PLATE-GIRDER BRIDGE WITH FLOORBEAMS

Two simply supported, welded, deck plate girders carry the four lanes of a highway bridge on a 137.5-ft span. The girders are spaced 35 ft c to c. Loads are distributed to the girders by longitudinal stringers and floorbeams (Fig. 12.28). The typical cross section in Fig. 12.29 shows a 48-ft roadway flanked by 3-ft-wide safety walks. Grade 50 steel is to be used for the girders and Grade 36 for stringers, floorbeams, and other components. Concrete to be used for the deck is class A, with 28-day strength $f'_c = 4{,}000$ psi and allowable compressive stress $f_c = 1{,}400$ psi. Appropriate design criteria given in Sec. 11 will be used for this structure.

12.9.1 Design of Concrete Slab

The slab is designed, to span transversely between stringers, in the same way as for rolled-beam stringers (Art. 12.2). A 7.5-in thick concrete slab will be used.

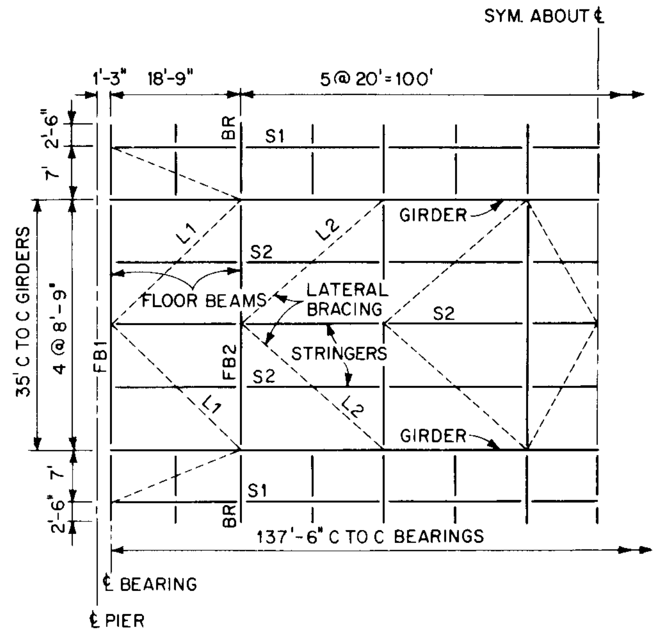


FIGURE 12.28 Framing plan for four-lane highway bridge with deck plate girders.

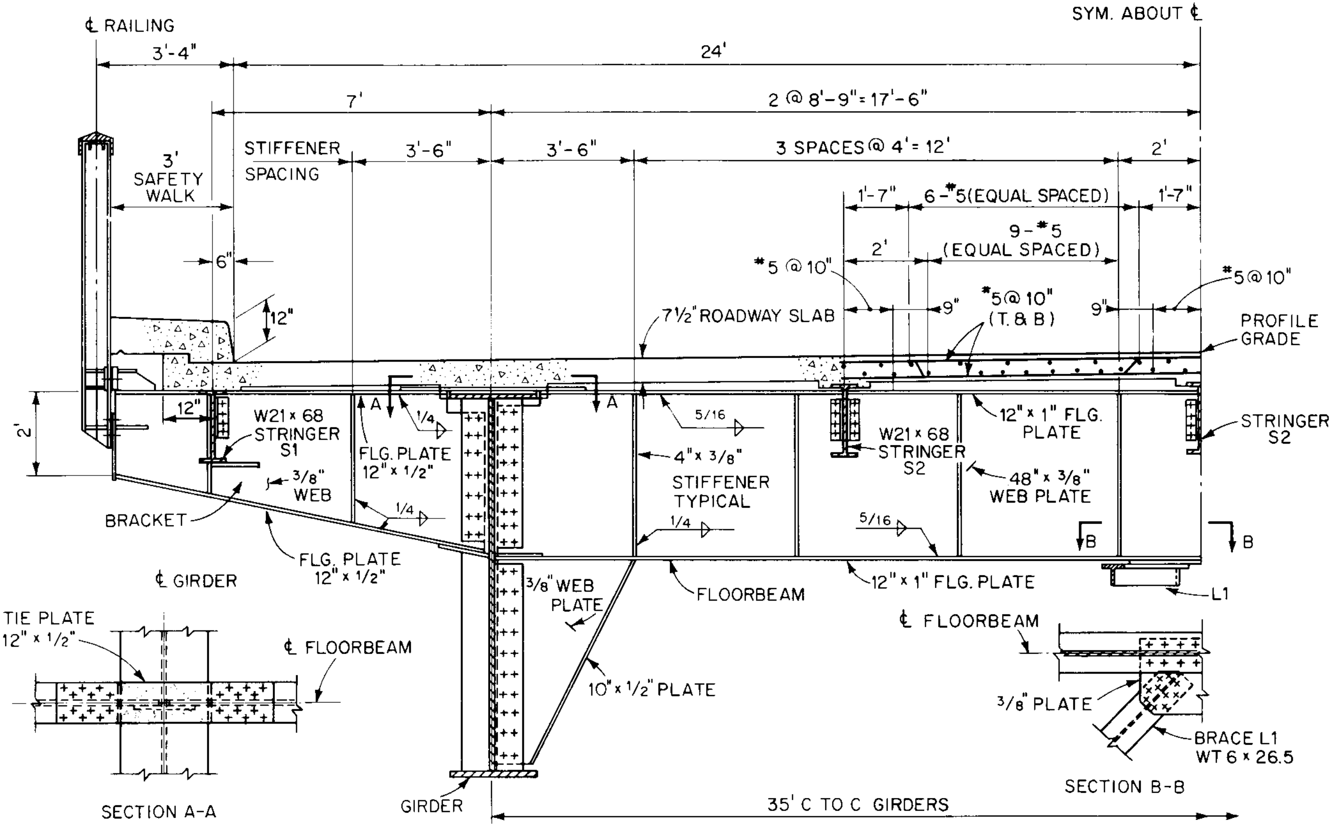


FIGURE 12.29 Typical cross section of deck-girder bridge at a floorbeam.

12.9.2 Design of Interior Stringer

Spacing of interior stringers c to c is 8.75 ft. Simply supported, a typical stringer S2 spans 20 ft. Table 12.43 lists the dead loads on S2. Maximum dead-load moment occurs at midspan and equals

$$M_{DL} = \frac{0.923(20)^2}{8} = 46.1 \text{ ft-kips}$$

Maximum dead-load shear occurs at the supports and equals

$$V_{DL} = \frac{0.923 \times 20}{2} = 9.2 \text{ kips}$$

The live load distributed to the stringer with spacing $S = 8.75$ ft is

$$\frac{S}{5.5} = \frac{8.75}{5.5} = 1.59 \text{ wheel loads} = 0.795 \text{ axle loads}$$

Maximum moment induced in a 20-ft span by a standard HS20 truck is 160 ft-kips. Hence, the maximum live-load moment in a stringer is

$$M_{LL} = 0.795 \times 160 = 127.2 \text{ ft-kips}$$

Maximum shear caused by the truck is 41.6 kips. Consequently, maximum live-load shear in the stringer is

$$V_{LL} = 0.795 \times 41.6 = 33.0 \text{ kips}$$

Impact is taken as 30% of live-load stress, because

$$I = \frac{50}{L + 125} = \frac{50}{20 + 125} = 0.35 > 0.30$$

So the maximum moment due to impact is

$$M_I = 0.30 \times 127.2 = 38.1 \text{ ft-kips}$$

and the maximum shear due to impact is

$$V_I = 0.30 \times 33.0 = 9.9 \text{ kips}$$

Maximum moments and shears in S2 are summarized in Table 12.44.

With an allowable bending stress $F_b = 20$ ksi for a stringer of Grade 36 steel, the section modulus required is

TABLE 12.43 Dead Load on S2, kips per ft

Slab: $0.150 \times 8.75 \times 7.5/12 =$	0.82
Haunch—assume:	0.033
Stringer—assume:	0.068
DL per stringer:	0.923

TABLE 12.44 Maximum Moments and Shears in S2

	DL	*LL*	*I*	Total
Moments, ft-kips	46.1	127.2	38.1	211.4
Shears, kips	9.2	33.0	9.9	52.1

$$S = \frac{M}{F_b} = \frac{211.4 \times 12}{20} = 127 \text{ in}^3$$

With an allowable shear stress $F_v = 12$ ksi, the web area required is

$$A_w = \frac{52.1}{12} = 4.33 \text{ in}^2$$

Use a W21 × 68. It provides a section modulus of 139.9 in^3 and a web area of 0.43 × 21.13 = 9.1 in^2.

12.9.3 Design of an Exterior Stringer

Simply supported, S1 spans 20 ft. It carries sidewalk as well as truck loads (Fig. 12.28). Dead loads are apportioned between S1 and the girder, 7 ft away, by treating the slab as simply supported at the girder. Table 12.45 lists the dead loads on S1.

Maximum dead-load moment occurs at midspan and equals

$$M_{DL} = \frac{1.15(20)^2}{8} = 57.5 \text{ ft-kips}$$

Maximum dead-load shear occurs at the supports and equals

$$V_{DL} = \frac{1.15 \times 20}{2} = 11.5 \text{ kips}$$

The live load from the roadway distributed to the exterior stringer with spacing $S = 7$ ft from the girder is

$$\frac{S}{4.0 + 0.25S} = \frac{7}{4.0 + 0.25 \times 7} = 1.22 \text{ wheel loads} = 0.61 \text{ axle loads}$$

Maximum moment induced in a 20-ft span by a standard HS20 truck load is 160 ft-kips. Hence, the maximum live-load moment in S1 is

TABLE 12.45 Dead Load on S1, kips per ft

Railing: 0.070 × 9.83/7	= 0.098
Sidewalk: 0.150 × 1 × 3 × 8/7	= 0.514
Slab: 0.150 × 8 × 7.5/12 × 4/7	= 0.428
Stringers, brackets, framing details—assume:	0.110
DL per stringer:	1.150

$$M_{LL} = 0.61 \times 160 = 97.7 \text{ ft-kips}$$

Maximum shear caused by the truck is 41.6 kips. Therefore, maximum live-load shear in S1 is

$$V_{LL} = 0.61 \times 41.6 = 25.4 \text{ kips}$$

Impact for a 20-ft span is 30% of live-load stress. Hence, maximum moment due to impact is

$$M_I = 97.7 \times 0.3 = 29.3 \text{ ft-kips}$$

and maximum shear due to impact is

$$V_I = 25.4 \times 0.3 = 7.6 \text{ kips}$$

Sidewalk loading at 85 psf on the 3-ft-wide sidewalk imposes a uniformly distributed load w_{SLL} on the stringer. With the slab assumed simply supported at the girder,

$$w_{SLL} = \frac{0.085 \times 3 \times 8}{7} = 0.29 \text{ kip per ft}$$

This causes a maximum moment of

$$M_{SLL} = \frac{0.29(20)^2}{8} = 14.5 \text{ ft-kips}$$

and a maximum shear of

$$V_{SLL} = \frac{0.29 \times 20}{2} = 2.9 \text{ kips}$$

Maximum moments and shears in S1 are summarized in Table 12.46.

If the exterior stringer has at least the capacity of the interior stringers, the allowable stress may be increased 25% when the effects of sidewalk live load are combined with those from dead load, traffic live load, and impact. In this case, the moments and shears due to sidewalk live load are less than 25% of the moments and shears without that load. Hence, they may be ignored.

With an allowable bending stress $F_b = 20$ ksi for Grade 36 steel, the section modulus required for S1 is

$$S = \frac{M}{F_b} = \frac{184.5 \times 12}{20} = 111 \text{ in}^3$$

With an allowable shear stress $F_v = 12$ ksi, the web area required is

TABLE 12.46 Maximum Moments and Shears in S1

	DL	*LL*	*I*	Total
Moments, ft-kips	57.5	97.7	29.3	184.5
Shears, kips	11.5	25.4	7.6	44.5

$$A_w = \frac{44.5}{12} = 3.7 \text{ in}^2$$

Use a W21 × 68, as for S2.

12.9.4 Design of an Interior Floorbeam

Floorbeam FB2 is considered to be a simply supported beam with 35-ft span and symmetrical 9.5-ft brackets, or overhangs (Fig. 12.29). It carries a uniformly distributed dead load due to its own weight and that of a concrete haunch, assumed at 0.21 kip per ft. Also, FB2 carries a concentrated load from S1 of 2 × 11.5 = 23.0 kips and a concentrated load from each of three interior stringers S2 of 2 × 9.2 = 18.4 kips (Fig. 12.30).

Moments and Shears in Main Span. Because of the brackets, negative moments occur and reach a maximum at the supports. The maximum negative dead-load moment is

$$M_{DL} = -0.21\left(\frac{9.5}{2}\right)^2 - 23.0 \times 7 = -171 \text{ ft-kips}$$

The reaction at either support under the symmetrical dead load is

$$R_{DL} = \frac{3 \times 18.4}{2} + 23 + \frac{0.21 \times 54}{2} = 56.3 \text{ kips}$$

Maximum dead-load shear in the overhang is

$$V_{DL} = 23 + 0.21 \times 9.5 = 25.0 \text{ kips}$$

Hence, the maximum shear between girders is

$$V_{DL} = 56.3 - 25.0 = 31.3 \text{ kips}$$

Maximum positive dead-load moment occurs at midspan and equals

$$M_{DL} = 31.3 \times 17.5 - 18.4 \times 8.75 - \frac{0.21(17.5)^2}{2} - 171 = 184 \text{ ft-kips}$$

Maximum live-load stresses in the floorbeam occur when the center truck wheels pass over it (Fig. 12.31). In that position, the wheels impose on FB2 a load

$$W = 16 + \frac{16 \times 6}{20} + \frac{4 \times 6}{20} = 22 \text{ kips}$$

For maximum positive moment, trucks should be placed in the two central lanes, as close to midspan as permissible (Fig. 12.32). Then, the maximum moment is

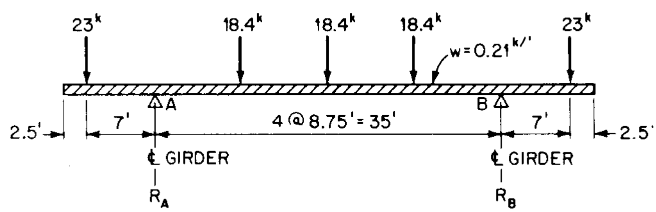


FIGURE 12.30 Dead loads on a floorbeam of the deck-girder bridge.

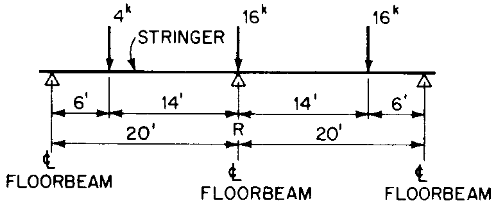


FIGURE 12.31 Positions of loads on a stringer for maximum live load on a floorbeam.

$$M_{LL} = 2 \times 22 \times 15.5 - 22 \times 6 = 550 \text{ ft-kips}$$

Maximum negative moment occurs at a support with a truck in the outside lane with a wheel 2 ft from the curb (Fig. 12.33). This moment equals

$$M_{LL} = -22 \times 4.5 = -99 \text{ ft-kips}$$

Maximum live-load shear between girders occurs at support A with three lanes closest to that support loaded, as indicated in Fig. 12.34. Because three lanes are loaded, the floorbeam need to be designed for only 90% of the resulting shear. The reaction at A is

$$R_{LL} = \frac{0.90 \times 22(39.5 + 33.5 + 27.5 + 21.5 + 15.5 + 9.5)}{35} = 83.2 \text{ kips}$$

Subtraction of the shear in the bracket for this loading gives the maximum liveload shear between girders:

$$V_{LL} = 83.2 - 0.9 \times 22 = 63.4 \text{ kips}$$

The maximum live-load shear in the overhang is produced by the loading in Fig. 12.33 and is V_{LL} in 22 kips.

Impact is taken as 30% of live-load stress, because

$$I = \frac{50}{L + 125} = \frac{50}{35 + 125} = 0.31 > 0.30$$

Sidewalk loading transmitted by exterior stringers S1 to the floorbeam equals $2 \times 2.9 = 5.8$ kips. This induces a shear in the overhang $V_{SLL} = 5.8$ kips. Also, it causes a reaction

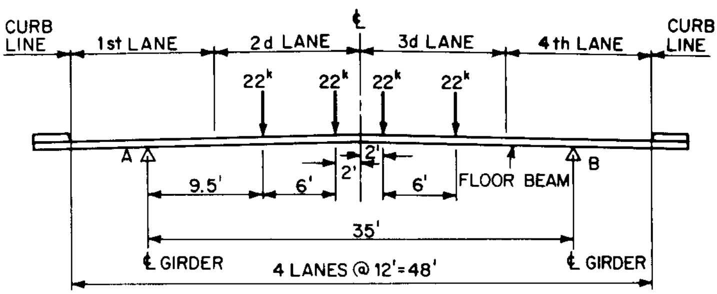


FIGURE 12.32 Positions of loads for maximum positive moment in a floorbeam.

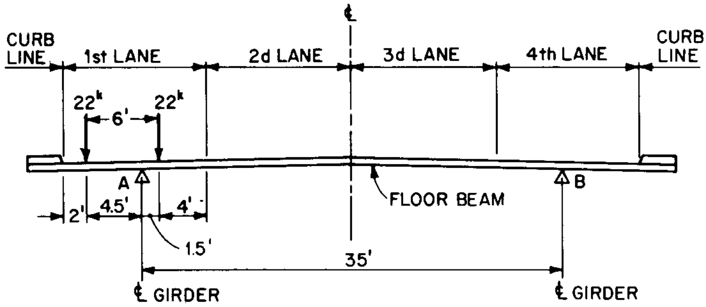


FIGURE 12.33 Positions of loads for maximum negative moment and maximum shear in the overhang of a floorbeam.

$$R_{SLL} = \frac{5.8 \times 42}{35} = 7.0 \text{ kips}$$

Subtraction of the overhang shear gives the maximum shear between girders

$$V_{SLL} = 7.0 - 5.8 = 1.2 \text{ kips}$$

Maximum negative moment due to the sidewalk live load is

$$M_{SLL} = -5.8 \times 7 = 41 \text{ ft-kips}$$

Results of preceding calculations are summarized in Table 12.47.

Main-Span Section. FB2 will be designed as a plate girder of Grade 36 steel. Assume a 48-in-deep web. If the floorbeam is not stiffened longitudinally, web thickness must be at least $t = D/170 = 48/170 = 0.283$ in. To satisfy the allowable shear stress of 12 ksi, with a maximum shear from Table 12.47 of 114.9 kips, web thickness should be at least $t = 114.9/(12 \times 48) = 0.20$ in. These requirements could be met with a $\frac{5}{16}$-in web, the minimum thickness required. But fewer stiffeners will be needed if a slightly thicker plate is selected. So use a $48 \times \frac{3}{8}$-in web.

Assume that the tension and compression flanges will be the same size and that each flange will have two holes for $\frac{7}{8}$ in-dia. high-strength bolts. To satisfy the allowable bending stress of 20 ksi, with a maximum moment of 899 ft-kips from Table 12.47, flange area should be about

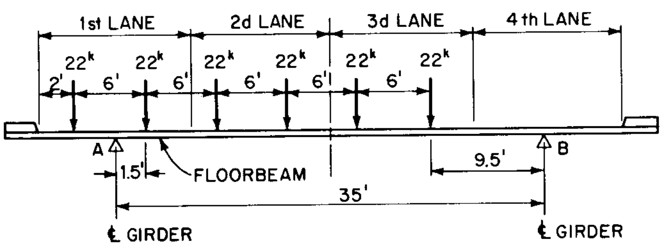


FIGURE 12.34 Position of loads for maximum shear at A in main span of floorbeam.

TABLE 12.47 Maximum Moments, Shears, and Reactions in Floorbeam FB2

	DL	*LL*	*I*	*SLL*	Total
Negative moments, ft-kips	−171	−99	−30	−41	−341
Positive moments, ft-kips	184	550	165	. . .	899
Shear in main span, kips	31.3	63.4	19.0	1.2	114.9
Shear in overhang, kips	25.0	22.0	6.6	5.8	59.4
Reaction, kips	56.3	83.2	25.0	7.0	171.5

$$A_f = \frac{899 \times 12}{20(48 + 1)} = 11 \text{ in}^2$$

With an allowance for the bolt holes, assume for each flange a plate 12 × 1 in. Width-thickness ratio of 12:1 for the compression flange is less than the 24:1 maximum and is satisfactory.

The trial section assumed is shown in Fig. 12.35. Moment of inertia and section modulus of the net section are calculated as shown in Table 12.48. Distance from neutral axis to top or bottom of the floorbeam is 25 in. Hence, the section modulus provided is

$$S_{net} = \frac{15{,}456}{25} = 618 \text{ in}^3$$

Maximum bending stress therefore is

$$f_b = \frac{899 \times 12}{618} = 17.5 < 20 \text{ ksi}$$

The section is satisfactory.

A check of the weight of the floorbeam is desirable to verify the assumptions made in dead-load calculations. Weight of slab haunch, beam, and details was assumed at 0.21 kip

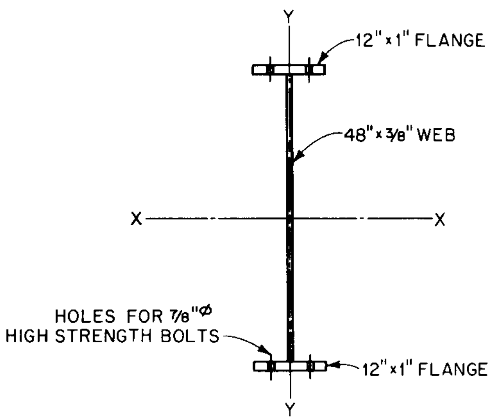


FIGURE 12.35 Cross section of floorbeam in main span.

TABLE 12.48 Moment of Inertia of Floorbeam FB2 at Midspan

Material	A	d	Ad^2 or I_o
2 flanges 12 × 1	24	24.5	14,400
Web 48 × ⅜	18		3,456
		I_g =	17,856
4 holes 1 × 1	4	24.5	−2,400
		I_{net} =	15,456 in^4

per ft. Average weight of haunch will be about 0.05 kip per ft. Thus, the assumed weight of floorbeam and details was about 0.16 kip per ft. If 8% of the weight is assumed in details, actual weight is $1.08(2 \times 40.8 + 61.2) = 154 < 160$ lb per ft assumed.

Flange-to-Web Welds. Each flange will be connected to the web by a fillet weld on opposite sides of the web. These welds must resist the horizontal shear between flange and web. The minimum size of weld permissible for the thickest plate at the connection usually determines the size of weld. In some cases, however, the size of weld may be determined by the maximum shear. In this example, shear does not govern, but the calculations are presented to illustrate the procedure.

The gross moment of inertia, 17,856 in^4, is used in computing the shear v, kips per in, between flange and web. From Table 12.47, maximum shear is 114.9 kips. Still needed is the static moment Q of the flange area:

$$Q = 12 \times 1 \times 24.5 = 294 \text{ in}^3$$

Then, the shear is

$$v = \frac{VQ}{I} = \frac{114.9 \times 294}{17{,}856} = 1.89 \text{ kips per in}$$

The allowable stress on the weld is not determined by fatigue. It is sufficient and necessary that the base metal in the flange be investigated for fatigue and the weld metal be checked for maximum shear stress. For fatigue, the stress category is B. On the assumption that the bridge is a nonredundant-load-path structure, the allowable stress range in FB2 for 500,000 cycles of loading is 23 ksi. The stress range due to live-load plus impact moments is $12[550 - (-99) + 165 - (-30)]/618 = 16.4 \text{ ksi} < 23$ ksi. The base metal is satisfactory for fatigue.

The allowable shear stress is $F_v = 0.27F_u = 0.27 \times 58 = 15.6$ ksi. Hence, the allowable load per weld is $15.6 \times 0.707 = 11.03$ kips per in, and for two welds, 22.06 kips per in. So the weld size required to resist the shear is $1.89/22.06 = 0.09$ in. The minimum size of weld permitted with the 1-in thick flange plate, however is ⁵⁄₁₆ in. Use two ⁵⁄₁₆-in welds at each flange.

Connection to Girder. Connection of the floorbeam to the girder is made with 18 A325 high-strength bolts. Each has a capacity in a slip-critical connection with Class A surface of 9.3 kips. For the maximum shear of 114.9 kips, the number of bolts required is $114.9/9.3 = 13$. The 18 provided are satisfactory.

Main-Span Stiffeners. Bearing stiffeners are not needed, because the web is braced at the supports by the connections with the girders. Whether intermediate transverse stiffeners are needed can be determined from Table 11.25. The compressive bending stress at the support is

$$f_b = \frac{341 \times 12 \times 25}{17{,}856} = 5.73 \text{ ksi}$$

For a girder web with transverse stiffeners, the depth-thickness ratio should not exceed

$$\frac{D}{t} = \frac{730}{\sqrt{5.73}} = 304 > 170$$

Hence, web thickness should be at least $48/170 = 0.28 < 0.375$ in. Actual $D/t_w = 48/0.375 = 128 < 170$. The average shear stress at the support is

$$f_v = \frac{114.9}{18} = 6.38 \text{ ksi}$$

The limiting shear stress for the girder web without stiffeners is, from Table 11.25,

$$F_v = \left(\frac{270}{128}\right)^2 = 4.45 \text{ ksi} < 6.38 \text{ ksi}$$

Hence, web thickness for shear should be at least

$$t_w = \frac{48}{270}\sqrt{6.38} = 0.45 \text{ in}$$

This is larger than the 3⁄8-in web thickness assumed. Therefore, intermediate transverse stiffeners are required. (A change in web thickness from 3⁄8 to 7⁄16 in would eliminate the need for the stiffeners.)

Stiffener spacing is determined by the shear stress computed from Eq. (11.25*a*). Assume that the stiffener spacing $d_o = 48$ in = the web depth D. Hence, $d_o/D = 1$. From Eq. (11.24*d*), for use in Eq. (11.25*a*), $k = 5(1 + 1^2) = 10$ and $\sqrt{k/F_y} = \sqrt{10/36} = 0.527$. Since $D/t_w = 128$, C in Eq. (11.24*a*) is determined by the parameter $128/0.527 = 243 > 237$. Hence, C is given by Eq. (11.24*c*):

$$C = \frac{45{,}000k}{(D/t_w)^2 F_y} = \frac{45{,}000 \times 10}{128^2 \times 36} = 0.763$$

From Eq. (11.25*a*), the maximum allowable shear for $d_o = 48$ in is

$$F'_v = F_v \left[C + \frac{0.87(1 - C)}{\sqrt{1 + (d_o/D)^2}}\right]$$

$$= 12\left[0.763 + \frac{0.87(1 - 0.763)}{\sqrt{1 + 1^2}}\right] = 10.9 > 6.38 \text{ ksi}$$

Since the allowable stress is larger than the computed stress, the stiffeners can be spaced 48 in apart. (Because of the brackets, the floorbeam can be considered continuous at the supports. Thus, the stiffener spacing need not be half the calculated spacing, as would be required for the first two stiffeners at simple supports.) Stiffener locations are shown in Fig. 12.29.

Stiffeners may be placed in pairs and welded on each side of the web with two ¼-in welds. Moment of inertia provided by each pair must satisfy Eq. (11.21). with J as given by Eq. (11.22).

$$J = 2.5\left(\frac{48}{48}\right)^2 - 2 = 0.5$$

$$I = 48(3/8)^3 0.5 = 1.27 \text{ in}^4$$

Use 4 × ⅜-in stiffeners. They satisfy minimum requirements for thickness of projection from the web and provide a moment of inertia

$$I = 0.375(2 \times 4 + 0.375)^3/12 = 18.36 > 1.27 \text{ in}^4$$

12.9.5 Design of Floorbeam Bracket

The floorbeam brackets are designed next. They can be tapered, because the rapid decrease in bending stress from the girder outward permits a corresponding reduction in web depth. To ensure adequate section throughout, the brackets are tapered from the 48-in depth of the floorbeam main span to 2 ft at the outer end (Fig. 12.29).

Splice at Girder. Bracket flanges are made the same size as the plate required for the moment splice to the main span. This plate is assumed to carry the full maximum negative moment of −341 ft-kips. With an allowable bending stress of 20 ksi, the splice plates then should have an area of at least

$$A_f = \frac{341 \times 12}{48.5 \times 20} = 4.2 \text{ in}^2$$

Use a 12 × ½-in plate, with gross area of 6 in^2. After deduction of two holes for ⅞-in dia bolts, it provides a net area of $6 - 2 \times 1 \times ½ = 5$ in^2. Hence, the bracket flanges also are 12 × ½-in plates. Use minimum-size ¼-in flange-to-web fillet welds.

The number of bolts required in the splice is determined by whichever is larger, 75% of the strength of the splice plate or the average of the calculated stress and strength of plate. The calculated stress is $20 \times 4.2/5 = 16.8$ ksi. The average stress is $(20 + 16.8)/2 = 18.4$ ksi. This governs, because 75% of 20 ksi is 15 ksi < 18.4. For A325 ⅞-in bolts with a capacity of 9.3 kips (slip-critical, Class A surface), the number of bolts needed is

$$n = \frac{18.4 \times 5}{9.3} = 10$$

Use 12 bolts.

Connection to Girder. The connection of each bracket to a girder must carry a shear of 59.4 kips. The number of bolts required is

$$n = \frac{59.4}{9.3} = 7$$

Use at least 8 bolts.

Bracket Stiffeners. Stiffener spacing on the brackets generally should not exceed the web depth. Locations of the stiffeners are shown in Fig. 12.29. Use pairs of 4 × ⅜-in plates, as in the main span, with ¼-in fillet welds.

Check of Bracket Section. Bending and shear stresses at an intermediate point on the bracket should be checked to ensure that, because of the reduction in depth, allowable stresses are not exceeded. For the purpose, a section midway between stringer S1 and the girder is selected. Depth of web there is $48 - 3.5(48 - 24)/9.5 = 39.15$ in. The dead load consists of 0.16 kip per ft from weight of bracket, 0.05 kip per ft from weight of concrete haunch, and 23 kips from the stringers. Thus, the dead-load moment is

$$M_{DL} = -\frac{0.16(6)^2}{2} - \frac{0.05(4.5)^2}{2} - 23 \times 3.5 = -83.9 \text{ ft-kips}$$

The dead-load shear is

$$V_{DL} = 0.16 \times 6 + 0.05 \times 4.5 + 23 = 24.2 \text{ kips}$$

Live load is 22 kips 1 ft from the section. Hence, the live-load and impact moments are

$$M_{LL} = -22 \times 1 = -22 \text{ ft-kips} \qquad M_I = -0.3 \times 22 = -6.6 \text{ ft-kips}$$

Live-load and impact shears are

$$V_{LL} = 22 \text{ kips} \qquad V_I = 0.3 \times 22 = 6.6 \text{ kips}$$

Moments and shears due to sidewalk live load are

$$M_{SLL} = -5.8 \times 3.5 = -20.3 \text{ ft-kips} \qquad V_{SLL} = 5.8 \text{ kips}$$

Hence, the total moments and shears at the section are

$$M = -132.8 \text{ ft-kips} \qquad V = 58.6 \text{ kips}$$

Shear stress in the web is

$$f_v = \frac{58.6}{39.15 \times 0.375} = 4.0 < 12 \text{ ksi}$$

The moment of inertia of the section is

$$I = 2 \times 6 \left(\frac{40.15}{2}\right)^2 + \frac{0.375(39.15)^3}{12} = 6{,}720 \text{ in}^4$$

and the section modulus is

$$S = \frac{6{,}720}{20.08} = 334 \text{ in}^3$$

So the maximum bending stress at the section is

$$f_b = \frac{132.8 \times 12}{334} = 4.8 < 20 \text{ ksi}$$

Therefore, the bracket section is satisfactory.

12.9.6 Design of a Girder Supporting Floorbeams

The girders will be made of Grade 50 steel. Simply supported, they span 137.5 ft, but have a loaded length of 140 ft. They will be made identical.

Loading. Most of the load carried by each girder is transmitted to it by the floorbeams as concentrated loads. Computations are simpler, however, if the floorbeams are ignored and then the girder treated as if it received loads only from the slab. Moments and shears computed with this assumption are sufficiently accurate for design purposes because of the relatively close spacing of the floorbeams. Thus, the dead load on the girders may be considered uniformly distributed (Table 12.49).

Sidewalk live load, because the span exceeds 100 ft, is determined from a formula, with loaded length of sidewalk $L = 140$ ft and sidewalk width $W = 3$ ft:

$$p = \frac{(0.03 + 3/L)(55 - W)}{50} = \frac{(0.03 + 3/140)(55 - 3)}{50}$$
$$= 0.0535 \text{ kip per ft}^2$$

Thus, the live load from the 3-ft sidewalk is

$$w_{SLL} = 0.0535 \times 3 = 0.160 \text{ kip per ft}$$

Live load, for maximum effect on a girder, should be placed as indicated in Fig. 12.36. Because of load reductions permitted in accordance with number of lanes of traffic loaded, the number of lanes to be loaded is determined by trial. Let W = wheel load, kips. Then, if two lanes are loaded, with no reduction permitted, the load P, kips, distributed to the girder is

$$P_2 = \frac{(39.5 + 33.5 + 27.5 + 21.5)W}{35} = \frac{122W}{35} = 3.48W$$

If three lanes are loaded, with 10% reduction,

$$P_3 = \frac{0.9(122 + 15.5 + 0.5)W}{35} = \frac{132.3W}{35} = 3.78W > P_2$$

And if all four lanes are loaded, with 25% reduction,

$$P_4 = \frac{0.75(147 + 3.5 - 2.5)W}{35} = \frac{111W}{35} = 3.17W < P_3$$

Therefore, loading in three lanes governs. The girder receives 3.78 wheel loads, or 1.89 axle loads.

Impact for loading over the whole span is taken as the following fraction of live-load stress:

TABLE 12.49 Dead Load on Girder, kips per ft

Railing:	0.07
Sidewalk: $0.150 \times 1 \times 3$	= 0.45
Slab: $0.150 \times 27 \times 7.5/12$	= 2.53
Floorbeams and stringers:	0.40
Girder—assume:	0.60
Lateral bracing—assume:	0.10
Utilities and miscellaneous:	0.10
DL per girder:	4.25

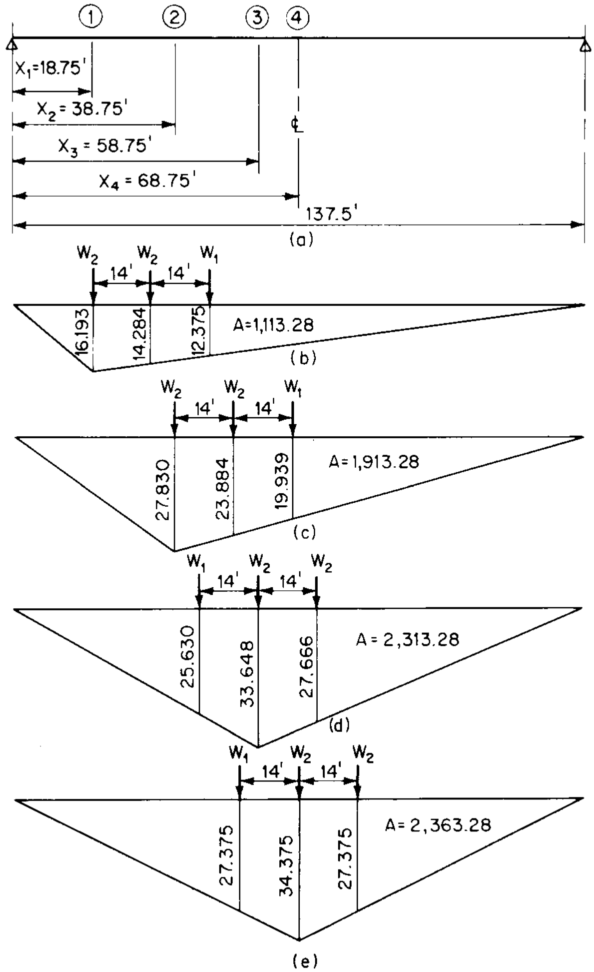


FIGURE 12.36 Moment influence lines for deck girder. (*a*) Location of four points on the girder for which influence diagrams are drawn. (*b*) Diagram for point 1. (*c*) Diagram for 2. (*d*) Diagram for 3. (*e*) Diagram for 4.

$$I = \frac{50}{L + 125} = \frac{50}{137.5 + 125} = 0.19$$

Moments. Curves for maximum moments at points along the span will be drawn by plotting maximum moments at midspan and at each floorbeam (points 1 to 4 in Fig. 12.36*a*). These moments are calculated with the aid of influence lines drawn for moment at these points (Fig. 12.36*b* to *e*).

Dead-load moments are obtained by multiplying the uniform load w_{DL} = 4.25 kips per ft by the area A of the appropriate influence diagram. Moments due to sidewalk live loading are similarly calculated with uniform load w_{SLL} = 0.16 kip per ft. Dead-load moments are summarized in Table 12.50.

TABLE 12.50 Dead-Load Moments and Sidewalk Live-Load Moments, ft-kips

	Distance from support, ft			
	18.75	38.75	58.75	68.75
Influence area	1,113.3	1,913.3	2,313.3	2,363.3
$M_{DL} = Aw_{DL}$	4,732	8,132	9,832	10,044
$M_{SLL} = Aw_{SLL}$	178	306	370	378

Maximum live-load moments are produced by truck loading on a 137.5-ft span. Since the girder receives 1.89 axle loads, it is subjected at 14-ft intervals to moving concentrated loads:

$$\text{Two } W_2 = 1.89 \times 32 = 60.48 \text{ kips} \qquad W_1 = 1.89 \times 8 = 15.12 \text{ kips}$$

For maximum moment at a point along the span, one load W_2 is placed at the point (Fig. 12.36*b* to *e*). The maximum moment then is the sum of the products of each load by the corresponding ordinate of the applicable influence diagram. Impact moments are 19% of the live-load moments. Table 12.51 summarizes maximum live-load and impact moments.

Total maximum moments are given and the curve of maximum moments (moment envelope) is plotted in Fig. 12.37.

Reaction. Maximum reaction occurs with full load over the entire span. For dead load, with $w_{DL} = 4.25$ kips per ft,

$$R_{DL} = \frac{4.25 \times 140}{2} = 297.5 \text{ kips}$$

For sidewalk live load, with $w_{SLL} = 0.16$ kip per ft,

$$R_{SLL} = \frac{0.16 \times 140}{2} = 11.2 \text{ kips}$$

Lane loading governs for live load. For maximum reaction and shear, the uniform load of 0.64 kip per ft should cover the entire span and the 26-kip concentrated load should be placed at the support, in each design lane.

TABLE 12.51 Maximum Live-Load and Impact Moments, ft-kips

	Distance from support, ft			
	18.75	38.75	58.75	68.75
M_{LL}	2,030	3,429	4,096	4,149
M_I	386	651	778	788

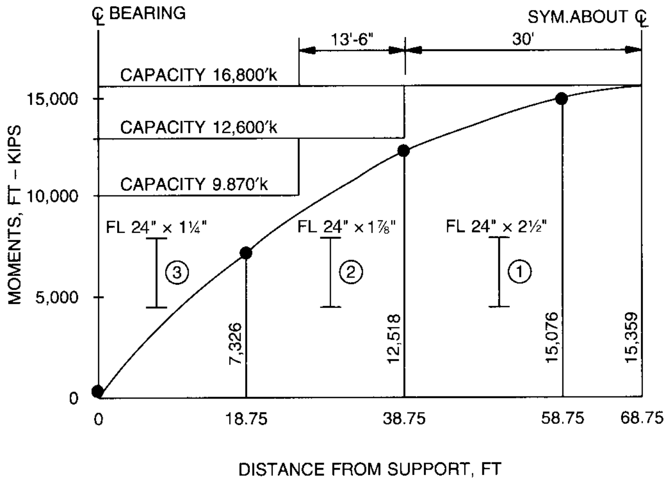


FIGURE 12.37 Moment diagram for deck girder and capacities of various sections.

$$R_{LL} = 1.89\left(26 + \frac{0.64 \times 140}{2}\right) = 134 \text{ kips}$$

$$R_I = 0.19 \times 134 = 25.4 \text{ kips}$$

The total maximum reaction is $R = 468.1$ kips, say 470 kips.

Shears. Maximum live-load shears at floorbeam locations occur with truck loading between the beam and the far support. A heavy wheel should be at the beam in each design lane. The shears are readily computed with influence diagrams. For example, the influence line for shear at point 1 (Fig. 12.38*a*) is shown in Fig. 12.38*b*. Dead-load shear is obtained as the product of the uniform dead load $w_{DL} = 4.25$ kips per ft by the area of the complete influence diagram.

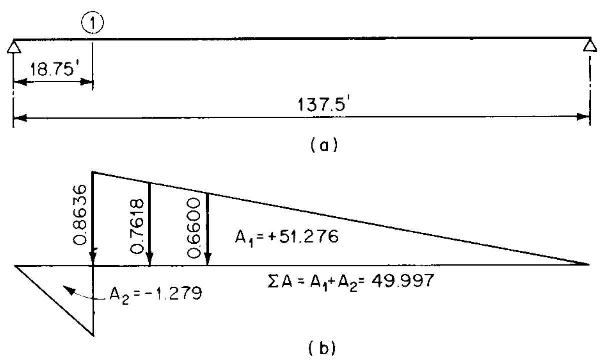


FIGURE 12.38 (*a*) Location of point 1 on girder. (*b*) Influence diagram for shear at point 1.

$$V_{DL} = 4.25(51.276 - 1.279) = 213 \text{ kips}$$

Sidewalk live-load shear is the product of the load $w_{SLL} = 0.16$ kip per ft and the larger of the positive or negative areas of the influence diagram.

$$V_{SLL} = 0.16 \times 51.276 = 8 \text{ kips}$$

Maximum live-load shear is the sum of the products of each load by the corresponding ordinate of the influence diagram (Fig. 12.38*b*).

$$V_{LL} = 60.48 \times 0.8636 + 60.48 \times 0.7618 + 15.12 \times 0.6600 = 108 \text{ kips}$$

The loaded length for impact is $137.5 - 18.75 = 118.75$ ft.

$$V_I = \frac{50}{118.75 + 125} 108 = 0.205 \times 108 = 22 \text{ kips}$$

Total maximum shear $V_1 = 351$ kips (Table 12.52).

Shears at other points are computed in the same way and are listed in Table 12.52.

Web Size. Minimum depth-span ratio for a girder is 1:25. Greater economy and a stiffer member are obtained, however, with a deeper member when clearances permit. In this example, the web is made 110 in deep, $1/15$ of the span. With an allowable stress of 17 ksi for thin Grade 50 steel, the web thickness required for shear is

$$t = \frac{470}{17 \times 110} = 0.25 \text{ in}$$

Without a longitudinal stiffener, according to Table 10.15 thickness must be at least

$$t = \frac{110\sqrt{50}}{990} = 0.8 \text{ in, say } 13/16 \text{ in}$$

Even with a longitudinal stiffener, however, to prevent buckling, web thickness, from Table 11.25, must be at least

$$t = \frac{110\sqrt{50}}{1{,}980} = 0.393 \text{ in, say } 7/16 \text{ in}$$

though transverse stiffeners also are provided.

TABLE 12.52 Maximum Shear, kips

	Distance from support, ft			
	0	18.75	38.75	58.75
Dead load	298	213	127	42
Sidewalk live load	11	8	6	4
Live load	134	108	89	69
Impact	(At 0.19) 26	(At 0.205) 22	(At 0.223) 20	(At 0.245) 17
Total	470	351	242	132

If the web were made $^{13}\!/_{16}$ in thick, it would weigh 304 lb per ft. If it were $^{7}\!/_{16}$ in thick, it would weigh 164 lb per ft, 140 lb per ft less. Since the longitudinal stiffener may weigh less than 10 lb per ft, economy favors the thinner web. Use a 110 × $^{7}\!/_{16}$-in web with a longitudinal stiffener.

Flange Size at Midspan. For Grade 50 steel 4 in thick or less, $F_y = 50$ ksi and the allowable bending stress is 27 ksi. With a maximum moment at midspan, from Fig. 12.37, of 15,359 ft-kips, and distance between flange centroids of about 113 in, the required area of one flange is about

$$A_f = \frac{15{,}359 \times 12}{113 \times 27} = 60.4 \text{ in}^2$$

Assume a 24 × 2½-in plate for each flange. It provides an area of 60 in² and has a width-thickness ratio

$$b/t = 24/2.50 = 9.6$$

which is less than 20 permitted.

The trial section is shown in Fig. 12.39. Moment of inertia is calculated in Table 12.53. Distance from neutral axis to top or bottom of the girder is 57.50 in. Hence, the section modulus is

$$S = \frac{428{,}200}{57.50} = 7447$$

Maximum bending stress therefore is

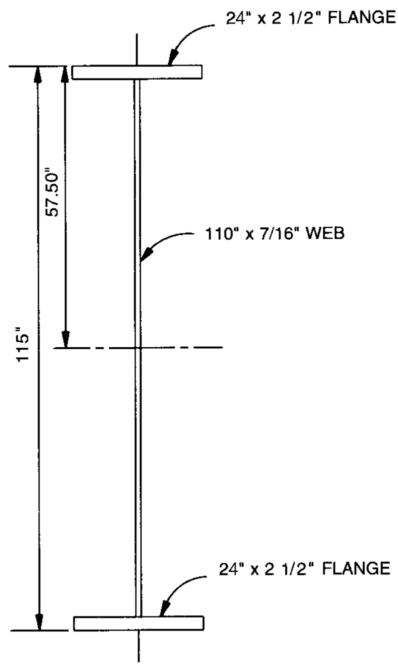


FIGURE 12.39 Cross section of deck girder at midspan.

TABLE 12.53 Moment of Inertia of Girder

Material	A	d	Ad^2 or I_o
2 flanges 24 × 2½	120.0	56.25	379,700
Web 110 × 7⁄16	48.1		48,500
			I = 428,200 in^4

$$f_b = \frac{15{,}359 \times 12}{7447} = 24.7 < 27 \text{ ksi}$$

The section is satisfactory. Moment capacity supplied is

$$M_C = \frac{27 \times 7{,}447}{12} = 16{,}760 \text{ ft-kips}$$

The flange thickness will be reduced between midspan and the supports, and the flange width will remain 24 in. Splices at the changes in thickness will be made with complete-penetration groove welds. For fatigue, the stress category at these splices is B. On the assumption that the structure supports a major highway with an ADTT less than 2500, the number of stress cycles of truck loading is 500,000. Since the bridge is supported by two simple-span girders and floorbeams, it is a non-redundant-path structure, and the allowable stress range is therefore 23 ksi at the splices.

Changes in Flange Size. At a sufficient distance from midspan, the bending moment decreases sufficiently to permit reducing the thickness of the flange plates to 1⅞ in. The moment of inertia of the section reduces to 330,150, and the section modulus to 5805. Thus, with 24 × 1⅞-in flange plates, the section has a moment capacity of

$$M_C = \frac{5805 \times 23}{12} = 11{,}100 \text{ ft-kips}$$

When this capacity is plotted in Fig. 12.37, the horizontal line representing it stays above the moment envelope until within 35 ft of midspan. Hence, flange size can be decreased before that point. Length of the 2½-in plate then is 75 ft. (See Fig. 12.40.)

At a greater distance from midspan, thickness of the flange plates can be reduced to a minimum of 1¼ in because $24/20 = 1.20$ in. The moment of inertia drops to 234,200, and the section modulus to 4,164. Consequently, with 24 × 1¼-in plates, the section has a moment capacity of

$$M_C = \frac{4{,}164 \times 23}{12} = 7{,}980 \text{ ft-kips}$$

When this capacity is plotted in Fig. 12.37, the horizontal line representing it stays above the moment envelope until within 49.5 ft of midspan. Economy should also be considered while determining a change in flange size. Total cost of a flange splice includes material and labor costs. Labor costs are a function of design, purchasing, and shop practices. For an economical splice, savings in material should exceed the labor associated with it. As a point of reference, an average of approximately 700-lb savings in flange material generally justifies the introduction of a shop splice in a flange. Using this as a guide, the length of 24-in × 1⅞ flange plates can be determined as follows:

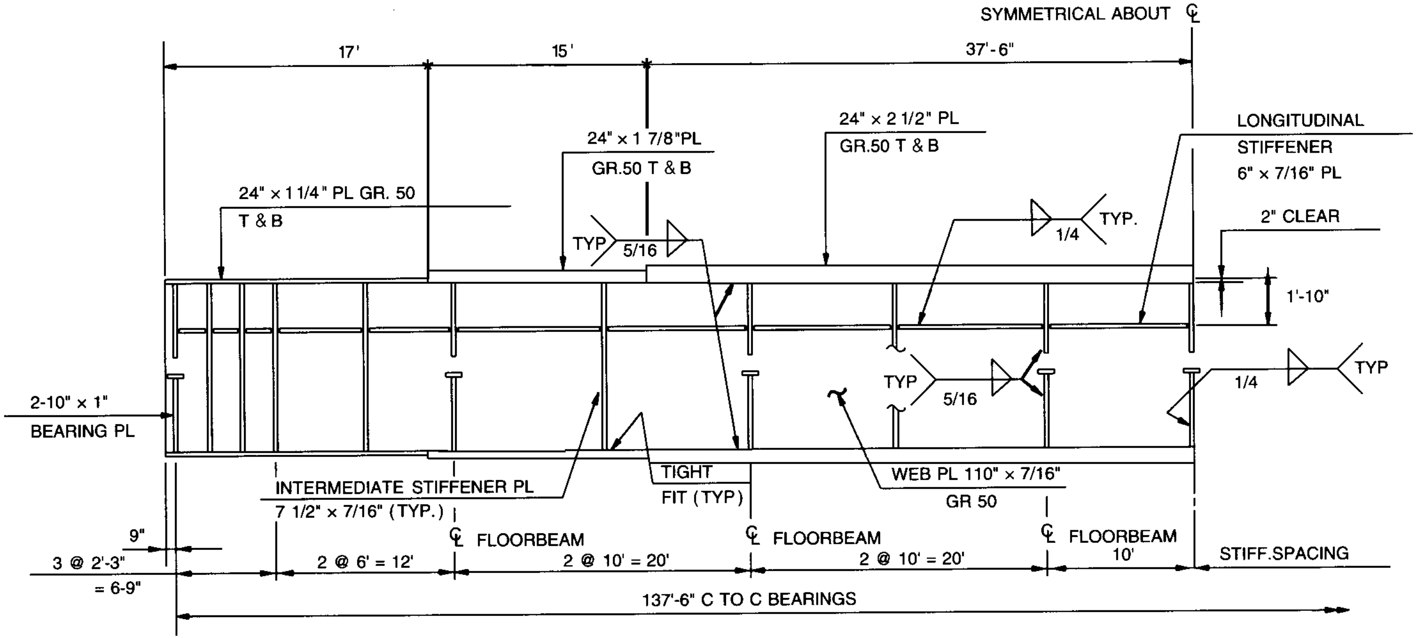


FIGURE 12.40 Details of deck plate girder for four-lane highway bridge.

$$L_1 = \frac{700}{(2\frac{1}{2} - 1\frac{7}{8}) \times 24 \times 490/144} = 13.7 \text{ ft (say, } L_1 = 15 \text{ ft)}$$

Then $75/2 + 15 = 52.50$ ft > 49.5 ft. The length of $24 \times 1\frac{1}{4}$-in plate which extends to the end of the girder is therefore $(137.50 + 2 \times 0.75 - 75.00 - 2 \times 15)/2 = 17$ ft. (Fig. 12.40).

Flange-to-Web Welds. Each flange will be connected to the web by a fillet weld on opposite sides of the web. These welds must resist the horizontal shear between flange and web. At the end section of the girder, for determination of the shear, the static moment is

$$Q = 24 \times 1.25 \times 55.63 = 1{,}669 \text{ in}^3$$

The shear stress then is

$$v = \frac{VQ}{I} = \frac{470 \times 1669}{234{,}200} = 3.35 \text{ kips per in}$$

The minimum size of fillet weld permissible, governed by the thickest plate at the section, is $\frac{5}{16}$ in. With an allowable shear stress $F_v = 0.27F_u = 0.27 \times 65 = 17.6$ ksi, the allowable load per weld $17.6 \times 0.707 = 12.44$ kips per in, and for two welds, 24.89 kips per in. Hence, the capacity of two $\frac{5}{16}$-in fillet welds is $24.89 \times \frac{5}{16} = 7.78$ kips per in > 3.35 kips per in. Use two $\frac{5}{16}$-in welds. (See also the design of fillet welds in Art. 12.9.4.)

Intermediate Transverse Stiffeners. Where required, a pair of transverse stiffeners of Grade 36 steel will be welded to the girder web. Minimum width of stiffener is $24/4 = 6.0$ in $>$ $(2 + 110/30 = 5.7$ in). Use a $7\frac{1}{2}$-in wide plate. Minimum thickness required is $\frac{7}{16}$ in. Try a pair of $7\frac{1}{2} \times \frac{7}{16}$-in stiffeners.

Maximum spacing of the transverse stiffeners can be computed from Eq. (11.25*a*). For the $110 \times \frac{7}{16}$-in girder web and a maximum shear at the support of 470 kips, the average shear stress is $470/48.1 = 9.77$ ksi. The web depth-thickness ratio $D/t_w = 110/(\frac{7}{16}) = 251$. Maximum spacing of stiffeners is limited to $110(270/251)^2 = 127$ in. Try a stiffener spacing $d_o = 80$ in. This provides a depth spacing ratio $D/d_o = 110/80 = 1.375$. From Eq. (11.24*d*), for use in Eq. (11.25*a*), $k = 5[1 + (1.375)^2] = 14.45$ and $\sqrt{k/F_y} = \sqrt{14.45/50} = 0.537$. Since $D/t_w = 251$, C in Eq. (11.24*a*) is determined by the parameter $251/0.537 = 467 > 237$. Hence, C is given by Eq. (11.24*c*):

$$C = \frac{45{,}000k}{(D/t_w)^2F_y} = \frac{45{,}000 \times 14.45}{251^2 \times 50} = 0.206$$

From Eq. (11.25*a*), the maximum allowable shear for $d_o = 80$ in is

$$F'_v = F_v \left[C + \frac{0.87(1 - C)}{\sqrt{1 + (d_o/D)^2}} \right]$$

$$= \frac{50}{3} \left[0.206 + \frac{0.87(1 - 0.206)}{\sqrt{1 + (80/110)^2}} \right] = 12.74 \text{ ksi} > 9.77 \text{ ksi}$$

Since the allowable stress is larger than the computed stress, the stiffeners may be spaced 80 in apart. The location of floorbeams. however, may make closer spacing preferable.

The AASHTO standard specifications limit the spacing of the first intermediate transverse stiffener to the smaller of $1.5D = 1.5 \times 110 = 165$ and the spacing for which the allowable shear stress in the end panel does not exceed

$$F_v = CF_y/3 = 0.206 \times 50/3 = 3.43 \text{ ksi} < 9.77 \text{ ksi}$$

Much closer spacing than 80 in is required near the supports. Try $d_o = 27$ in, for which $k = 88$ and $C > 1$. Hence, $F_v = 50/3 = 17$ ksi > 9.62 ksi. Spacing selected for intermediate transverse stiffeners between the supports and the first floorbeam is shown in Fig. 12.40.

At that beam, the shear stress is $f_v = 351/48.1 = 7.28$ ksi. Try a stiffener spacing $d_o = 10$ ft $= 120$ in, which is less than the 127-in limit. This provides $D/d_o = 0.917$, $k = 9.20$, and $C = 0.131$. The allowable shear for this spacing then is

$$F_v = \frac{50}{3}\left[0.131 + \frac{0.87(1 - 0.131)}{\sqrt{1 + (120/110)^2}}\right] = 10.69 \text{ ksi} > 7.28 \text{ ksi}$$

The 10-ft spacing is satisfactory. Actual spacing throughout the span is shown in Fig. 11.40.

The moment of inertia provided by each pair of stiffeners must satisfy Eq. (11.21), with J as given by Eq. (11.22).

$$J = 2.5\left(\frac{110}{27}\right)^2 - 2 = 39.5$$

$$I = 27(7/16)^3\, 39.5 = 89.3 \text{ in}^4$$

The moment of inertia furnished by a pair of 7½-in-wide stiffeners is

$$I = \frac{(7/16)(15.437)^3}{12} = 134 > 89.3 \text{ in}^4$$

Hence, the pair of $7\frac{1}{2} \times \frac{7}{16}$-in stiffeners is satisfactory.

Longitudinal Stiffener. One longitudinal stiffener of Grade 36 steel will be welded to the web. It should be placed with its centerline at a distance $110/5 = 22$ in below the bottom surface of the compression flange (Fig. 12.40).

Assume a 6-in wide stiffener. Then, by Eq. (11.28*b*), the thickness required is

$$t = \frac{6\sqrt{36}}{95.94} = 0.375 \text{ in, say } 7/16 \text{ in}$$

Moment of inertia furnished with respect to the edge in contact with the web is

$$I = \frac{0.437(6)^3}{3} = 31.5 \text{ in}^4$$

With transverse stiffeners spaced 120 in apart, the moment of inertia required by Eq. (11.28*a*), is

$$I_{min} = 110(0.437)^3\left[2.4\left(\frac{120}{110}\right)^2 - 0.13\right] = 25.1 < 31.5 \text{ in}^4$$

Therefore, use a $6 \times \frac{7}{16}$-in plate for the longitudinal stiffener. A $6 \times \frac{3}{8}$-in plate would also check.

Bearing Stiffeners. A pair of bearing stiffeners of Grade 50 steel is provided at each support. They are designed to transmit the 470 kip end reaction between bearing and girder.

Try 10×1-in plates. With provision for clearing the flange-to-web fillet weld, the effective width of each plate is $10 - 0.75 = 9.25$ in. The effective bearing area is $2 \times 1 \times 9.25 = 18.5$ in^2. Allowable bearing stress is 40 ksi. Actual bearing stress is

$$f_p = \frac{470}{18.5} = 25.4 < 40 \text{ ksi}$$

The width-thickness ratio of the assumed plate $b/t = 10/1 = 10$ satisfies Eq. (11.20), with $F_y = 50$ ksi.

$$\frac{b}{t} = \frac{69}{\sqrt{50}} = 9.75 < 10$$

The pair of stiffeners is designed as a column acting with a length of web equal to 18 times the web thickness, or 7.88 in. Area of the column is

$$2 \times 10 \times 1 + 7.88 \times \frac{7}{16} = 23.44 \text{ in}^2$$

Buckling is prevented by the floorbeam connecting to the stiffeners. Hence, the stress in the stiffeners must be less than the allowable compressive stress of 27 ksi and need not satisfy the column formulas. For the 470-kip reaction, the compressive stress is

$$f_a = \frac{470}{23.44} = 20.1 < 27 \text{ ksi}$$

Therefore, the pair of 10 × 1-in bearing stiffeners is satisfactory.

Stiffener-web welds must be capable of developing the entire reaction. With fillet welds on opposite sides of each stiffener, four welds are used. They extend the length of the stiffeners, from the bottom of the 48-in-deep floorbeam to the girder tension flange. Thus, total length of the welds is $4(110 - 48 - 7/16) = 241$ in. Average shear on the welds is

$$v = \frac{470}{241} = 1.95 \text{ kips per in}$$

Weld size required to carry this shear is, with allowable stress $F_v = 0.27F_o = 17.6$ ksi,

$$\frac{1.95}{0.707 \times 17.6} = 0.157 \text{ in}$$

This, however, is less than the 5⁄16-in minimum size of weld required for a 1-in-thick plate. Therefore, use 5⁄16-in fillet welds.

12.9.7 Design of Horizontal Lateral Bracing

Each girder flange is subjected to half the transverse wind load. The top flange is assisted by the concrete deck in resisting the load and requires no lateral bracing. The following illustrates design of lateral bracing for the bottom.

Figure 12.41 shows the layout of the lateral truss system, which lies in a plane at the bottom of the floorbeams. The girders comprise the chords of the truss, and the floorbeams the transverse members, or posts. The truss must be designed to resist a wind load of 50 psf, but not less than 300 lb per lin ft, on the exposed area. The wind is considered a uniformly distributed, moving load acting perpendicular to the girders and reversible in direction.

The uniform load on the girder for an exposed depth of 12.14 ft (Table 12.54) is

$$w = 0.050 \times 12.14 = 0.61 \text{ kip per ft}$$

It is resolved into a concentrated load at each panel point (Fig. 12.41):

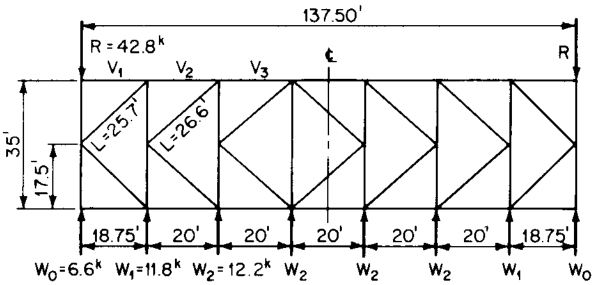


FIGURE 12.41 Lateral bracing system for deck-girder bridge.

$$W_2 = 0.61 \times 20 = 12.2 \text{ kips}$$

$$W_1 = \frac{0.61(20 + 18.75)}{2} = 11.8 \text{ kips}$$

$$W_0 = 0.61 \left(\frac{18.75}{2} + 1.5\right) = 6.6 \text{ kips}$$

The reaction at each support is

$$R = 2 \times 12.2 + 11.8 + 6.6 = 42.8 \text{ kips}$$

With the wind considered a moving load, maximum shear in each panel is:

$$V_1 = 42.8 - 6.6 = 36.2 \text{ kips}$$

$$V_2 = 36.2 - 11.8 \times \frac{118.75}{137.5} = 25.9 \text{ kips}$$

$$V_3 = 25.9 - 12.2 \times \frac{98.75}{137.5} = 17.1 \text{ kips}$$

$$V_4 = 17.1 - 12.2 \times \frac{78.75}{137.5} = 10.1 \text{ kips}$$

The shear is assumed to be shared equally by the two diagonals in each panel. Since the direction of the wind is reversible, the stress in each diagonal may be tension or compression. Design of the members is governed by compression.

The diagonals, being secondary compression members, are permitted a slenderness ratio L/r up to 140. (The effective length factor K is taken conservatively as unity.) For the end panel, the length c to c of connections is

TABLE 12.54 Exposed Area, ft^2 per lin ft

Railing	0.91
Slab	1.83
Girder	9.40
Total	12.14

$$L = 25.7 - 3 = 22.7 \text{ ft}$$

Hence, the radius of gyration should be at least $r = 22.7 \times 12/140 = 1.95$ in. Similarly, for interior panels, minimum $r = 23.6 \times 12/140 = 2.02$ in.

Assume for the diagonals a WT6 $\times$ 26.5 (Fig. 12.42). It has the following properties:

$$S_x = 3.54 \text{ in}^3 \qquad r_x = 1.51 \text{ in} \qquad r_y = 2.48 \text{ in} \qquad A = 7.80 \text{ in}^2 \qquad y = 1.02 \text{ in}$$

To permit the slenderness ratio about the vertical axis to govern the design, provide a vertical brace at midlength of each diagonal. The minimum slenderness ratio then is

$$\frac{L}{r_y} = \frac{22.7 \times 12}{2.48} = 110 < 140$$

Horizontal Buckling. For a column of Grade 36 steel with this slenderness ratio and with bolted ends, the allowable compressive stress is

$$F_a = 16.98 - 0.00053 \left(\frac{L}{r}\right)^2 = 16.98 - 0.00053(110)^2 = 10.57 \text{ ksi}$$

Maximum stress occurs in the end panel where wind shear is a maximum, 36.2 kips. Each diagonal is assumed to carry half this, 18.1 kips. Thus, it is subjected to an axial force of

$$F = \frac{18.1 \times 25.7}{17.5} = 26.6 \text{ kips}$$

This causes an average compressive stress in the diagonal of

$$f_a = \frac{26.6}{7.80} = 3.4 < 10.57 \text{ ksi}$$

Hence, the WT6 $\times$ 26.5 is adequate for resisting buckling in the horizontal direction.

Vertical Buckling. Because of the T shape of the WT, its end connections load it eccentrically. Therefore, the diagonal should be checked for combined axial plus bending stresses and buckling in the vertical direction. The eccentricity and c distance from the neutral axis to the top of the compression flange is 1.02 in (Fig. 12.42). The slenderness ratio for buckling in the vertical direction, with a conservative value of $K = 1.0$ and provision for a midlength brace, is

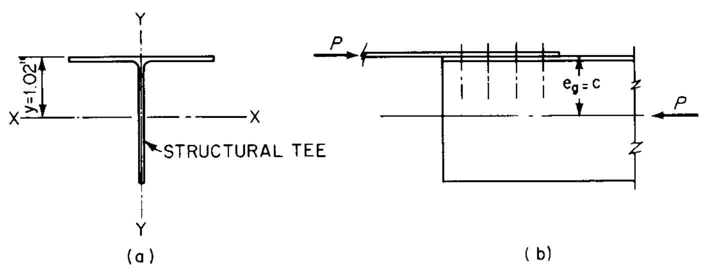


FIGURE 12.42 Diagonal brace. (*a*) Cross section. (*b*) Eccentric loading on the end connection of the diagonal.

$$\frac{L}{r_x} = \frac{12 \times 22.7/2}{1.51} = 90.2$$

Members subjected to combined axial compression and bending must satisfy

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - f_a/F'_{ex})F_{bx}} + \frac{C_{my} f_{by}}{(1 - f_a/F'_{ey})F_{by}} \leq 1.0$$

where $F'_e = \dfrac{\pi^2 E}{FS(K_b L_b / r_b)^2}$
$FS = 2.12$
C_m = coefficient defined in Art. 6.19.1 (1.0 is conservative)

The axial stress f_a is 3.4 ksi and the allowable stress is

$$F_a = 16.98 - 0.00053(KL/r_x)^2 = 16.98 - 0.00053(90.1)^2 = 12.7 \text{ ksi}$$

The bending stress f_b is $26.6 \times 1.02/3.54 = 7.66$ ksi. The allowable bending stress for Grade 36 steel in this case is $F_b = 20.0$ ksi.

$$F'_e = \frac{\pi^2(29{,}000)}{2.12(90.2)^2} = 16.6 \text{ ksi}$$

Substitution in the interaction equation gives

$$\frac{3.4}{12.7} + \frac{1.0 \times 7.66}{[1 - (3.4/16.6)]20.0} = 0.27 + 0.48 = 0.75 < 1.0\text{—OK}$$

Use the WT6 × 26.5 for all the diagonals.

Bracing Connections. End connections of the laterals are to be made with A325 ⅞-in-dia. high-strength bolts. These have a capacity of 9.3 kips in slip-critical connections with Class A surfaces. The number of bolts required is determined by whichever is larger, 75% of the strength of the diagonal or the average of the calculated stress and strength of the diagonal.

In the computation of the tensile strength of the T section, the effective area should be taken as the net area of the connected flange plus half the area of the outstanding web (Table 12.55). With an allowable stress of 20 ksi, the tensile capacity is

$$T = 5.71 \times 20 = 114 \text{ kips}$$

Compressive capacity with $F_a = 8.5$ ksi on the gross area is

$$C = 7.80 \times 8.5 = 66 < 114 \text{ kips}$$

Tensile capacity governs. Hence, the number of bolts required is determined by

TABLE 12.55 Net Area of Diagonal, in^2

Gross area:	7.80
Half web area: $-5.45 \times 0.345/2$ =	-0.94
Two holes: $-2 \times 1 \times 0.576$ =	-1.15
Net area:	5.71

$$0.75 \times 114 = 86 \text{ kips} > \left(\frac{26.6 + 114}{2} = 70 \text{ kips} \right)$$

and equals 86/9.3 = 9.2. Use ten ⅞-in high-strength bolts.

12.9.8 Expansion Shoes

Each girder transmits its end reactions to piers through one fixed and one expansion shoe. For a moderate climate, the latter should be designed to permit movements resulting from variations of temperature between +50 and −70°F, and to allow rotation of the girder ends under live loads. Both shoes are weldments, fabricated of Grade 36 steel. They must be capable of transmitting the maximum girder reaction of 470 kips and their own weight, assumed at 10 kips, or a total of 480 kips.

The expansion shoe incorporates a rocker, to permit the required movements, a sole plate attached to the girder bottom to transmit the reaction to the rocker, and a base plate, to distribute the load to the concrete pier (Fig. 12.43). AASHTO specifications require that the shoe permit a thermal movement of 1.25 in per 100 ft of span, or a total of $1.25 \times 140/100 = 1.75$ in. For the temperature range specified, the thermal movements expected are

$$\begin{aligned} &\text{Expansion: } 0.0000065 \times 50 \times 140 \times 12 = 0.547 \text{ in} \\ &\text{Shortening: } 0.0000065 \times 70 \times 140 \times 12 = \underline{0.763} \text{ in} \\ &\text{Total: } \qquad\qquad\qquad\qquad\qquad\qquad\quad 1.310 \text{ in} \end{aligned}$$

The maximum live-load deflection preferred by AASHTO specifications is 1/800 of the span, or $140 \times 12/800 = 2.1$ in. If this deflection occurred, and if the elastic curve is assumed parabolic, the end rotation under live load of the girder would be

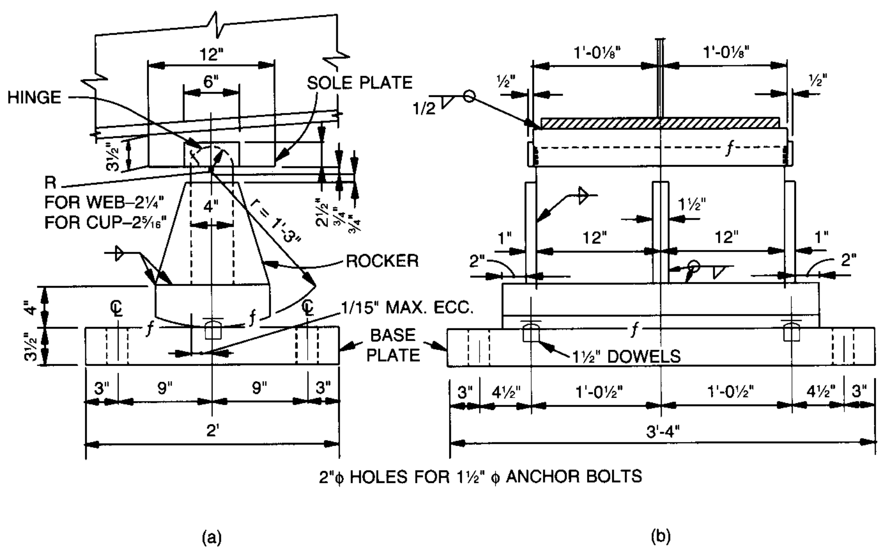


FIGURE 12.43 Expansion shoe. (*a*) Side view. (*b*) End view.

$$\theta = \frac{4 \times 2.1}{137.5 \times 12} = 0.005 \text{ radian}$$

The distance from the neutral axis to the center of curvature of the rocker is

$$R_r = 110/2 + 5 = 60 \text{ in}$$

With R_r as radius, live load causes an expansion of $60 \times 0.005 = 0.30$ in at each end of the girder. Thus, the rocker must be capable of accommodating a live-load expansion of $2 \times 0.3 = 0.60$ in.

Addition of this to the thermal expansion yields a total expansion of

$$e = 0.60 + 0.55 = 1.15 \text{ in}$$

Maximum shortening is 0.76 in. The 4-in web of the rocker (Fig. 12.43) permits movements up to $4/2 = 2$ in, expansion or contraction.

Rocker. The rocker has a radius of 15 in, diameter (d) of 30 in. From previous AASHTO specifications, the allowable bearing, for $25 \leq d \leq 125$, if Grade 36 steel is used, is

$$P = \frac{3\sqrt{d}(F_y - 13)}{20} = \frac{3\sqrt{30}(36 - 13)}{20} = 19 \text{ kips per in}$$

To carry the 480-kip load, length of bearing should be at least $480/19 = 25.3$ in. Since the base of the rocker incorporates two 1⅝-in-dia. holes for 1½-in-dia. pintles, the computed length should be increased to $25.3 + 2 \times 1.625 = 28.6$ in. Use 30 in (Fig. 12.43*b*).

Web and Hinge. The 4-in-thick rocker web rests on a curved steel plate with 4-in maximum thickness. Maximum eccentricity of loading on the web is $0.76 + 0.60 = 1.36$ in, less than half the web thickness. Therefore, the loading cannot cause bending in the curved plate.

The curved top of the web serves as a hinge (Fig. 12.43*a*), which bears against a cup in the sole plate. Thus, the compressive stress in the 24-in-long web equals the bearing stress in the hinge. The allowable bearing stress of 14 ksi for Grade 36 steel pins subject to rotation applies also to such hinges. The compressive stress in the web is

$$f_p = \frac{480}{4 \times 24} = 5.0 < 14 \text{ ksi}$$

Hence, the web and hinge are satisfactory.

Base Plate. The rocker is seated on a base plate, which distributes the 480-kip load to the concrete pier. Allowable bearing stress on the concrete is $0.3f'_c = 0.3 \times 3 = 0.9$ ksi. Hence, the minimum net area of the plate is $480/0.9 = 533$ in^2. Since the plate incorporates four 2-in-dia. holes for 1½-in-dia. anchor bolts, the required area should be increased to $533 + 4 \times 3.14 = 546$ in^2. For a width of 24 in (Fig. 12.42*a*), length of plate should be at least $546/24 = 22.7$ in. Use 40 in to obtain adequate space beyond the rocker for the anchor bolts.

$$\text{Net area of base plate} = 40 \times 24 - 4 \times 3.14 = 947.4 > 533 \text{ in}^2$$

Thickness of plate must be large enough to keep bending stresses caused by the bearing pressure within the allowable. Under dead load, live load, and impact, the pressure may be considered substantially uniform (Fig. 12.44*a*). If thermal movements also occur, the pressure

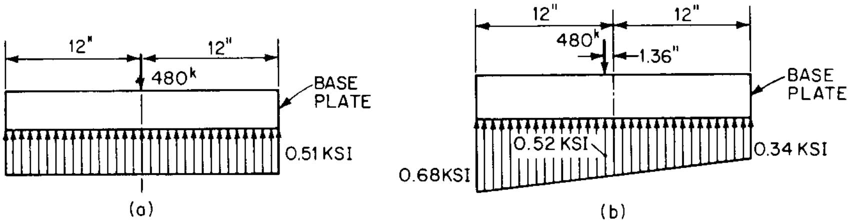


FIGURE 12.44 Base plate at expansion shoe. (*a*) Uniform pressure on the plate under dead load plus live load plus impact. (*b*) Linearly varying pressure on the plate with thermal loading added.

will be nonuniform and is usually assumed to vary linearly (Fig. 12.44*b*). The allowable bending stresses may be increased 25% when temperature stresses are included.

For a maximum movement of 1.36 in, including 0.76 in thermal contraction and 0.60 live load shortening, the base pressures are

$$p = \frac{P}{A} \pm \frac{6P_e}{bd^2} = \frac{480}{947} \pm \frac{6 \times 480 \times 1.36}{40(24)^2} = 0.51 \pm 0.17$$

Therefore, the maximum pressure is 0.68 ksi and the minimum 0.34 ksi. Pressure directly under the load is 0.34 + (0.68 − 0.34)13.15/24 = 0.52 ksi. Consider now a 1-in-wide strip of the base plate under this linearly varying pressure. The bending moment in the plate at the bearing point, 10.64 in from the nearest edge of the plate, is

$$M = \frac{0.52(10.64)^2}{2} + \frac{1}{2}(0.68 - 0.52)\frac{2}{3}(10.64)^2 = 35.7 \text{ in-kips}$$

With the allowable stress increased 25%, the effective moment is

$$M_{\text{eff}} = \frac{35.7}{1.25} = 28.6 \text{ in-kips}$$

For dead load, live load, and impact, with the basic allowable stress $f_b = 20$ ksi, the base pressure is

$$p = \frac{480}{947} = 0.51 \text{ ksi}$$

The bending moment in a 1-in-wide strip of plate at the bearing point, 12 in from either plate edge, is

$$M = \frac{0.51(12)^2}{2} = 36.7 > 28.6 \text{ in-kips}$$

This moment governs. Thickness of base plate required then is

$$t = \sqrt{\frac{6M}{f_b}} = \sqrt{\frac{6 \times 36.7}{20}} = 3.3, \text{ say } 3.5 \text{ in}$$

Use a base plate 24 × 3½ in by 3 ft 4 in long.

12.9.9 Fixed Shoes

A fixed shoe is placed at the opposite end of each girder from the expansion shoe. The fixed shoe precludes translation at the girder end but permits rotation in the plane of the web. Like the expansion shoe, the fixed shoe is a weldment, fabricated of Grade 36 steel. Major components are a bearing bar with curved top to serve as a hinge, a sole plate attached to the girder bottom to transmit the reaction to the bearing bar, a base plate to distribute the load to the concrete pier, and three ribs attached to the bearing bar to improve stability and load distribution (Fig. 12.45).

Loadings. Several loading combinations must be investigated:

Group I. Dead load (DL) + live load (LL) + impact (I) at 100% of the basic allowable stress

Group II. DL + wind (W) at 125% of the basic allowable stress

Group III. $DL + LL + I + 0.3W$ + longitudinal force (LF) + wind on moving live load (WL) at 125% of the basic allowable stress

The group I loading is a vertical load of 480 kips, as for the expansion shoe.

For the group II loading, the dead load is a vertical load from the girder of 297.5 kips plus the weight of the shoe, or a total of 307 kips. The wind imposes horizontal and vertical forces. The horizontal force is contributed by the 42.8-kip reaction of the horizontal lateral bracing system, which is assumed to be shared equally by the fixed shoes of the two girders. Each shoe therefore takes $42.8/2 = 21.4$ kips horizontally, perpendicular to the girders.

The vertical wind loading is caused by overturning effects of the wind. The wind forces are the same as those used in the design of the lateral system.

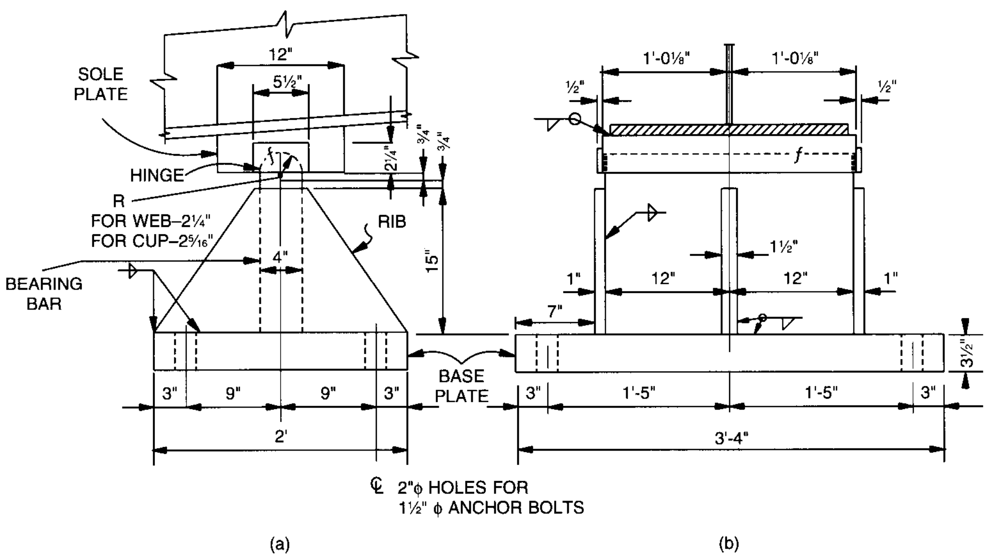


FIGURE 12.45 Fixed shoe. (*a*) Side view. (*b*) End view.

$$\text{Railing force:} \qquad W_1 = 0.05 \times 0.91 \times {}^{140}\!/_2 = 3.2 \text{ kips}$$

$$\text{Slab and girder force: } W_2 = 0.05(1.83 + 9.40)^{140}\!/_2 = 39.5 \text{ kips}$$

W_1 acts 13.1 ft above the hinge of the shoe, and W_2, 6 ft above the hinge. Thus, these forces produce a moment about the hinge of

$$M_h = 3.2 \times 13.1 + 39.5 \times 6 = 279 \text{ ft-kips}$$

The moment is resisted by a couple consisting of vertical forces at each shoe, 35 ft apart. Hence, the moment induces an upward or downward force of 279/35 = 8 kips.

Total downward vertical force at the hinge then is 8 + 307 = 315 kips.

Similarly, the wind forces cause a moment about the bottom of the shoes, or top of pier, of

$$M_p = 279 + (3.2 + 39.5)1.3 = 335 \text{ ft-kips}$$

This induces vertical forces at top of pier of 335/35 ≈ 10 kips.

Total vertical downward force at top of pier then is 10 + 307 = 317 kips per shoe. The 21.4-kip horizontal force acts simultaneously with this.

For group III loading, the vertical load $DL + LL + I = 480$ kips. From the preceding calculation, $0.3W$ causes a horizontal force of $0.3 \times 21.4 = 6.4$ kips transversely and a vertical force of $0.3 \times 8 = 2$ kips at the hinge and $0.3 \times 10 = 3$ kips at the top of pier. LF is 5% of the live load (lane loading for moment, including concentrated load but not impact) for two lanes of traffic heading in the same direction.

$$LL = 2(0.64 \times 140 + 18) = 216 \text{ kips}$$

The longitudinal force acting on one fixed shoe then is

$$LF = \frac{0.05 \times 216 \times 30.5}{35} = 10 \text{ kips}$$

WL is 0.1 kip per ft applied 6 ft above the roadway surface.

$$WL = 0.1 \times 140 = 14 \text{ kips}$$

Acting 17.1 ft above the hinge, it is shared equally by the four shoes. Thus, each shoe receives a transverse horizontal load of 14/4 = 4 kips. Also, WL produces a moment about the hinge of $17.1 \times 14/2 = 120$ ft-kips and vertical forces on the shoes of 120/35 = 3 kips.

Maximum downward vertical load on the hinge therefore is 480 + 2 + 3 = 485 kips.

Similarly, WL causes a moment about the top of pier of $120 + 7 \times 1.3 = 129$ ft-kips and vertical forces at top of pier of 129/35 = 4 kips. Total vertical downward force at top of pier then is 480 + 3 + 4 = 487 kips per shoe. The 10-kip longitudinal force and a transverse wind force of 6.4 + 14/4 = 10 kips act simultaneously with the vertical force.

Because the group II and III loadings are allowed a 25% increase in allowable stress, design of the shoe for vertical loading is governed by group I loading with basic allowable stresses.

Bearing Bar and Hinge. Width and thickness of the bearing bar and radius of the hinge are made the same as the width and thickness of the rocker web and radius of the hinge, respectively, of the expansion shoe (Figs. 12.43 and 12.45).

Base Plate. With a 40 × 24-in base plate, the same size as used for the expansion shoe, the maximum pressure on the concrete pier, 0.51 ksi, is the same as that under the base plate of the expansion shoe. Thickness of the plate for the fixed shoe, however, is governed by bending moment about the longitudinal axis at the outer face of the exterior rib, 7 in from the edge of the base plate. This moment is

$$M = \frac{0.51(7)^2}{2} = 12.5 \text{ in-kips}$$

Required thickness of base plate, with an allowable bending stress of 20 ksi, then is

$$t = \sqrt{\frac{6M}{f_b}} = \sqrt{\frac{6 \times 12.5}{20}} = 1.94 \text{ in}$$

Use the same size base plate as for the expansion shoe, 24 × 3½ in by 3 ft 4 in long.

12.9.10 Overturning Forces

The structure should be checked for resistance to overturning under group II or group III loading, plus an upward force. This force should be taken as 20 psf for group II and 6 psf for group III on deck and sidewalk. The force is assumed to act at the windward quarter point of the width of the structure, 54/4 = 13.5 ft from the windward edge of the sidewalk, or 13.5 − 9.5 = 4.0 ft from the windward girder.

With the larger uplift forces and lesser downward loads, group II loading governs the design. For this loading, the uplift force 4 ft from the windward girder is

$$P = 0.02 \times 54 \times 140 = 151 \text{ kips}$$

It causes an uplift reaction at each of the two windward shoes of

$$R_u = \frac{151}{2} \frac{35 - 4}{35} = 67 \text{ kips}$$

For group II loading, as calculated previously, wind produces an uplift of 10 kips at top of pier. Hence, the total uplift is 67 + 10 = 77 kips. This is counteracted by the 307-kip downward dead-load reaction. Net force is 307 − 77 = 230 kips downward.

Since there is no uplift at the shoes, tie-down bolts are not required. The horizontal forces, being small, will be resisted by friction. Minimum-size anchor bolts permitted by AASHTO specifications may be used. Use four 1½-in-dia. anchor bolts per shoe.

12.9.11 Seismic Evaluation of Bearings

Since this is a single-span bridge, only the connection between the bridge and the abutments must be designed to resist the longitudinal and transverse gravity reactions. If the bridge is assigned to AASHTO Seismic Performance Category A, the connection should be designed to resist a horizontal seismic force equal to 0.20 times the dead-load reaction. For allowable-stress design, a 50% increase in allowable stress is permitted for structural steel and a 33⅓% increase for reinforced concrete.

Expansion Shoe. The total dead-load reaction is 297.5 kips. The transverse seismic (EQ) load = 0.20 × 297.5 kips = 59.5 kips. For the sole-plate connection to the girder, try a 5⁄16-in fillet weld with a capacity of 5⁄16 × 0.707 × 0.27 × 58 × 1.5 = 5.2 kips per in. The required length of weld is 59.5/5.2 = 11.5 in ≤ (2 × 12 in = 24 in). For the cap-plate connection to the end of the sole plate, try a 5⁄16-in fillet weld. Length = 11.5 in ≅ (6.0 + 2 × 2.50 = 11 in).

The rocker will be subjected to a transverse moment equal to 17.25 × 59.5 = 1026 in-kips and a vertical dead load of 297.5 kips. The eccentricity e = 1.026/297.5 = 3.45 in ≤ 30/6. There will be no uplift. Since the rocker base is 30 in long, it carries a load

$$P_{\max} = \frac{297.5}{30}\left(1 + \frac{6 \times 3.45}{30}\right)$$

$$= 16.76 \text{ kips per in} < (1.50 \times 19 = 28.5 \text{ kips per in})$$

where 19 kips per in is the allowable bearing pressure (Art. 12.9.8).

The masonry plate will be subjected to a transverse moment of 20.75 × 59.5 = 1235 in-kips. The eccentricity e = 1,235/297.5 = 4.15 in < (40/6 = 6.7 in). There will be no uplift. The bearing pressure on the 24 × 40-in masonry plate is

$$P_{\max} = \frac{297.5}{24 \times 40}\left(1 + \frac{6 \times 4.15}{40}\right) = 0.50 \text{ ksi} < (1.33 \times 0.90 = 1.20 \text{ ksi})$$

This is less critical than the service loads.

Two 1½-in dowels, each with an area of 1.76 in², secure the rocker to the masonry plate. The shear in these dowels is

$$f_v = \frac{59.5}{2 \times 1.76} = 16.9 \text{ ksi} \le (1.50 \times 12 = 18 \text{ ksi})$$

where 12 ksi is the allowable shear stress. Four 1½-in anchor bolts will provide sufficient connection to the abutment.

Fixed Shoe. The total dead-load reaction is 297.5 kips. However, the fixed shoe must resist a longitudinal EQ component of 20% of the dead-load reaction, 59.5 kips, plus a transverse EQ component equal to 30% of that, 17.9 kips. Hence, the shoe will be subjected to a total force equal to

$$P = \sqrt{(59.5)^2 + (17.9)^2} = 62.1 \text{ kips}$$

The shoe will be anchored with four 1½-in anchor bolts. The shear on the bolts is

$$f_v = \frac{62.1}{4 \times 1.76} = 8.8 \text{ ksi} \le (1.50 \times 12 = 18 \text{ ksi})$$

The fixed-shoe masonry plate will be subjected to a longitudinal moment M_L and a transverse moment M_T.

$$M_L = 59.5 \times 20.75 = 1{,}235 \text{ in-kips}$$

which induces an eccentricity e_L = 1,235/297.5 = 4.15 in.

$$M_T = 17.9 \times 20.75 = 371 \text{ in-kips}$$

which induces an eccentricity e_T = 371/297.5 = 1.25 in. The bearing pressure on the plate is

$$P_{\max} = \frac{297.5}{24 \times 40}\left(1 + \frac{6 \times 4.15}{24} + \frac{6 \times 1.25}{40}\right)$$
$$= 0.69 \text{ ksi} \leq (1.33 \times 0.90 = 1.20 \text{ ksi})$$

The masonry-plate reaction is less than factored design load; therefore, the plate is satisfactory.

The AASHTO Seismic Performance Category A has lower seismic design requirements than Categories B to D. While rocker bearings may be adequate for Category A, other types of bearings, which are low profile, such as pot or elastomeric bearings, are generally preferred. In extreme cases base-isolation bearings are used.

12.10 THROUGH PLATE-GIRDER BRIDGES WITH FLOORBEAMS

For long or heavily loaded bridge spans, restrictions on depth of structural system imposed by vertical clearances under a bridge generally favor use of through construction. Through girders support the deck near their bottom flange. Such spans preferably should contain only two main girders, with the railway or roadway between them (Fig. 12.46). In contrast, deck girders support the deck on the top flange (Art. 12.8).

The projection of the girders above the deck in through bridges may be objectionable for highway structures, because they obstruct the view from the bridge of pedestrians or drivers. But they may offer the advantage of eliminating the need for railings and parapets. For railroad bridges over highways, streets, or other facilities from which the bridges are highly visible to the general public, through girders provide a more attractive structure than through trusses.

The projection of the girders above the deck also has the disadvantage of requiring special provisions for bracing the compression flange of the girders. Deck girders usually require no special provision for this purpose, because when a rigid deck is used, it provides the needed lateral support. Through girders should be laterally braced with gusset plates or knee braces with solid webs connected to the stiffeners.

In railroad bridges, spacing of the through girders should be at least 1⁄20 of the span, or should be adequate to ensure that the girders and other structural components provide required clearances for trains, whichever is greater.

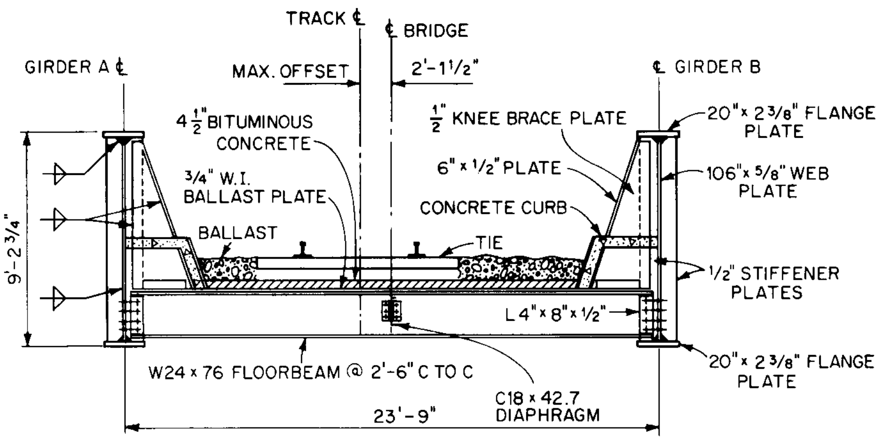


FIGURE 12.46 Cross section of through-girder railroad bridge.

Article 12.11 presents an example to indicate the design procedure for a through girder bridge with floorbeams. Because the example in Art. 12.9 dealt with highway loading, additional information is provided by designing a railroad bridge in the following example. Also, a curved alignment is selected, whereas the girders are kept straight, to illustrate the application of centrifugal forces to the structure. Note that because the girders are straight, the centerline of the track is offset from the centerline of the bridge. Design procedures not discussed in the example generally are the same as for deck girders (Art. 12.9) or plate-girder stringers (Art. 12.4).

12.11 EXAMPLE—ALLOWABLE-STRESS DESIGN OF A THROUGH PLATE-GIRDER BRIDGE

Two simply supported, welded, through plate girders carry the single track of a railroad bridge on an 86-ft span (Fig. 12.46). The girders are spaced 23.75 ft c to c. The track is on an 8° curve, for which the maximum design speed is 30 mph. Maximum offset of centerline of track for centerline of bridge is 2.12 ft. Live loads from the trains are distributed by ties, ballast, and a Grade 50W steel ballast plate to rolled-steel floorbeams spaced 2.5 ft c to c. These beams transmit the loads to the girders. Steel to be used is Grade 36. Loading is Cooper E65.

12.11.1 Design of Floorbeams

For convenience in computing maximum moment, the dead load on a floorbeam may be considered to consist of three parts: weight of track and load-distributing material, spread over about 18.5 ft; weight of floorbeam and connections, distributed over the span, which is taken as 23.5 ft; and weight of concrete curb, which is treated as a concentrated load (Table 12.56). This loading produces a reaction

$$R_{DL} = \frac{0.553 \times 18.5}{2} + \frac{0.080 \times 23.5}{2} + 1.9 = 7.9 \text{ kips}$$

Maximum bending moment occurs at midspan and equals

$$\begin{aligned} M_{DL} &= 7.9 \times 11.75 - 1.9 \times 10.50 - 0.553 \times 9.25 \times 4.63 - 0.080 \times 11.75 \times 5.88 \\ &= 44 \text{ ft-kips} \end{aligned}$$

The live load P, kips, carried by the floorbeam can be computed from

TABLE 12.56 Dead Load on Floorbeam, kips per ft

Track: 0.200 × 2.5/18.5	= 0.027
Tie: 0.160/18.5	= 0.009
Ballast: 0.120 × 1 × 2.5	= 0.300
Bituminous concrete: 0.150 × 2.5 × 4.5/12	= 0.140
¾-in ballast plate: 0.0306 × 2.5	= 0.077
Load over 18.5 ft:	0.553
Beam—assume:	0.080
Concrete curb: 0.150 × 2.5 × 2.5 × 2 = 1.9 kips	

$$P = 1.15AD/S \qquad S \geq d \tag{12.42}$$

where A = axle load, kips
S = axle spacing, ft
D = effective beam spacing, ft
d = actual beam spacing, ft

with D taken equal to d. The axle load $A = 65$ kips, and the axle spacing $S = 5$ ft.

$$P = \frac{1.15 \times 65 \times 2.5}{5} = 37.4 \text{ kips}$$

$P/2 = 18.7$ kips is applied as a concentrated load at each rail. Then loads cause a reaction

$$R_{LL} = \frac{18.7(11.98 + 7.27)}{23.5} = 15.3 \text{ kips}$$

Maximum moment occurs under a rail and equals

$$M_{LL} = 15.3 \times 11.52 = 176.5 \text{ ft-kips}$$

Impact for this example is taken as 36% of wheel live-load stresses. (See Art. 12.35.5 for current requirements.) Impact moment then is

$$M_I = 0.36 \times 176.5 = 63.5 \text{ ft-kips}$$

The total moment is 284 ft-kips. This requires a section modulus

$$S = \frac{284 \times 12}{20} = 170 \text{ in}^3$$

Use a W24 × 76, with $S = 176$ in^3.

Maximum live-load floorbeam reaction is $37.4 - 15.3 = 22.1$ kips. The maximum floorbeam reaction is

$$R = 7.9 + 22.1 + 22.1 \times 0.36 = 38.0 \text{ kips}$$

12.11.2 Design of Girders

The girders will be made of Grade 36 steel. Simply supported, they span 86 ft. They will be made identical.

Dead Load. Most of the load carried by each girder is transmitted to it by the floorbeams as concentrated loads. Computations are simpler, however, if the floorbeams are ignored and the girder is treated as if it received load from the ballast plate. Moments and shears computed with this assumption are sufficiently accurate for design purposes because of the relatively close spacing of the floorbeams. Thus, the dead load on the girder may be considered uniformly distributed. It is computed to be 3.765 kips per ft.

Maximum dead-load moment occurs at midspan and equals

$$M_{DL} = \frac{3.765(86)^2}{8} = 3{,}500 \text{ ft-kips}$$

Dead-load moments along the span are listed in Table 12.57. Maximum dead-load shear and the reaction is

TABLE 12.57 Moments, ft-kips, in Girders

Distance from supports, ft L_1	L_2	Dead load, M_{DL}	Equivalent E10 loading q	$3.83q$	Live load M_{LL}, $3.83qL_1L_2/2$	Impact M_r, $0.175 \times 6.5qL_1L_2/2$	Centrifugal force C
15	71	2,010	1.40	5.37	2,860	850	180
30	56	3,170	1.33	5.09	4,280	1,270	280
40	46	3,470	1.34	5.12	4,720	1,400	310
43	43	3,500	1.33	5.09	4,720	1,400	310

$$R_{DL} = \frac{3.765 \times 86}{2} = 162 \text{ kips}$$

Live Load. Computation of live-load moments, shears, and reactions is simplified with the aid of tables or charts. (See, for example, D. B. Steinman, ‘‘Locomotive Loadings for Railway Bridges,’’ *ASCE Transactions,* vol. 86, pp. 606–723, and J. E. Roberts and S. L. Mellon, ‘‘Bridge Engineering, Standard Handbook for Civil Engineers,’’ 4th ed., McGraw-Hill Book Company, New York.) If figures are available for any magnitude of Cooper E loading, those for any other magnitude can be obtained by proportion.

Since the tracks are not centered between the girders, one girder will be more heavily loaded than the other. The amount of live load transmitted to the more heavily loaded girder may be obtained by taking moments about the other girder. Let q be the equivalent uniform live load for E10 loading, kips per ft. Then, $6.5q$ is the equivalent load for E65, and the girder receives $6.5q \times 14.0/23.75 = 3.83q$. Live-load moments along the span are listed in Table 12.57.

Maximum reaction and shear under E10 loading is 66.1 kips. Hence, the maximum reaction for E65 loading is

$$R_{LL} = 66.1 \times 3.83 = 254 \text{ kips}$$

Impact is taken as 17.5% of the axle live-load stresses. Therefore, the impact moment is

$$M_I = 0.175 \times 6.5 \times 1.33 \times 43^2/2 = 1{,}400 \text{ ft-kips}$$

and the impact reaction is

$$R_I = 0.175 \times 6.5 \times 1.33 \times 86/2 = 65 \text{ kips}$$

Centrifugal Force. This is computed as a percentage of live load, with speed $S = 30$ mph and degree of curve $D = 8°$.

$$C = 0.00117S^2D = 0.00117(30)^2 8 = 8.43\%$$

Application of this percentage to the equivalent load producing maximum moment yields the equivalent centrifugal force

$$C_e = 0.0843 \times 1.33 \times 6.5 = 0.73 \text{ kip per ft}$$

This force acts 6 ft above top of rail, or 10.8 ft above bottom of girder. It is resisted by a couple consisting of a vertical force at each girder equivalent to

$$q_C = \frac{0.73 \times 10.8}{23.75} = 0.332 \text{ kip per ft}$$

The maximum moment produced by these forces can be obtained by proportion from the maximum live-load moment:

$$M_C = \frac{4{,}720 \times 0.332}{5.09} = 310 \text{ ft-kips}$$

Similarly, the maximum shear and reaction equal

$$R_C = \frac{254 \times 0.332}{5.09} = 17 \text{ kips}$$

Longitudinal Force. The longitudinal force from trains should be taken as 15% of the live load, without impact, acting at base of rail. Application of this percentage to the equivalent load producing maximum reaction yields the equivalent longitudinal force

$$q_L = 0.15 \times 1.54 \times 6.5 = 1.50 \text{ kips per ft}$$

Since the rails will be continuous across the bridge, the longitudinal forces will be $1.50 \times 86/1{,}200 = 0.11$ kips per ft. This imposes on the girder a horizontal force of

$$H_L = \frac{0.11 \times 86 \times 14}{23.75} = 5.6 \text{ kips}$$

Because this force acts at top of rail, about 4 ft above the bottom of the girder, it causes a moment. This is resisted by a couple composed of a force at each support of the girder:

$$R_L = \frac{5.6 \times 4}{86} = 0.3 \text{ kips}$$

Wind Transverse to Bridge. The wind may act on live load and structure in any horizontal direction. The wind load on the train should be taken as a moving load of 0.3 kip per ft, acting 8 ft above top of rail. Wind load on the structure should be taken as 0.03 ksf, acting on 1.5 times the vertical projection of the span.

Transverse to the bridge, wind on the live load, acting 12.8 ft above the bottom of the girder, imposes vertical forces on the girder of $0.3 \times 12.8/23.75 = 0.162$ kip per ft. This causes a midspan bending moment of

$$M_{WLL} = \frac{0.162(86)^2}{8} = 150 \text{ ft-kips}$$

Maximum shear and reaction equal

$$R_{WLL} = \frac{0.162 \times 86}{2} = 7 \text{ kips}$$

In addition, acting at each of the four girder supports is a transverse horizontal force

$$H_{WLL} = \frac{0.3 \times 86}{4} = 6.5 \text{ kips}$$

Transverse wind on a projection of 9.3 ft of structure imposes a load of

$$0.030 \times 9.3 \times 1.5 = 0.420 \text{ kip per ft}$$

It acts about 4.7 ft above the bottom of the girder. The resulting overturning moment causes vertical forces in the girders of $0.420 \times 4.7/23.75 = 0.083$ kip per ft. These forces produce a midspan bending moment

$$M_W = \frac{0.083(86)^2}{8} = 77 \text{ ft-kips}$$

The reaction is

$$R_W = \frac{0.083 \times 86}{2} = 3.6 \text{ kips}$$

Also, a transverse horizontal force acts at each of the four girder supports:

$$H_W = \frac{0.42 \times 86}{4} = 9 \text{ kips}$$

Longitudinal Wind. Longitudinal wind on the live load transmitted to the girder equals $0.3 \times 14/23.75 = 0.18$ kip per ft. Acting 12.8 ft above the bottom of the girder, it imposes vertical and horizontal longitudinal forces at the supports:

$$R_{WL} = \frac{0.18 \times 86 \times 12.8}{86} = 2.3 \text{ kips}$$

$$H_{WL} = 0.18 \times 86 = 15.5 \text{ kips}$$

Similarly, the longitudinal wind on the structure is $0.420/2 = 0.210$ kip per ft per girder. It imposes vertical and horizontal longitudinal forces at the supports of

$$R_{WL} = \frac{0.21 \times 86 \times 4.7}{86} = 1.0 \text{ kip}$$

$$H_{WL} = 0.21 \times 86 = 18 \text{ kips}$$

Wind on Unloaded Bridge. The structure also should be investigated for a transverse wind load of 50 psf on 1.5 times the vertical projection of the span. Moments and shears caused can be obtained by proportion from those previously computed.

$$M_W = \frac{77 \times 50}{30} = 128 \text{ ft-kips}$$

$$R_W = \frac{3.6 \times 50}{30} = 6 \text{ kips}$$

Loading Combinations. Three loading combinations are investigated:

Case I. $DL + LL + I + C$ at full basic allowable stresses (Table 12.58)

Case II. Case I + wind on loaded bridge + longitudinal force at 125% of basic allowable stresses (Table 12.59)

Case III. Dead load + wind on unloaded bridge at 125% of basic allowable stresses (Table 12.60).

TABLE 12.58 Loading Case I—Maximum Moments and Shears

Type of loading	Moment, ft-kips	Shear, kips
DL	3,500	162
LL	4,720	254
I	1,400	65
C	310	17
Total	9,930	498

TABLE 12.59 Loading Case II—Maximum Moments and Shears

Type of loading	Moment, ft-kips	Shear, kips
Case I	9,930	498
Wind	230	11
LF	. . .	4
Total	10,160	513

Moments and shears for case I are larger than those for case III and, when allowance is made for a 25% increase in allowable stresses for case II, also larger than those for case II. Hence, case I at basic allowable stresses governs the design.

The curve of maximum moments at various points of the span, or moment envelope (Fig. 12.47), can now be plotted for case I.

Web Size. While depth of web has no effect on vertical clearances under through-girder bridges, it has several effects on economy. The deeper the girders, the less flange material required and the stiffer the members. But web thickness and number of stiffeners required usually increase. Also, girder spacing may have to be increased, because of wider gussets or knee braces needed for lateral bracing.

In this example, a web depth of 106 in is assumed. With an allowable stress of 12.5 ksi, the web thickness required for shear is

$$t = \frac{504}{12.5 \times 106} = 0.380 \text{ in}$$

To prevent buckling, however, even with transverse stiffeners, thickness should be at least $1/160$ of the clear distance between flanges.

$$t = \frac{106}{160} = 0.663 \text{ in, say } 11/16 \text{ in}$$

Use a $106 \times 11/16$-in web.

TABLE 12.60 Loading Case III—Moments and Shears

Type of loading	Moment, ft-kips	Shear, kips
DL	3,500	162
Wind on bridge	128	6
Total	3,628	168

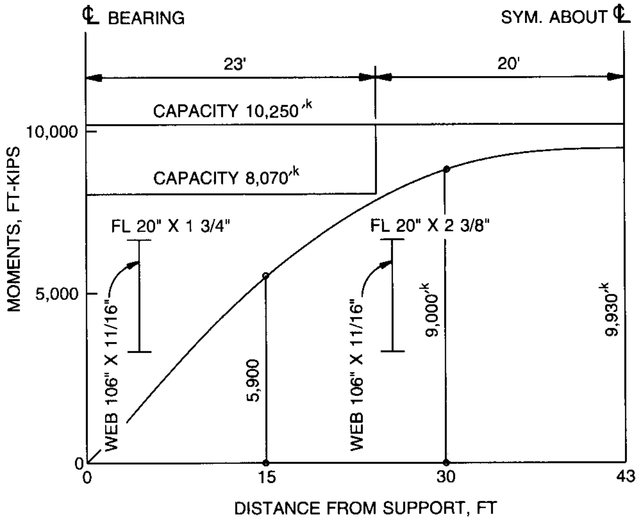


FIGURE 12.47 Moment envelope for through girder and capacities of cross sections.

Flange Size at Midspan. To select a trial size for the flange, assume an allowable bending stress $F_b = 20$ ksi and distance between flange centroids of about 110 in. Then, for a maximum moment of 9,930 ft-kips, the required area of one flange is about

$$A_f = \frac{9{,}930 \times 12}{110 \times 20} = 54 \text{ in}^2$$

Assume a $20 \times 2\frac{3}{8}$-in plate for each flange, with an area of 47.5 in^2. Width-thickness ratio of outstanding portion is $10/2.38 = 4.2$, which is less than 12 permitted. The trial section is shown in Fig. 12.48. Moment of inertia of the section about the X axis is calculated in Table 12.61. Distance from neutral axis to top or bottom of the girder is 55.4 in. Hence, the gross and net section moduli are

$$S_g = \frac{347{,}100}{55.4} = 6{,}270 \text{ in}^3 \qquad S_{net} = \frac{340{,}100}{55.4} = 6{,}150 \text{ in}^3$$

The allowable tensile bending stress is 20 ksi. The actual tensile stress is

$$f_b = \frac{9{,}930 \times 12}{6{,}150} = 19.4 < 20 \text{ ksi}$$

The allowable compressive bending stress is a function of l, the distance, in, between points of lateral support of the compression flange, and r_y, the radius of gyration, in, of the compression flange and that portion of the web area on the compression side of the axis of bending, about the axis in the plane of the web. For a rectangular section, $r = 0.289d$, where d = depth of section perpendicular to axis. Hence, for the compression flange,

$$r_y = 0.289 \times 20 = 5.78 \text{ in}$$

AREMA specifications limit the spacing of lateral supports for the compression flange to a

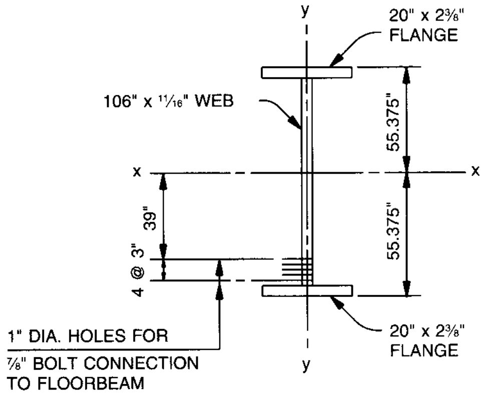


FIGURE 12.48 Cross section of through girder at midspan.

maximum of 12 ft for through girders. Since the knee braces are placed at floorbeam locations, which are 30 in apart, space the knee braces 10 ft = 120 in c to c. Then, the allowable compressive stress is the larger of the following:

$$F_b = 20 - 0.0004\left(\frac{l}{r_y}\right)^2 = 20 - 0.0004\left(\frac{120}{5.78}\right)^2 = 19.83 \text{ ksi}$$

$$F_b = \frac{10{,}500A_f}{ld} = \frac{10{,}500 \times 47.5}{120 \times 110.75} = 37.5 \text{ ksi}$$

but not to exceed 20 ksi. The actual compressive stress is

$$f_b = \frac{9{,}930 \times 12}{6{,}270} = 19.0 < 19.83 \text{ ksi}$$

The section is satisfactory. Moment capacity supplied is

$$M_C = \frac{20 \times 6{,}150}{12} = 10{,}250 \text{ ft-kips}$$

Intermediate Transverse Stiffeners. For the web, the depth-thickness ratio $d/t = 106/(^{11}\!/_{16}) = 154$. This exceeds the AREMA limit of 60 for an unstiffened web. Transverse stiffeners are required and spacing should not exceed $d = 332t/\sqrt{f_v} \leq 72$ in, where f_v is the shear stress, ksi.

TABLE 12.61 Moment of Inertia of Through Plate Girder at Midspan

Material	A	d	Ad^2 or I_o
2 flanges 20 × 2⅜	95.0	54.19	278,900
Web 106 × 11⁄16			68,200
			I_g = 347,100 in^4
4 holes: $-1(^{11}\!/_{16})(39^2 + 42^2 + 45^2 + 48^2 + 51^2)$			= −7,000
			I_{net} = 340,100 in^4

$$f_v = \frac{498}{72.9} = 6.83 \text{ ksi}$$

For this shear, $d = 332(^{11}\!/_{16})/\sqrt{6.83} = 87$ in > 72 in. Use a stiffener spacing of 60 in. Try a pair of plates at each location, with the width equal at least to $D/30 + 2 = {}^{106}\!/_{30} + 2 = 5.5$ in < 16 in. Use $6 \times {}^{3}\!/_{8}$-in plates welded to the web for the intermediate stiffeners.

Change in Flange Size. At a sufficient distance from midspan, the bending moment decreases sufficiently to permit reducing the thickness of the flange plates to $1^{3}\!/_{4}$ in. The net moment of inertia reduces to 265,000 in^4 and the section modulus to 4,840 in^3. Thus, with $20 \times 1^{3}\!/_{4}$-in flange plates, the section has a moment capacity of

$$M_C = \frac{20 \times 4{,}840}{12} = 8{,}070 \text{ ft-kips}$$

When this is plotted in Fig. 12.47, the horizontal line representing it stays above the moment envelope until within 20 ft of midspan. Hence, flange size can be decreased at that point. Length of the $2^{3}\!/_{8}$-in plate then is 40 ft and of the $1^{3}\!/_{4}$-in plates, which extend to the end of the girder, 23 ft.

The flange plates will be spliced with complete-penetration groove welds. For calculation of fatigue stresses, the welded connection is Stress Category B and for this span of less than 100 ft should be designed for 2,000,000 cycles of loading. The allowable stress range is 14.5 ksi. The actual stress range for live loads plus impact is estimated to be

$$f_r = \frac{5{,}200 \times 12}{4{,}840} = 12.9 \text{ ksi} < 14.5 \text{ ksi—OK}$$

Flange-to-Web Welds. AREMA specifications require that the flange plates be connected to the web with continuous, full-penetration groove welds.

Knee Braces. A knee brace with solid web (Fig. 12.49*a*) braces the compression flange of the girder at 10-ft intervals. Attached with bolts to the top of the floorbeam and welded to a girder stiffener, the brace extends from the floorbeam to the top flange of the girder, and from the web of the girder outward a maximum of 36 in. The outer edge is cut to a slope of 3 on 1. (Some railroads prefer a maximum slope of 2.8 on 1.) The length of this edge is 75 in.

Assume a $^{1}\!/_{2}$-in-thick plate for the web. Since the 75-in length of the edge exceeds $60 \times {}^{1}\!/_{2} = 30$ in, the edge is stiffened with a $6 \times {}^{1}\!/_{2}$-in plate. This plate is considered to act with 6 in of the web in transmitting the buckling force to the floorbeam (Fig. 12.49*b*). This force is assumed horizontal and equal to 2.5% of the force in the $20 \times 2^{3}\!/_{8}$-in compression flange. With a compressive stress in the flange of 19.0 ksi, the force to be resisted is

$$F = 0.025 \times 20 \times 2.375 \times 19.0 = 23 \text{ kips}$$

The T section in Fig. 12.49*b* therefore is subjected, because of the 3 on 1 slope, to a force

$$P = 23 \times {}^{79}\!/_{25} = 72.7 \text{ kips}$$

Area of the T is $2 \times 6 \times {}^{1}\!/_{2} = 6$ in^2. Distance of the neutral axis from the outer surface of the flange is

$$y = \frac{3 \times 0.25 + 3 \times 3.5}{6} = 1.88 \text{ in}$$

Moments of inertia are computed to be $I_x = 24.83$ in^4 and $I_y = 9.00$ in^4. The latter governs. Thus, the least radius of gyration is

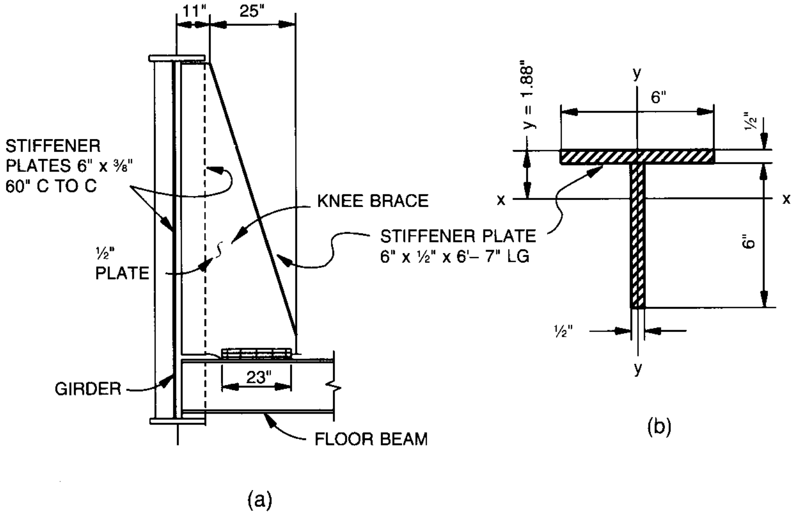


FIGURE 12.49 Knee brace for compression flange of through girder. (*a*) Elevation. (*b*) Cross section assumed effective.

$$r_y = \sqrt{\frac{9.0}{6}} = 1.227 \text{ in}$$

The slenderness ratio then is 79/1.227 = 65. Hence, treated as a column, the T section has an allowable compressive stress of

$$F_a = 21.5 - \frac{0.1kL}{r} = 21.5 - 0.1 \times 65 = 15.0 \text{ ksi}$$

And the brace has a capacity of

$$P = 15.0 \times 6 = 90.0 > 72.7 \text{ kips}$$

Therefore, the knee brace is satisfactory.

The number of ⅞-in-dia. high-strength bolts required to transmit the 23-kip horizontal force to the floorbeam, with a capacity of 9.3 kips per bolt, is $23/(2 \times 9.3) = 2$. For sealing, however, the maximum bolt spacing is $4 + 4t = 4 + 4 \times ½ = 6$ in. Sealing controls. Use five ⅞-in bolts 5 in c to c and two angles 4 × 4 × ½ in by 23 in long.

Other Details. Stiffeners are designed and located in the same way as for deck plate girders (Art. 12.9). They should be placed at floorbeams, but need not be at every beam. Other details also are treated in the same way as for plate girders.

12.12 COMPOSITE BOX-GIRDER BRIDGES

Box girders have several favorable characteristics that make their use desirable for spans of about 120 ft and up. Structural steel is employed at high efficiency, because a high percentage can be placed in wide flanges where the metal is very effective in resisting bending. Corrosion resistance is higher than in plate-girder and rolled-beam bridges. For, with more than half the steel surface inside the box, less steel, especially corners, which are highly susceptible, is exposed to corrosive influences. Also, the box shape is more effective in resisting torsion than the I shape used for plate girders and rolled beams. In addition, box girders offer an attractive appearance.

The high torsional rigidity of box girders makes this type of construction preferable for bridges with curved girders. Also, the high rigidity assists the deck in distributing loads transversely. This is illustrated in Fig. 12.50. A single load placed off center on a bridge with single-web girders is carried mainly by nearby girders. But similarly placed on a box-girder bridge, the load is supported nearly equally by all the girders. The effect of the deck is ignored in this illustration.

Depending on its width, a bridge may be supported on one or more box girders. Each girder may comprise one or more cells. For economy in long-span construction, the cells may be made wide and deep. Width, for example, may be 12 ft or more. Usual thickness of the concrete deck, however, generally limits spacing of the girder webs to about 10 ft and cantilevers to about 5 ft. Consequently, thicker slabs are justified to take advantage of the economies accruing from wider girder cells.

Some designers have found it advantageous to use an alternative scheme with narrow box girders. They place a pair of boxes near the roadway edges and distribute the loads to these girders through longitudinal stringers and transverse floorbeams, as is done in plate-girder construction (Art. 12.9). See, for example, Fig. 12.58.

Box girders may be simply supported or continuous. Since they generally are used principally in long spans, continuity is highly desirable for economy and increased stiffness. Also, use of high-strength steels is advantageous in the longer spans.

Box girders are adaptable to composite and orthotropic-plate construction. (The latter is discussed in Arts. 12.14 and 12.15.) With composite construction, only a narrow top flange is needed with each web. The flanges usually need be only wide enough for load distribution to the web and to provide required clearances and edge distances for welded shear connectors. Figures 12.51 and 12.52 show several types of box-girder bridges that have been constructed with and without composite construction.

Boxes may be rectangular or trapezoidal. (Triangular boxes with apex down have been used, but they have several disadvantages. They usually have to be deeper than rectangular

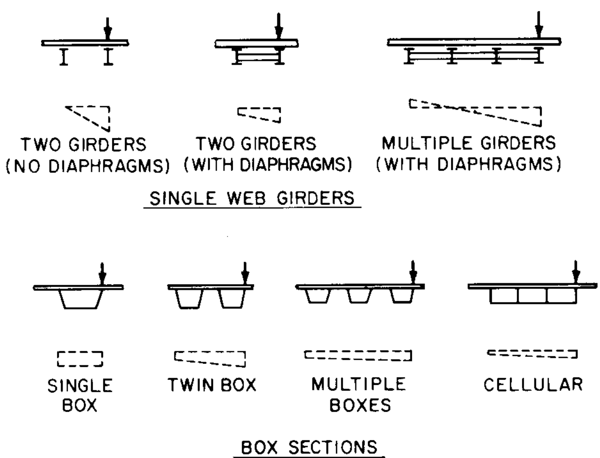


FIGURE 12.50 Comparison of lateral load distribution for single-web girders and box girders.

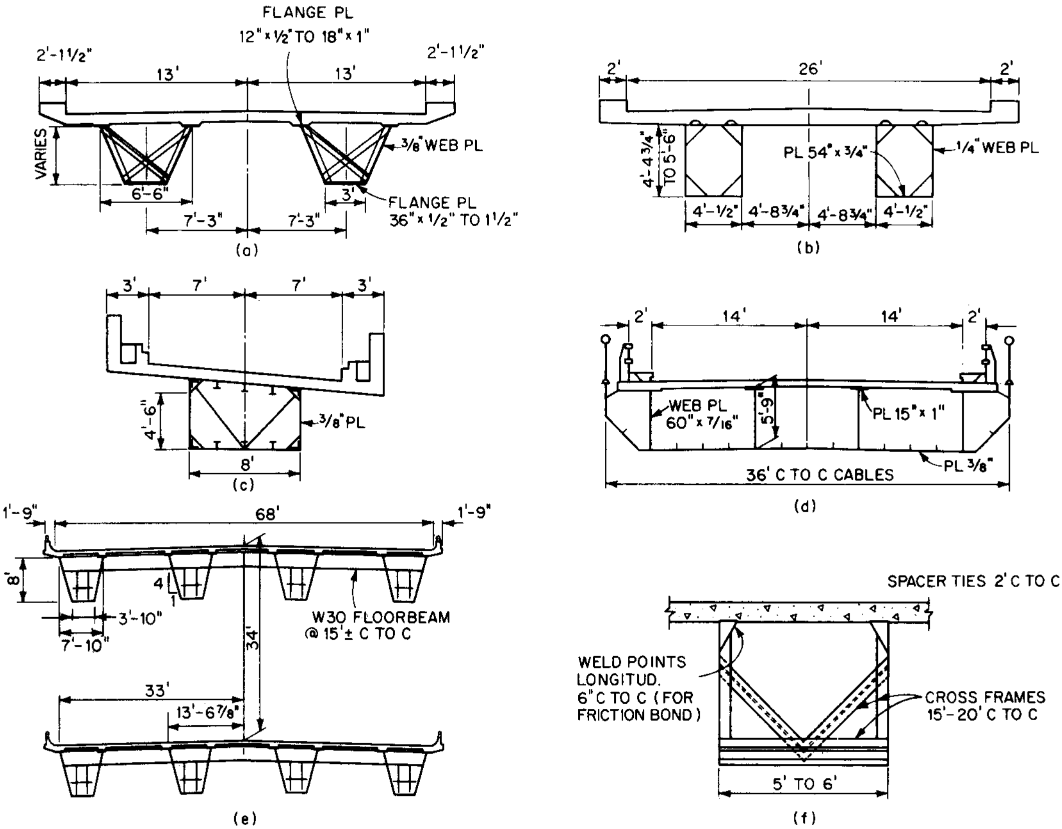


FIGURE 12.51 Examples of cross sections of composite box-girder highway bridges. (*a*) Rigidframe construction with incined legs, over Stillaguamish River. Spans are 50–160–85 ft and 200 ft c to c of leg pins. (*b*) Boxes with corners trussed for rigidity, in 110-ft span. King County, Wash. (*c*) Ramp with minimum horizontal radius of 67 ft and continuous spans of 58.5–52–73 ft, in Port Authority of New York and New Jersey Bus Terminal. (*d*) Suspension-bridge spans of 170–430–170 ft over Klamath River, Orleans, Calif. (*e*) Double-deck approach spans of 170–170 ft, Fremont Bridge, Portland, Ore. (*f*) Box UV girder proposed by Homer Hadley. V-shaped troughs formed by corner plates atop the webs are filled with concrete to secure composite action with concrete deck.

or trapezoidal boxes. Also, because of smaller area, triangular boxes have less torsional resistance. Furthermore, the bottom flange often has to be a heavy built-up section, complicated by bent plates for connecting to the webs.) With trapezoidal boxes, fewer girders may be required, but a thicker bottom plate or thicker concrete slab may be needed than for rectangular boxes. Fabrication costs for either shape are about the same.

Construction costs for box-girder bridges often are kept down by shop fabrication of the boxes. Thus, designers should bear in mind the limitations placed by shipping clearances on the width of box girders as well as on length and depths. If the girders are to be transported by highway, and single box girders with widths exceeding about 12 ft are required, use of more but narrower girders may be more economical.

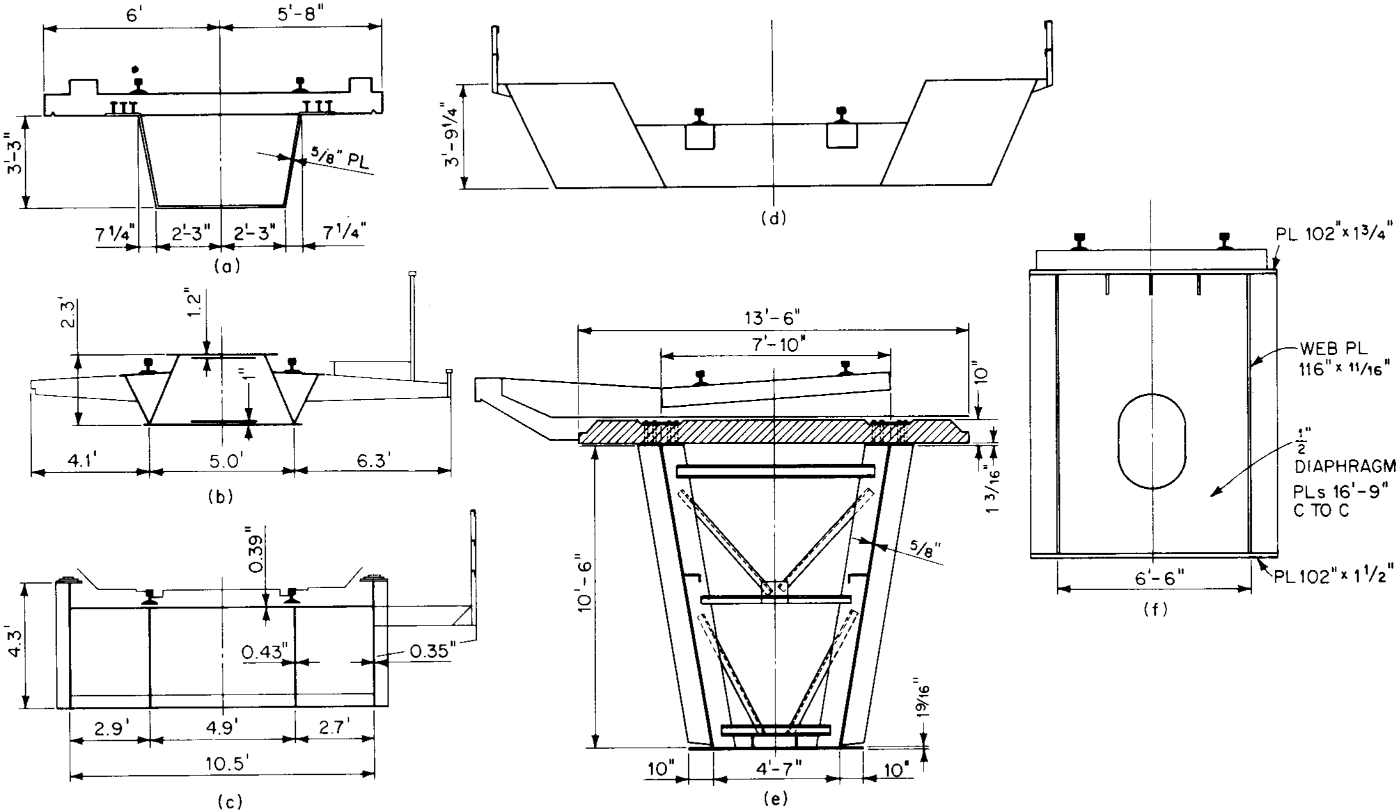


FIGURE 12.52 Examples of cross sections of railroad box-girder bridges. (*a*) Composite girder with 85-ft span for elevated rapid-transit system, San Francisco, Calif. (*b*) Rails bear directly on steel box of 56-ft span over Autobahn, Kirchweyhe, Bremen, Germany. (*c*) Three-cell box over Alsstrasse, M-Gladbach, Germany, with 80-ft span. (*d*) Rigid-frame construction with 117-ft span carries track with radius of 1780 ft, near Frankfurt, Germany. (*e*) Precast-concrete deck bolted to girder for composite action in 152.5-ft span, Czechoslavakia. (*f*) Typical section of four 122-ft spans in Chester, Pa.

12.13 EXAMPLE—ALLOWABLE-STRESS DESIGN OF A COMPOSITE, BOX-GIRDER BRIDGE

Following is an example to indicate the design procedure for a bridge with box girders composite with a concrete deck. The procedure does not differ greatly from that for a single-web plate girder with composite deck (Art. 12.4). The example incorporates the major differences.

A two-lane highway bridge with simply supported, composite box girders will be designed. The deck is carried by two trapezoidal girders (Fig. 12.53). Top width of each box is 8 ft 6 in, as is the distance c to c of adjacent top flanges of the girders. Bottom width is 5 ft 10 in. Thus, the webs have a slope of 4 on 1. The girders span 120 ft. Structural steel to be used is Grade 36. Loading is HS20-44. Appropriate design criteria given in Sec. 11 will be used for this structure.

Concrete Slabs. The general design procedure outlined in Art. 12.2 for slabs on rolled beams also holds for slabs on box girders. A 7.5-in-thick concrete slab will be used with the box girders.

Design Criteria. AASHTO "Standard Specifications for Highway Bridges" apply to single-cell box girders where width c to c between top steel flanges is approximately equal to the distance c to c of adjacent top steel flanges of adjacent box girders. (The distance c to c of flanges of adjacent boxes should be between 0.8 and 1.2 times the distance c to c of the flanges of each box.) In this example, both the width and spacing equal 8 ft 6 in. Also, the deck overhang must not exceed 6 ft or 60% of the spacing. In this example, the overhang

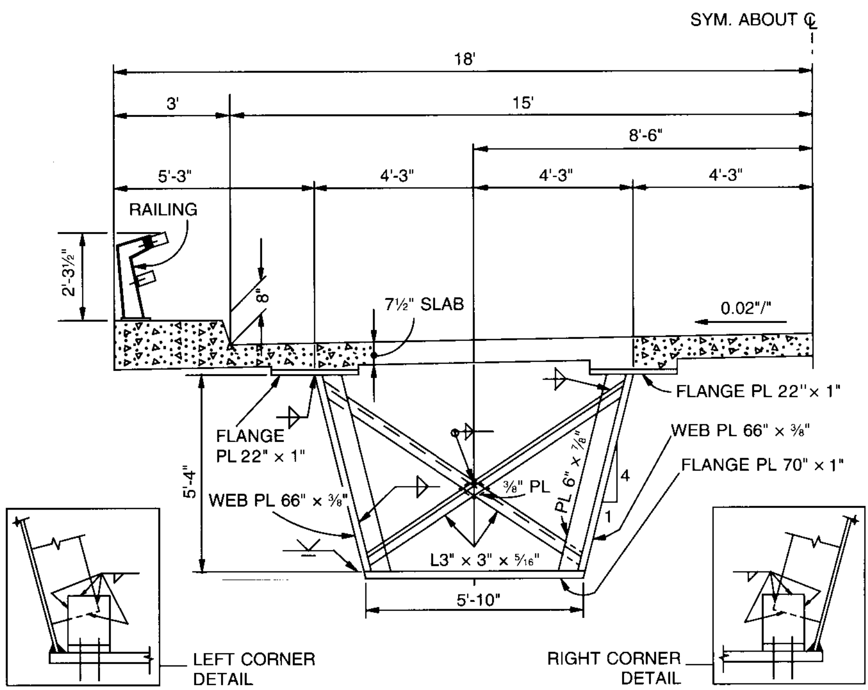


FIGURE 12.53 Half cross section of composite box girder with 120-ft span.

of 5.25 ft is nearly equal to $0.60 \times 8.5 = 5.1$ ft. Hence, AASHTO specifications for composite box girders may be used.

Loads, Moments, and Shears. Assume that the girders will not be shored during casting of the concrete slab. Hence, the dead load on each girder includes the weight of the 18-ft-wide half of the deck as well as weights of steel girders and framing details. This dead load will be referred to as *DL* (Table 12.62*a*). Maximum moment occurs at the center of the 120-ft span and equals

$$M_{DL} = \frac{2.34(120)^2}{8} = 4{,}210 \text{ ft-kips}$$

Maximum shear occurs at the supports and equals

$$V_{DL} = \frac{2.34 \times 120}{2} = 140.4 \text{ kips}$$

Railings and safety walks will be placed after the concrete slab has cured. This superimposed dead load will be designated *SDL* (Table 12.62*b*). Maximum moment occurs at midspan and equals

$$M_{SDL} = \frac{0.71(120)^2}{8} = 1{,}280 \text{ ft-kips}$$

Maximum shear occurs at supports and equals

$$V_{SDL} = \frac{0.71 \times 120}{2} = 42.6 \text{ kips}$$

The HS20-44 live load imposed may be a truck load or lane load. But for this span, truck loading governs. The center of gravity of the three axles lies between the two heavier loads and is 4.66 ft from the center load. Maximum moment occurs under the center axle load when its distance from midspan is the same as the distance of the center of gravity of the loads from midspan, or $4.66/2 = 2.33$ ft. Thus, the center load should be placed $^{120}\!/_2 - 2.33 = 57.67$ ft from a support. Then, the maximum moment is

$$M_T = \frac{72(120/2 + 2.33)^2}{120} - 32 \times 14 = 1{,}880 \text{ ft-kips}$$

Under AASHTO specifications, the live-load bending moment for each girder is determined by applying to the girder the fraction W_L of a wheel load (both front and rear) as given by

TABLE 12.62 Dead Load, kips per ft, on Box Girder

(*a*) Dead load on steel box		(*b*) Dead load on composite section	
Slab: $0.150 \times 18 \times 7.5/12$	= 1.69	Future overlay: 0.025×15	= 0.38
Haunches: $2 \times 0.150 \times 2 \times 1/12$	= 0.05	Railing:	0.02
Girder and framing details—assume:	0.60	Safety walk: $0.150 \times 3 \times 8.38/12$ =	0.31
DL per girder:	2.34	*SDL* per girder:	0.71

$$W_L = 0.1 + 1.7R + \frac{0.85}{N_w} \tag{12.43}$$

where $R = N_w/N$, with $0.5 \leq R \leq 1.5$
$N_w = W_c/12$, reduced to nearest whole number
N = number of box girders
W_c = roadway width, ft, between curbs or between barriers if curbs are not used

In this example, $W_c = 30$, $N_w = 30/12 = 2$, $N = 2$, and $R = 2/2 = 1$. Therefore,

$$W_L = 0.1 + 1.7 \times 1 + \frac{0.85}{2} = 2.225 \text{ wheels} = 1.113 \text{ axles}$$

AASHTO standard specifications do not allow reduction of load intensity where W_L is obtained using the preceding equation. Therefore, the maximum live-load moment is

$$M_{LL} = 1.113 \times 1{,}880 = 2{,}100 \text{ ft-kips}$$

Though this moment does not occur at midspan as do the maximum dead-load moments, stresses due to M_{LL} may be combined with those from M_{DL} and M_{SDL} to produce the maximum stress, for all practical purposes.

For maximum shear with the truck load, the outer 32-kip load should be placed at the support. Then, the shear is

$$V_T = \frac{72(120 - 14 + 4.66)}{120} = 66.4 \text{ kips}$$

On the assumption that the live-load distribution is the same as for bending moment, the maximum live-load shear is

$$V_{LL} = 1.113 \times 66.4 = 73.8 \text{ kips}$$

Impact is taken as the following fraction of live-load stress:

$$I = \frac{50}{L + 125} = \frac{50}{120 + 125} = 0.204$$

Hence, the maximum moment due to impact is

$$M_1 = 0.204 \times 2{,}100 = 430 \text{ ft-kips}$$

and the maximum shear due to impact is

$$V_1 = 0.204 \times 73.8 = 15.1 \text{ kips}$$

Midspan Bending Moments, ft-kips			End shear, kips			
M_{DL}	M_{SDL}	$M_{LL} + M_I$	V_{DL}	V_{SDL}	$V_{LL} + V_I$	Total V
4,210	1,280	2,530	140.4	42.6	88.9	271.9

Properties of Composite Section. The 7.5-in thick roadway slab includes an allowance of 0.5 in for a wearing surface. Hence, the effective thickness of the concrete slab for composite action is 7 in. Half the width of the deck, 18 ft = 216 in, is considered to participate in the composite action with each box girder.

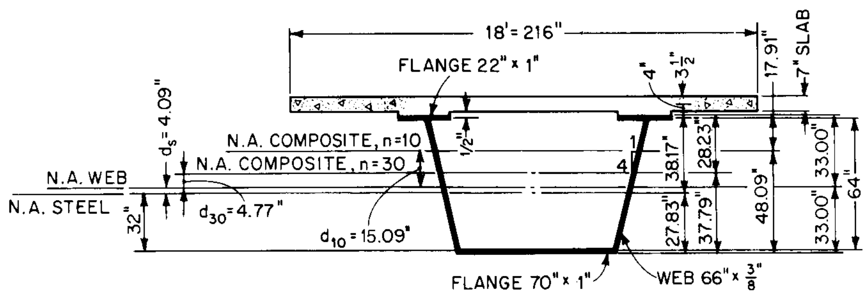


FIGURE 12.54 Locations of neutral axes of steel box alone and of composite box section.

A trial section for a girder is assumed as shown in Fig. 12.54. Its neutral axis can be located by taking moments of web and flange areas about a horizontal axis at middepth of the web. This computation and those for the section moduli S_{st} and S_{sb} of the steel section alone are conveniently tabulated in Table 12.63. The moment of inertia of each inclined web I_x may be computed from

$$I_x = \frac{s^2}{s^2 + 1} I \tag{12.44}$$

where s = slope of web with respect to horizontal axis
I = moment of inertia of web with respect to axis at middepth normal to the web = $ht^3/12$
h = depth of web in its plane
t = web thickness normal to its plane

In computation of the properties of the composite section, the concrete slab, ignoring the

TABLE 12.63 Steel Section for Maximum Moment in Box Girder

Material	A	d	Ad	Ad^2	I_o	I
Two top flanges 22 × 1	44.0	32.50	1,430	46,500		46,500
Two webs 66 × ⅜	49.5				16,900	16,900
Bottom flange 70 × 1	70.0	−32.50	−2,275	73,900		73,900
	163.5		−845			137,300
$d_s = -845/163.5 = -5.17$ in					$-5.17 \times 845 =$	−4,300
					$I_{NA} =$	134,000

Distance from neutral axis of steel section to:

Top of steel = 32 + 1.00 + 5.17 = 38.17 in

Bottom of steel = 32 + 0.88 − 5.17 = 27.83 in

Section moduli	
Top of steel	Bottom of steel
$S_{st} = 134{,}000/38.17 = 3{,}510 \text{ in}^3$	$S_{sb} = 134{,}000/27.83 = 4{,}810 \text{ in}^3$

haunch area, is transformed into an equivalent steel area. For the purpose, for this bridge, the concrete area is divided by the modular ratio $n = 10$ for short-time loading, such as live loads and impact. For long-time loading, such as dead loads, the divisor is $3n = 30$, to account for the effects of creep. The computations of neutral-axis location and section moduli for the composite section are tabulated in Table 12.64. To locate the neutral axis, moments are taken about middepth of the girder webs.

Stresses in Composite Section. Since the girders will not be shored when the concrete is cast and cured, the stresses in the steel section for load *DL* are determined with the section moduli of the steel section alone (Table 12.63). Stresses for load *SDL* are computed with the section moduli of the composite section when $n = 30$ (Table 12.64*a*). And stresses in the steel for live loads and impact are calculated with section moduli of the composite section when $n = 10$ (Table 12.64*b*). Calculations for the stresses are given in Table 12.65.

The width-thickness ratio of the unstiffened compression flanges now can be checked, as for an I shape, by the general formula applicable for any stress level:

$$\frac{b}{t} = \frac{194}{\sqrt{F_y}} = \frac{194}{\sqrt{36}} = 32 > \frac{22}{1}$$

Hence, the trial section is satisfactory.

Stresses in the concrete are determined with the section moduli of the composite section with $n = 30$ for *SDL* (Table 12.54*a*) and $n = 10$ for $LL + I$ (Table 12.54*b*). Since the inclination of web plates to a plane normal to the bottom flange is not greater than 1 to 4, and the width of the bottom flange is not greater than 20% of the span (70 in $<$ 0.20 (120) (12) $=$ 288 in), secondary stresses (transverse bending stresses) resulting from distortion of the span, and from distortion of the girder cross section, and from vibrations of the bottom plate need not be considered. Therefore, the composite section is satisfactory. With the thickness specified for maximum moment, no changes in flange thicknesses are desirable. Use the section shown in Fig. 12.54 throughout the span.

Check of Web. The 64-in vertical projection of the webs satisfies the requirements that the depth-span ratio for girder plus slab exceed 1:25 and for girder alone 1:30. The depth-thickness ratio of each web is $66/0.375 = 176$. This is close enough to the AASHTO specifications limiting requirement of $D/t \leq 170$ to be acceptable without longitudinal stiffeners. For the maximum compressive bending stress of 18.24 ksi, the maximum depth-thickness ratio permitted with transverse stiffeners but without longitudinal stiffeners is

$$\frac{D}{t} = \frac{727}{\sqrt{f_b}} = \frac{727}{\sqrt{18.24}} = 170$$

The design shear for the inclined web V_w equals the vertical shear V_v divided by the cosine of the angle of inclination θ of the web plate to the vertical. For a maximum shear of 271.9 kips and a slope of 4 on 1 ($\cos\theta = 0.97$), the design shear is

$$V_w = \frac{V_v}{\cos\theta} = \frac{271.9}{0.97} = 280 \text{ kips}$$

With a cross-sectional area of 49.5 in^2, the web will be subjected to shearing stress considerably below the 12 ksi permitted.

$$f_v = \frac{280}{49.5} = 5.7 < 12 \text{ ksi}$$

Maximum shears at sections along the span are given in Table 12.66.

TABLE 12.64 Composite Section for Maximum Moment in Box Girder

(*a*) For dead loads, $n = 30$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	163.5		−845			134,000
Concrete 216 × 7/30	50.4	37.0	1,865	69,000	200	69,200
	213.9		1,020			203,200
$d_{30} = 1{,}020/213.9 = 4.77$ in					$-4.77 \times 1{,}020 =$	−4,900
					$I_{NA} =$	198,300

Distance from neutral axis of composite section to:

Top of steel = 33.00 − 4.77 = 28.23 in

Bottom of steel = 33.00 + 4.77 = 37.77 in

Top of concrete = 28.23 + 0.50 + 7 = 35.73 in

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 198{,}300/28.23$ $= 7{,}020\ \text{in}^3$	$S_{sb} = 198{,}300/37.77$ $= 5{,}250\ \text{in}^3$	$S_c = 198{,}300/35.73$ $= 5{,}550\ \text{in}^3$

(*b*) For live loads, $n = 10$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	163.5		−845			134,000
Concrete 216 × 7/10	151.2	37.0	5,594	207,000	600	207,600
	314.9		4,749			341,600
$d_{10} = 4{,}749/314.7 = 15.09$ in					$-15.09 \times 4{,}749 =$	−71,600
					$I_{NA} =$	270,000

Distance from neutral axis of composite section to:

Top of steel = 33.00 − 15.09 = 17.91 in

Bottom of steel = 33.00 + 15.09 = 48.09 in

Top of concrete = 17.91 + 0.50 + 7 = 25.41 in

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 270{,}000/17.91$ $= 15{,}070\ \text{in}^3$	$S_{sb} = 270{,}000/48.09$ $= 5{,}610\ \text{in}^3$	$S_c = 270{,}000/25.41$ $= 10{,}630\ \text{in}^3$

TABLE 12.65 Stresses in Composite Box Girder, ksi

(a) Steel stresses	
Top of steel (compression)	Bottom of steel (tension)
DL: $f_b = 4{,}210 \times 12/3{,}570 = 14.39$	$f_b = 4{,}210 \times 12/4{,}810 = 10.50$
SDL: $f_b = 1{,}280 \times 12/7{,}020 = 2.19$	$f_b = 1{,}280 \times 12/5{,}250 = 2.93$
$LL + I$: $f_b = 2{,}530 \times 12/15{,}070 = \underline{2.02}$	$f_b = 2{,}530 \times 12/5{,}610 = \underline{5.41}$
Total: 18.60 < 20	18.84 < 20

(b) Stresses at top of concrete	
SDL: $f_c = 1{,}280 \times 12/(5{,}550 \times 30)$	$= 0.09$
$LL + I$: $f_c = 2{,}530 \times 12/(10{,}630 \times 10)$	$= \underline{0.29}$
Total:	0.38 < 1.0

Flange-to-Web Welds. Fillet welds placed on opposite sides of each girder web to connect it to each flange must resist the horizontal shear between flange and web. In this example, as is usually the case (see Art 12.4, for example), the minimum size of weld permissible for the thickest plate at the connection determines the size of weld. For both the ⅞in bottom flange and the 1-in top flanges, the minimum size of weld permitted is 5⁄16 in. Therefore, use a 5⁄16-in fillet weld on opposite sides of each web at each flange.

Intermediate Transverse Stiffeners. To determine if transverse stiffeners are required, the allowable shear stress F_v will be computed and compared with the average shear stress f_v = 5.03 ksi at the support.

$$F_v = [270(D/t)]^2 = (270/170)^2 = 2.52 \text{ ksi} < 5.03 \text{ ksi}$$

Therefore, transverse intermediate stiffeners are required.

Maximum spacing of stiffeners may not exceed $3 \times 64 = 192$ in or $D[260/(D/t)]^2 = 64(260/170)^2 = 150$ in. Try a stiffener spacing $d_o = 90$ in. This provides a depth-spacing ratio $D/d_o = 64/90 = 0.711$. From Eq. (11.24d), for use in Eq. (11.25a), $k = 5[1 + (0.711)^2] = 7.53$ and $\sqrt{k/F_y} = \sqrt{7.53/36} = 0.457$. Since $D/t = 170$, C in Eq. (11.24a) is determined by the parameter $170/0.457 = 372 > 237$. Hence, C is given by

TABLE 12.66 Maximum Shear in Composite Box Girder

	Distance from support, ft						
	0	10	20	30	40	50	60
DL, kips	183	153	124	93	62	31	0
$LL + I$, kips	89	81	73	65	57	49	41
Total, kips	272	234	197	158	119	80	41
f_v, ksi	5.49	4.73	3.98	3.19	2.40	1.62	0.83

$$C = \frac{45{,}000k}{(D/t)^2F_y} = \frac{45{,}000 \times 7.53}{170^2 \times 36} = 0.326$$

From Eq. (11.25*a*), the maximum allowable shear for $d_o = 90$ in is

$$F'_v = \frac{F_y}{3}\left[C + \frac{0.87(1 - C)}{\sqrt{1 + (d_o/D)^2}}\right]$$

$$= \frac{36}{3}\left[0.326 + \frac{0.87(1 - 0.326)}{\sqrt{1 + (90/64)^2}}\right] = 7.99 \text{ ksi} > 5.03 \text{ ksi}$$

Since the allowable stress is larger than the computed stress, the stiffeners may be spaced 90 in apart.

The AASHTO standard specifications limit the spacing of the first intermediate stiffener to the smaller of $1.5D = 1.5 \times 64 = 96$ in and the spacing for which the allowable shear stress in the end panel does not exceed

$$F_v = CF_y/3 = 0.326 \times 36/3 = 3.91 \text{ ksi} < 5.03 \text{ ksi}$$

Therefore, closer spacing is needed near the supports. Try $d_o = 45$ in, for which $k = 15.11$, $C = 0.654$, and $F_v = CF_y/3 = 0.654 \times 36/3 = 7.85$ ksi > 5.03. Therefore, 45-in spacing will be used near the supports and 90-in spacing in the next 22.5 ft of girder, as shown in Fig. 12.55. Transverse stiffeners are omitted from the central 60 ft of girder, except at midspan.

Where required, a single plate stiffener of Grade 36 steel will be welded inside the box girder to each web. Minimum width of stiffeners is one-fourth the flange width, or $21/4 = 5.25 > 2 + 66/30 = 4.2$ in. Use a 6-in-wide plate. Minimum thickness required is $6/16 = 3/8$ in. Try $6 \times 3/8$-in stiffeners.

The moment of inertia provided by each stiffener must satisfy Eq. (11.21), with J as given by Eq. (11.22).

$$J = 2.5\left(\frac{64}{90}\right)^2 - 2 = -0.73 \qquad \text{use } 0.5$$

$$I = 90(3/8)^3 0.5 = 2.37$$

The moment of inertia furnished is

$$I = \frac{(3/8)6^3}{3} = 27 > 2.37 \text{ in}^4$$

Hence, the $6 \times 3/8$-in stiffeners are satisfactory. Weld them to the webs with a pair of 1/4-in fillet welds.

Bearing Stiffeners. Instead of narrow-plate stiffeners and a cross frame over the bearings, a plate diaphragm extending between the webs is specified. The plate diaphragm has superior resistance to rotation, displacement, and distortion of the box girder. Assume for the diaphragm a bearing length of 20 in at each web, or a total of 40 in. The allowable bearing stress is 29 ksi. Then, the thickness required for bearing is

$$t = \frac{271.9}{40 \times 29} = 0.23 \text{ in}$$

But the thickness of a bearing stiffener also is required to be at least

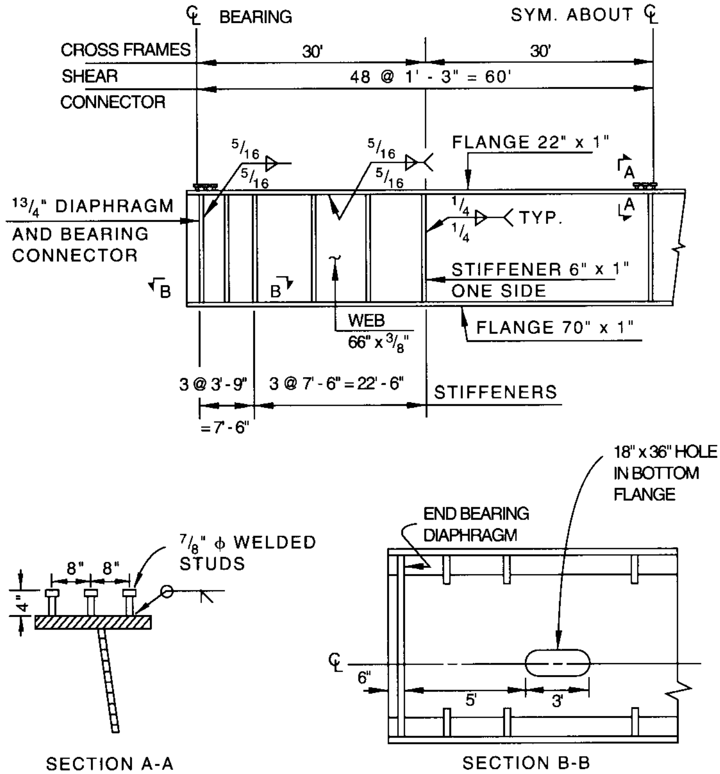


FIGURE 12.55 Locations of stiffeners, cross frames, and shear connectors for composite box girder.

$$t = \frac{b'}{12}\sqrt{\frac{F_y}{33}} = \frac{20}{12}\sqrt{\frac{36}{33}} = 1.74 \text{ in}$$

Therefore, use a plate 64 × 1¾ in extending between the webs at the supports, with a 30-in-square access hole.

The welds to the webs must be capable of developing the entire 271.9-kip reaction. Minimum-size fillet weld for the 1¾-in diaphragm is 5⁄16 in. With two such welds at each web, their required length, with an allowable stress of 15.7 ksi, is

$$\frac{271.9}{4(5/16)0.707 \times 15.7} = 19.6 \text{ in}$$

Weld the full 66-in depth of web.

Shear Connectors. To ensure composite action of concrete deck and box girders, shear connectors welded to the top flanges of the girders must be embedded in the concrete (Art 11.16). For this structure, 7⁄8-in-dia. welded studs are selected. They are to be installed in groups of three at specified locations to resist the horizontal shear between the steel section and the concrete slab (Fig. 12.55). With height $H = 4$ in, they satisfy the requirement $H/d \geq 4$, where d = stud diameter, in.

With $f'_c = 2{,}800$ psi for the concrete, the ultimate strength of a ⅞-in welded stud is, from Eq. (12.26),

$$S_v = 0.4d^2\sqrt{f'_c E_c} = 0.4(7/8)^2\sqrt{2.8 \times 2{,}900} = 27.6 \text{ kips}$$

This value is needed for determining the number of shear connectors required to develop the strength of the steel girder or the concrete slab, whichever is smaller. With an area $A_s = 163.5$ in^2, the strength of the girder is

$$P_1 = A_s F_y = 163.5 \times 36 = 5{,}890 \text{ kips}$$

The compressive strength of the concrete slab is

$$P_2 = 0.85 f'_c bt = 0.85 \times 2.8 \times 216 \times 7 = 3{,}600 < 5{,}890 \text{ kips}$$

Concrete strength governs. Hence, from Eq. (12.25), the number of studs provided between midspan and each support must be at least

$$N_1 = \frac{P_1}{\phi S_v} = \frac{3{,}600}{0.85 \times 27.6} = 153$$

With the studs placed in groups of three on each top flange, there should be at least 153/6 = 26 groups on each half of the girder.

Pitch is determined by fatigue requirements. The allowable load range, kips per stud, may be computed from Eq. (12.4). With $\alpha = 10.6$ for 500,000 cycles of load (AASHTO specifications),

$$Z_r = 10.6(0.875)^2 = 8.12 \text{ kips per stud}$$

At the supports, the shear range $V_r = 89$ kips, the shear produced by live load plus impact. Consequently, with $n = 10$ for the concrete, and the transformed concrete area equal to 151.2 in^2, and $I = 252{,}300$ in^4 from Table 12.64*b*, the range of horizontal shear stress is

$$S_r = \frac{V_r Q}{I} = \frac{89 \times 151.2 \times 20.70}{252{,}300} = 1.107 \text{ kips per in}$$

Hence, the pitch required for stud groups near the supports is

$$p = \frac{6Z_r}{S_r} = \frac{6 \times 8.12}{1.107} = 44 \text{ in}$$

Use a pitch of 15 in to satisfy both this requirement and that for 26 groups of studs between midspan and each support (Fig. 12.55).

Intermediate Cross Frames. Though intermediate cross frames or diaphragms are not required by standard specifications, it is considered good practice by many designers to specify such interior bracing in box girders to help maintain the shape under torsional loading. So in addition to the end bearing diaphragm, cross frames will be installed at 30-ft intervals. Minimum-size angles can be used (Fig. 12.56).

Camber. The girders should be cambered to compensate for dead-load deflections under *DL* and *SDL*. For computation for *DL*, the moment of inertia I of the steel section alone should be used. For *SDL*, I should apply to the composite section with $n = 30$ (Table 12.64*a*). Both deflections can be computed from Eq. (12.5) with $w_{DL} = 2.34$ kips per ft and $w_{SDL} = 0.33$ kip per ft.

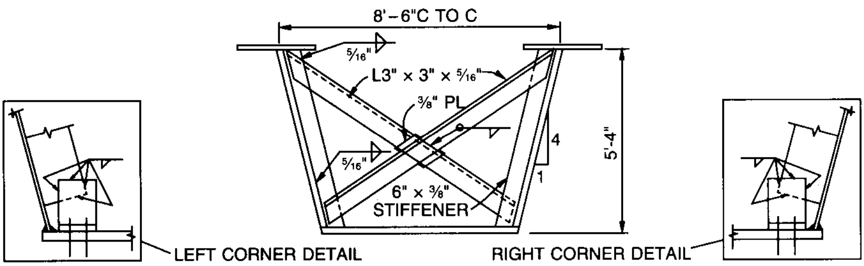


FIGURE 12.56 Intermediate cross frame.

$$\begin{array}{ll} DL: & \delta = 22.5 \times 2.34(120)^4/(29{,}000 \times 123{,}300) = 3.04 \text{ in} \\ SDL: & \delta = 22.5 \times 0.33(120)^4/(29{,}000 \times 187{,}500) = \underline{0.29} \text{ in} \\ \text{Total:} & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 3.33 \text{ in} \end{array}$$

Live-Load Deflection. Maximum live-load deflection should be checked to ensure that it does not exceed $12L/800$. This deflection may be obtained with acceptable accuracy from Eq. (12.6), with

$$P_T = 8 \times 1.113 + 0.204 \times 8 \times 1.113 = 10.73 \text{ kips}$$

From Table 12.64*b*, for $n = 10$, $I = 270{,}000$ in^4. Therefore,

$$\delta = \frac{324 \times 10.73}{29{,}000 \times 270{,}000}(120^3 - 555 \times 120 + 4{,}780) = 0.74 \text{ in}$$

And the deflection-span ratio is

$$\frac{0.74}{120 \times 12} = \frac{1}{1{,}800} < \frac{1}{800}$$

Thus, the live-load deflection is acceptable.

Other Details. These may be treated in the same way as for I-shaped plate girders.

12.14 ORTHOTROPIC-PLATE GIRDER BRIDGES

In orthotropic-plate construction, a steel-plate deck is used instead of concrete. The plate is topped with a wearing surface that may or may not be concrete. The steel-plate deck serves the usual deck function of distributing loads to main carrying members, but it also acts as the top flange of those members (Art. 11.20). Because the deck provides a large area, orthotropic-plate construction is very efficient in resisting bending. With a lightweight wearing surface, furthermore, bridges of this type have relatively low dead load, a characteristic particularly important for keeping down the costs of long spans. Figure 12.57 shows some examples of cross sections that have been used for orthotropic-plate bridges.

These examples indicate that orthotropic plates often are used with box girders. In addition to low dead weight, this type of construction offers many of the advantages of composite box girders discussed in Art. 12.12. The examples, however, are all long-span bridges. It may also be economical for medium spans to use orthotropic plates with girders with inverted-T shapes.

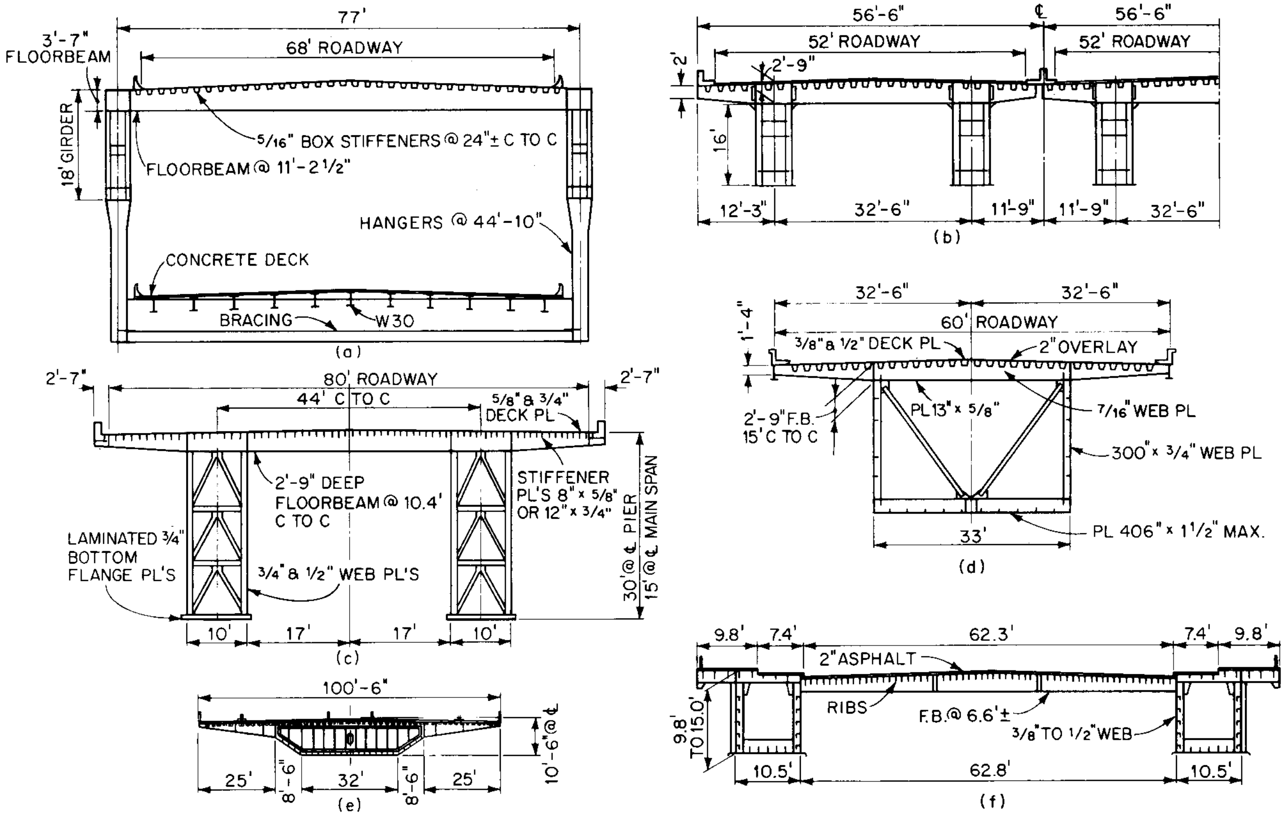


FIGURE 12.57 Examples of cross sections of orthotropic-plate highway bridges. (*a*) Fremont Bridge, Portland, Ore, incorporates continuous tied-arch spans of 448.3–255.3–448.3 ft. (*b*) Poplar St. Bridge over the Mississippi River at St. Louis has continuous spans of 300–500–600–500–265 ft. (*c*) San Mateo-Heyward Bridge over San Francisco Bay, Calif., contains three cantilever–type spans of 375–750–375 ft with a 375-ft suspended span and 187.5 cantilevers. (*d*) San Diego-Coronado Bridge over San Diego Bay, Calif., provides continuous spans of 600–600–500 ft. (*e*) Wye River Bridge, England, cable-stayed, spans 285–770–285 ft. (*f*) Severin River Bridge over the Rhine River, Cologne, Germany, also cable-stayed, has spans of 161–292–157–990–494–172 ft.

For relatively simple types of orthotropic-plate bridges, such as those with the type of cross section shown in Fig. 12.58, approximate analyses by the Pelikan Esslinger method (Art. 4.12) or similar methods give sufficiently accurate results for practical purposes. For more complex types, more accurate analyses may be desirable. These can be executed with the aid of computers. Special attention should be given to the stability of deep webs and wide flanges and to the effect of shear lag on wide decks.

(T. G. Galambos, "Guide to Stability Design Criteria for Metal Structures," John Wiley & Sons, Inc., New York.)

12.15 EXAMPLE—DESIGN OF AN ORTHOTROPIC-PLATE BOX-GIRDER BRIDGE

Following is an example to indicate the design procedure for a bridge with box girders and an orthotropic-plate deck. The example incorporates the major differences in method from that for a single-web plate girder (Art. 12.4) and composite box girder (Art. 12.13).

A two-lane highway bridge with two simply supported box girders, inverted-T floorbeams, and trapezoidal, longitudinal ribs will be designed. The girders are rectangular in section, with webs 30 in c to c. They span 120 ft. The typical cross section in Fig. 12.58 shows a 30-ft roadway flanked by 3-ft-wide safety walks. The transverse floorbeams, spanning 30 ft between the girders, are spaced 15 ft c to c. The ribs, spaced 2 ft c to c, are continuous. Structural steel to be used for the box girders is Grade 36. Loading is H20-44. Appropriate design criteria in Sec. 11 will be used for this structure.

Design Procedure. The Pelikan-Esslinger method of approximate analysis will be used. The bridge will be considered to comprise three systems, as defined in Art. 11.20. They contain four members (Art. 4.12): I—the plate; Il—plate and rib; Ill—plate and floorbeam; and IV—girder and plate, including ribs. Member IV can be designed by conventional procedure and is treated first. The other members are designed in accordance with special theory applicable to orthotropic-plate construction.

12.15.1 Design of Girder with Orthotropic-Plate Flange

Member IV consists of one steel box section spanning 120 ft, seven ribs, and half the width of the steel-plate deck (Fig. 12.58). The ribs and this ⅜-in-thick plate act as part of the top flange of the girder. In addition, a 42 × ½ in plate atop the two webs of the box section also forms part of the top flange. Note that the ⅜-in plate, 180 in wide, slopes upward from the girder toward midspan at 0.02 in per in, or a total of 3.60 in. Consequently, because the ribs are welded to this plate, the center of gravity of the ribs rises an average of 2.30 in.

A 60 × ⅜-in plate is assumed for each web, and a 42-in plate, 1⅛ in thick, is selected for the bottom flange at midspan. Each rib is trapezoidal, a 5⁄16-in bent plate, 12 in wide at the top, 6 in wide at the bottom (Figs. 12.58 and 12.61).

Loads, Moments, and Shears. Member IV is subjected to a dead load consisting of its own weight and the weights of framing details, floorbeams, railing, curb, and wearing course (Table 12.67).

Dead-load moments and shears at 10-ft intervals along the span are listed in Tables 12.68 and 12.69.

The H20-44 live load imposed may be a truck load or a lane load. But for this span, lane loading governs. For bending moment, the 0.64-kip-per-ft uniform load should cover the entire span. For maximum moment at any point on the span, an 18-kip concentrated load

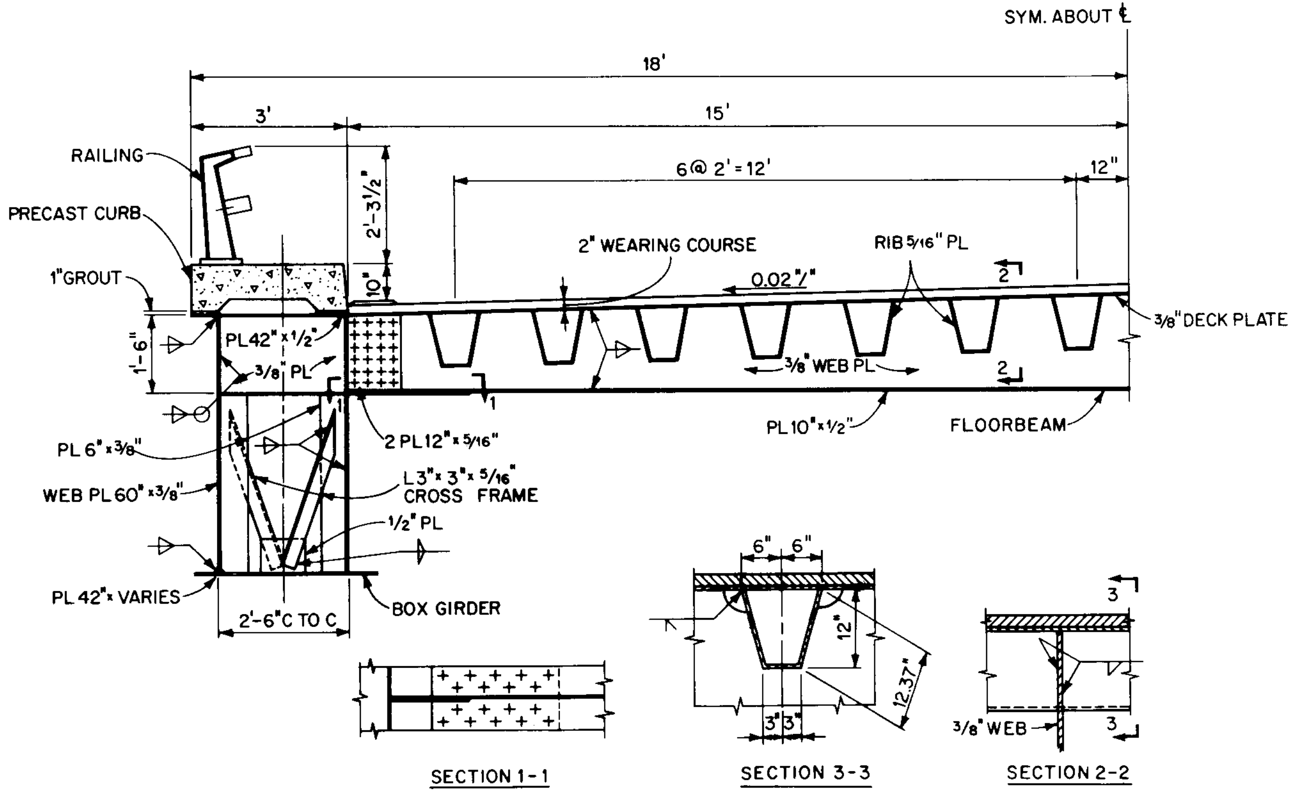


FIGURE 12.58 Half cross section of orthotropic-plate bridge with 120-ft span.

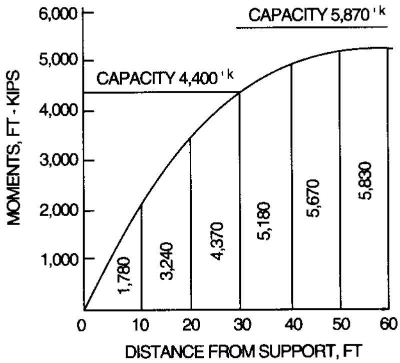


FIGURE 12.59 Moment envelope for girder (member IV).

should be placed at that point. For maximum shear at any point, a 26-kip concentrated load should be placed there, and the uniform load should extend from the point to the farthest support. For example, maximum moment occurs at midspan with the 18-kip load placed there and the uniform load over the entire span:

$$M_T = \frac{0.64(120)^2}{8} + \frac{18 \times 120}{4} = 1{,}152 + 540 = 1{,}692 \text{ ft-kips}$$

Maximum shear occurs at a support with the 26-kip load placed there and the uniform load over the entire span:

$$V_T = \frac{0.64 \times 120}{2} + 26 = 38.4 + 26 = 64.4 \text{ kips}$$

In distributing the two lanes of live load to the girders, the deck is restrained against rotation to some extent by the girders. It may be assumed to be somewhere between simply supported and fixed. In this example, either assumption gives about the same result. For the assumption of a simple support, with one truck wheel 2 ft from a girder, the load distributed by the deck to that girder is

$$W = \frac{28 + 22 + 16 + 10}{30} = 2.53 \text{ wheel}$$

$$= 1.27 \text{ axles}$$

Then, the maximum moment at midspan due to live load is

$$M_{LL} - 1.27 \times 1{,}692 = 2{,}150 \text{ ft-kips}$$

Similarly, maximum live-load shear at the support and maximum reaction is

$$V_{LL} = 1.27 \times 64.4 = 81.8 \text{ kips}$$

Maximum live-load moments and shears at 10-ft intervals along the span are listed in Tables 12.68 and 12.69.

Impact for maximum moments and maximum shear and reaction at the supports may be taken as the following fraction of live-load stress:

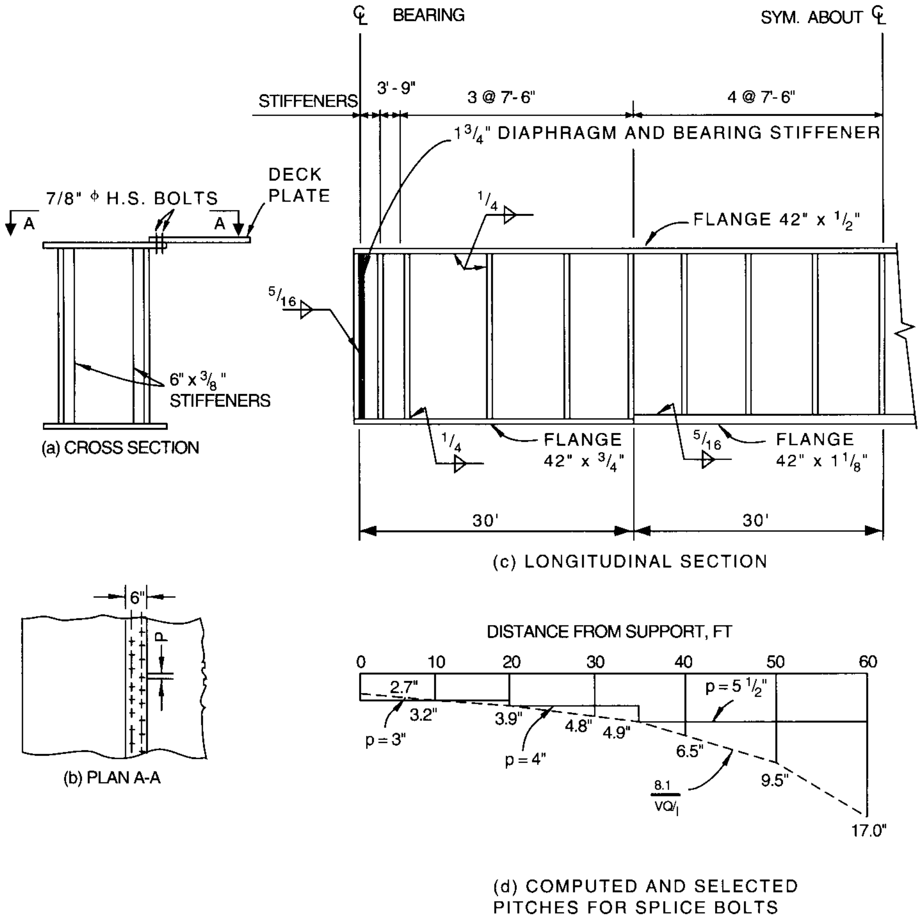


FIGURE 12.60 Details of box girder with orthotropic-plate top flange.

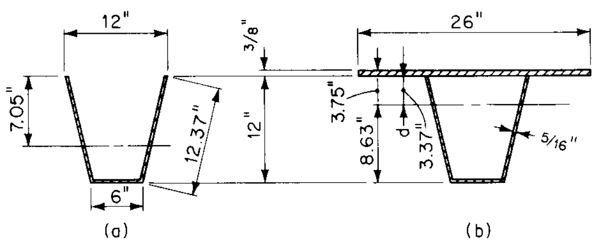


FIGURE 12.61 Sections through rib of orthotropic-plate deck. (*a*) Rib alone. (*b*) Rib with part of plate as top flange.

TABLE 12.67 Dead Load on Member IV, kips per ft

Railing:	0.025
Curb: $0.150 \times 3.0 \times 0.83$	= 0.375
Wearing course: $0.150 \times 15 \times 2/12$	= 0.375
Deck plate: 0.153×15	= 0.230
Ribs: 7×0.0329	= 0.230
Floorbeams—assume:	0.050
Girder and details—assume:	0.500
DL per girder:	1.785, say 1.8

$$I = \frac{50}{L + 125} = \frac{50}{120 + 125} = 0.204$$

Impact moments and shears also are given in Tables 12.68 and 12.69.

These tables, in addition, give the total moments and shears at 10-ft intervals. The moments are used to plot the curve of maximum moments (moment envelope) in Fig. 12.59.

Web Size. Minimum depth-span ratio for a girder is 1:25. For a 120-ft span, therefore, the girders should be at least $120/25 = 4.8$ ft $= 58$ in deep. Use webs 60 in deep.

With an allowable shear stress of 12 ksi for Grade 36 steel, the thickness required for shear in each web is

$$t = \frac{207}{2 \times 12 \times 60} = 0.144 \text{ in}$$

With stiffeners, the web thickness, however, to prevent buckling, should be at least $1/165$ of the depth, or 0.364 in. Use two $60 \times 3/8$-in webs, with an area of 45 in^2.

Check of Section at Midspan. The assumed girder section at midspan is shown in Fig. 12.58, with the bottom flange taken as $1\frac{1}{8}$ in thick. (See also Fig. 12.60.) The neutral axis is located by taking moments of areas of the components about middepth of the webs. This computation and those for moment of inertia and section moduli are given in Table 12.70. The neutral axis is found to be 12.36 in above middepth of web, based on the gross section. (Area of bolt holes is less than 15% of the flange area.)

Maximum bending stress occurs in the bottom flange and equals

TABLE 12.68 Moments in Orthotropic-Plate Box Girder, ft-kips

	Distance from support					
	10	20	30	40	50	60
M_{DL}	990	1,800	2,430	2,880	3,150	3,240
M_{LL}	660	1,200	1,610	1,910	2,090	2,150
M_I	130	240	330	390	430	440
Total	1,780	3,240	4,370	5,180	5,670	5,830

TABLE 12.69 Shears in Orthotropic-Plate Box Girder, kips

	Distance from support						
	0	10	20	30	40	50	60
V_{DL}	108	90	72	54	36	18	0
V_{LL}	82	71	60	99	38	27	17
V_I	17	15	12	10	8	6	3
Total	207	176	144	113	82	51	20
f_v, ksi	4.60	3.91	3.20	2.52	1.82	1.13	0.44

$$f_b = \frac{5{,}830 \times 12}{3{,}520} = 19.9 < 20 \text{ ksi}$$

The section is satisfactory. Moment capacity supplied is

$$M_C = \frac{20 \times 3{,}520}{12} = 5{,}870 \text{ ft-kips}$$

Change in Bottom Flange. At a sufficient distance from midspan, the bending moment decreases enough to permit reducing the thickness of the bottom flange to ¾ in. As indicated in Table 12.71, the moment of inertia reduces to 121.400 in^4 and the section modulus of the bottom flange to 2,640 in^3.

TABLE 12.70 Maximum Moment at Midspan of Box Girder with Orthotropic-Plate Flange

Material	A	d	Ad	Ad^2	I_o	I
Deck plate 180 × ⅜	67.50	32.49	2,192	71,300		71,300
Seven ribs	65.87	25.25	1,664	42,100		42,100
Top flange 42 × ½	21.00	30.25	635	19,200		19,200
Two webs 60 × ⅜	45.00				13,500	13,500
Bottom flange 42 × 1⅛	47.25	−30.56	−1,443	44,400		44,400
	246.62		3,048			190,500

$d = 3{,}048/246.62 = 12.36$ in $\qquad -12.36 \times 3{,}048 = -37{,}600$

$I_{NA} = 152{,}900$

Distance from neutral axis to:

$$\text{Top of steel} = 30.50 + 0.38 + 0.02 \times 180 - 12.36 = 22.12 \text{ in}$$
$$\text{Bottom of steel} = 31.13 + 12.36 = 43.49 \text{ in}$$
$$\text{Bottom of rib} = 22.12 - 0.38 - 12 - 0.02 \times 12 = 9.50 \text{ in}$$

Section moduli		
Top of steel	Bottom of steel	Bottom of rib
$S_t = 152{,}900/22.12 = 6{,}910$ in^3	$S_b = 152{,}900/43.49 = 3{,}520$ in^3	$S_r = 152{,}900/9.50 = 16{,}100$ in^3

TABLE 12.71 Section near Supports of Box Girder with Orthotropic-Plate Flange

Material	A	d	Ad	Ad^2	I
Midspan section	246.62		3,048		190,500
Flange decrease 42 × ⅜	−15.75	−30.94	487	−15,100	−15,100
	230.87		3,535		175,400
$d = 3{,}535/230.87 = 15.30$ in				$-15.30 \times 3{,}535 =$	−54,000
				$I_{NA} =$	121,400

Distance from neutral axis to:

$$\text{Top of steel} = 30.50 + 0.38 + 0.02 \times 180 - 15.30 = 19.18 \text{ in}$$

$$\text{Bottom of steel} = 30.75 + 15.30 = 46.05 \text{ in}$$

$$\text{Bottom of rib} = 19.18 - 0.38 - 12 - 12 \times 0.02 = 6.56 \text{ in}$$

Section moduli		
Top of steel	Bottom of steel	Bottom of rib
$S_t = 121{,}400/19.18$ $= 6{,}330 \text{ in}^3$	$S_b = 121{,}400/46.05$ $= 2{,}640 \text{ in}^3$	$S_r = 121{,}400/6.56$ $= 18{,}500 \text{ in}^3$

With the ¾-in bottom flange, the section has a moment capacity of

$$M_C = \frac{20 \times 2{,}640}{12} = 4{,}400 \text{ ft-kips}$$

When this is plotted on Fig. 12.59, the horizontal line representing it stays above the moment envelope until within 30 ft of midspan. Hence. flange size can be decreased at that point. Length of the 1⅛-in bottom-flange plate then is 60 ft and of the ¾-in plate, which extends to the end of the girder, 30 ft.

The flange plates will be spliced with complete-penetration groove welds. If these welds are ground smooth in the direction of stress and a transition slope of 1 to 2½ is provided for change in plate thickness, the connection falls in Stress Category B for fatigue calculations. The bridge may be treated as a redundant-load-path structure subjected to 500,000 cycles of loading. Hence, the allowable stress range $F_r = 29$ ksi. The stress range for live loads plus impact is

$$f_r = \frac{1{,}940 \times 12}{2{,}640} = 8.82 \text{ ksi} < 29 \text{ ksi—OK}$$

Flange-to-Web Welds. Each flange will be connected to each web by a fillet weld on opposite sides of the web. These welds must resist the horizontal shear between flange and web. Computations can be made, as for the plate-girder stringers in Art. 12.4, but minimum size of weld permissible for the thickest plate at the connection governs. Therefore, use ¼-in welds with the ½-in top flange, the ¾-in bottom flange, and 5⁄16-in welds with the 1⅛-in bottom flange (Fig. 12.60*c*).

Bending Stresses in Deck. As part of member IV, the deck plate is subjected to the following maximum bending stresses:

At midspan	At flange change
$f_b = 5{,}830 \times 12/6{,}910 = 10.12$ ksi	$f_b = 4{,}370 \times 12/6{,}330 = 8.28$ ksi

At midspan, maximum stress at the bottom of a rib is

$$f_b = \frac{5{,}830 \times 12}{16{,}100} = 4.35 \text{ ksi}$$

Intermediate Transverse Stiffeners. To determine if transverse stiffeners are required, the allowable shear stress F_v will be compared with the average shear stress $f_v = 4.60$ ksi at the support. Web depth-thickness ratio $D/t = 60/(3/8) = 160$.

$$F_v = [270/(D/t)]^2 = (270/160)^2 = 2.85 \text{ ksi} < 4.60 \text{ ksi}$$

Therefore, transverse intermediate stiffeners are required.

Maximum spacing of stiffeners may not exceed $3 \times 60 = 180$ in or $D[260/(D/t)]^2 = 60(260/160)^2 = 158$ in. Try a stiffener spacing $d_o = 90$ in. This provides a depth-spacing ratio $D/d_o = 60/90 = 0.667$. From Eq. (11.24*d*), for use in Eq. (11.25*a*), $k = 5[1 + (0.667)^2] = 7.22$ and $\sqrt{k/F_y} = \sqrt{7.22/36} = 0.445$. Since $D/t = 160$, C in Eq. (10.30*a*) is determined by the parameter $160/0.445 = 357 > 237$. Consequently, C is given by

$$C = \frac{45{,}000k}{(D/t)^2 F_y} = \frac{45{,}000 \times 7.22}{160^2 \times 36} = 0.352$$

From Eq. (11.25*a*), the maximum allowable shear for $d_o = 90$ in is

$$F'_v = F_v \left[C + \frac{0.87(1 - C)}{\sqrt{1 + (d_o/D)^2}} \right]$$

$$= \frac{36}{3} \left[0.352 + \frac{0.87(1 - 0.352)}{\sqrt{1 + (90/60)^2}} \right] = 7.98 \text{ ksi} > 4.60 \text{ ksi}$$

Since the allowable stress is larger than the computed stress, the stiffeners may be spaced 90 in apart.

The AASHTO standard specifications limit the spacing of the first intermediate stiffener to the smaller of $1.5D = 1.5 \times 60 = 90$ in and the spacing for which the allowable shear stress in the end panel does not exceed

$$F_v = CF_y/3 = 0.352 \times 36/3 = 4.22 \text{ ksi} < 4.60 \text{ ksi}$$

Therefore, closer spacing is needed near the supports. Try $d_o = 45$ in, for which $k = 13.89$, $C = 0.674$, and $F_v = CF_y/3 = 0.674 \times 36/3 = 8.09$ ksi > 4.60 ksi. Therefore, 45-in spacing will be used near the supports and 90-in spacing for the rest of the girder (Fig. 12.60*c*).

Where required, a single, vertical plate stiffener of Grade 36 steel will be welded inside each box girder to each web. (Longitudinal stiffeners are not required, since the 3/8-in web thickness exceeds $D\sqrt{f_b}/727 = 60\sqrt{19.1}/727 = 0.362$ in.) Width of transverse stiffeners should be at least

$$2 + \frac{D}{30} = 2 + \frac{60}{30} = 4 \text{ in}$$

Use a 6-in-wide plate. Minimum thickness required is $6/16 = 3/8$ in. Try $6 \times 3/8$-in stiffeners.

The moment of inertia provided by each stiffener must satisfy Eq. (11.21), with J as given by Eq. (11.22).

$$J = 2.5 \left(\frac{60}{90}\right)^2 - 2 = -0.89\text{—Use } 0.5$$

$$I = 90(3/8)^3 0.5 = 2.37 \text{ in}^4$$

The moment of inertia furnished is

$$I = \frac{(3/8)6^3}{3} = 27 > 2.37 \text{ in}^4$$

Hence, the $6 \times 3/8$-in stiffeners are satisfactory. Weld them to the webs with a pair of 1/4-in fillet welds.

Bearing Stiffeners. Use a bearing diaphragm. See Art 12.13.

Intermediate Cross Frames. Cross frames should be provided at the floorbeams (Fig. 12.58) to maintain the cross-sectional shape of the box girders.

Longitudinal Splice of Deck and Box Girder. The 3/8-in deck plate is to be attached to the 1/2-in top flange of the box girder with A325 7/8-in-dia, high-strength bolts in slip-critical connections with Class A surfaces (Fig. 12.60*a*). With an allowable stress of 15.5 ksi, each bolt has a capacity of 9.3 kips. The bolts must be capable of resisting the horizontal shear between the top flange and the deck plate. For determination of the pitch of the bolts along the longitudinal splice (Fig. 120.60*b*), the statical moment Q of the deck-plate area, including the ribs about the neutral axis of the girder is needed.

For the girder section at midspan,

$$Q = 67.5 \times 20.13 + 65.87 \times 12.89 = 1{,}360 + 850 = 2{,}210 \text{ in}^3$$

Also, for this section, $Q/I = 2{,}210/152{,}900 = 0.0145$. For the section near the supports,

$$Q = 67.5 \times 17.19 + 65.87 \times 9.95 = 1{,}162 + 656 = 1{,}818 \text{ in}^3$$

And for this section, $Q/I = 1{,}818/121{,}400 = 0.0150$.

Multiplication of Q/I by the shear V, kips, yields the shear, kips per in, to be resisted by the bolts. The pitch required then is found by dividing VQ/I into the bolt capacity 9.3. The shears V can be obtained from Table 12.69. The computed pitch is shown by the dash line in Fig. 12.60*d*, while the pitch to be used is indicated by the solid line. For sealing, the maximum pitch, in, is

$$4 + 4t = 4 + 4 \times 3/8 = 5\tfrac{1}{2} \text{ in}$$

(See also "*F1oorbeam Connections to Girders*" in Art. 12.15.3.)

Camber. The girders should be cambered to compensate for dead-load deflection. In computation of this deflection, an average moment of inertia may be used, in this case, 140,000 in^4. The deflection can be computed from Eq. (12.5) with $w_{DL} = 1.8$ kips per ft.

$$\delta = \frac{22.5 \times 1.8(120)^4}{29{,}000 \times 140{,}000} = 2.1 \text{ in}$$

Live-Load Deflection. Maximum live-load deflection should be checked to ensure that it does not exceed $12L/800$. This deflection may be computed with acceptable accuracy for lane loading from

$$\delta = \frac{144ML^2}{EI} \tag{12.45}$$

where $M = M_{LL} + M_I$ at midspan, ft-kips
L = span, ft
I = average moment of inertia, in^4

For $M_{LL} + M_I = 2{,}380$ ft-kips, from Table 12.68, and an average $I = 140{,}000 \text{ in}^4$,

$$\delta = \frac{144 \times 2{,}380(120)^2}{29{,}000 \times 140{,}000} = 1.21 \text{ in}$$

And the deflection-span ratio is

$$\frac{1.21}{120 \times 12} = \frac{1}{1{,}190} < \frac{1}{800}$$

Thus, the live-load deflection is acceptable.

Other Details. These may be treated in the same way as for I-shaped plate girders.

12.15.2 Design of Ribs

Select Grade 50W steel for the ribs and deck plate for atmospheric corrosion resistance. This steel has a yield strength of 50 ksi in thicknesses up to 4 in. The trapezoidal section chosen for the ribs is shown in Fig. 12.61.

Stresses in the ribs, and in the deck plate as part of the ribs (member II). may be determined by orthotropic-plate theory (Art. 4.12). In the first stage of the calculations, the ribs are assumed to be continuous and supported by rigid floor-beams. In the second stage, midspan bending moments are increased, because the floorbeams actually are flexible. The decrease in rib moments at the supports, however, is ignored.

Rib Dead Load. Each rib supports its own weight and the weights of framing details, a 24-in-wide strip of deck plate. and a 2-in-thick wearing course (Table 12.72). Because ribs in adjoining spans are subjected to the same uniform loading, each rib may be treated as a fixed-end beam.

TABLE 12.72 Dead Load, kips per ft, on Rib

Rib: 30.74×0.00106	= 0.0327
Deck: 24×0.00128	= 0.0306
Wearing course: $0.150 \times 2 \times 2/12$	= 0.0500
Details:	0.0032
DL per rib:	0.1165, say 0.12

Dead-load moment at the support is

$$M_{DL} = -\frac{0.12(15)^2 12}{12} = -27 \text{ in-kips}$$

And at midspan,

$$M_{DL} = \frac{0.12(15)^2 12}{24} = 14 \text{ in-kips}$$

Shear will not be computed because, with two webs per rib, shear stresses are negligible.

Effective Width of Rib Top Flange. Before live-load moments can be determined for the ribs, certain properties of member II must be computed:

$$\text{Effective span } s_e = 0.7s = 0.7 \times 15 \times 12 = 126 \text{ in [Eq. (4.166)]}$$

For determination of the effective width of the deck plate as the top flange of member II, take the effective width a_e at top of rib equal to the actual width a, and the effective rib spacing e_e equal to the actual spacing e between ribs. Then, $a_e/s_e = 12/126 = 0.1$ and $e_e/s_e = 12/126 = 0.1$. From Table 4.6, $a_o/a_e = 1.08$ and $e_o/e_e = 1.08$. Hence, the effective width of the top flange is

$$a'_o + e'_o = 1.08 \times 12 + 1.08 \times 12 = 26 \text{ in}$$

The resulting rib cross section is shown in Fig. 12.61*b*. The neutral axis can be located by taking moments of the areas of rib and deck plate about the top of rib. This computation and those for moment of inertia and section moduli are given in Table 12.73*b*.

The effective top-flange width for computing the relative rigidities of floorbeam and rib is 1.1 $(a + e) = 26.4$ in. The section properties given in Table 12.73*b* will be used, however, because the effect on stresses, in this case, is negligible.

Slenderness of Rib. From Table 12.73*b*, the radius of gyration of the rib $r = \sqrt{I_{NA}/A} = \sqrt{376/19.16} = 4.43$ in, and the slenderness ratio is $L/r = 15 \times 12/4.43 = 41$. The yield strength F_y of the rib steel is 50 ksi. The maximum compressive stress in the deck plate F is 10.12 ksi (Art. 12.15.1). The maximum permissible slenderness ratio is

$$\frac{L}{r} = 1000\sqrt{\frac{1.5}{F_y} - \frac{2.7F}{F_y^2}} = 1000\sqrt{\frac{1.5}{50} - \frac{2.7 \times 10.12}{50^2}} = 138 > 41\text{—OK}$$

Torsional Rigidity. The basic differential equation for an orthotropic plate with closed ribs [Eq. (4.178)] contains two parameters, the flexural rigidity D_y in the longitudinal direction and the torsional rigidity H. The latter may be computed from Eq. (4.181). For that computation, for the trapezoidal rib, by Eq. (4.183),

$$K = \frac{(12 + 6)^2 12^2}{(6 + 2 \times 12.37)/0.3125 + 12/0.375} = 357.5 \text{ in}^4$$

With the shearing modulus $G = 11{,}200$ ksi,

$$GK = 11{,}200 \times 357.5 = 4.01 \times 10^6 \text{ kip-in}^2$$

Also needed for computing H is the reduction factor v. It can be obtained approximately from Eq. (4.184), with

TABLE 12.73 Properties of Ribs

(*a*) Rib without deck plate

Material	A	d	Ad	Ad^2	I_o	I
Two sides 12.37 × 5⁄16	7.73	6.00	46.4	278	79	357
Flange 5.38 × 5⁄16	1.68	11.84	19.9	236		236
	9.41		66.3			593
$d = 66.3/9.41 = 7.05$ in					$-7.05 \times 66.3 =$	-468
					$I_{NA} =$	125

(*b*) Rib with 26-in plate flange

Material	A	d	Ad	Ad^2	I
Rib alone	9.41		66.3		593
Top flange 26 × 3⁄8	9.75	−0.1875	−1.8	0.34	0
	19.16		64.5		593
$d = 64.5/19.16 = 3.37$ in				$-3.37 \times 64.5 =$	-217
				$I_{NA} =$	376

Distance from neutral axis to:

Top of deck plate = 0.375 + 3.37 = 3.75 in

Bottom of rib = 12 − 3.37 = 8.63 in

Section moduli

Top of deck plate	Bottom of rib
$S_t = 376/3.75 = 100 \text{ in}^3$	$S_b = 376/8.63 = 43.6 \text{ in}^3$

$$EI_p = \frac{29{,}000(0.375)^3}{10.92} = 140 \text{ kip-in}^2$$

Thus, the reciprocal of the reduction factor may be taken as

$$\frac{1}{v} = 1 + \frac{4.01 \times 10^6}{140} \frac{12^2}{12(12 + 12)^2} \left(\frac{\pi}{145.8}\right)^2 \times \left[\left(\frac{12}{12}\right)^3 + \left(\frac{12 - 6}{12 + 6} + \frac{6}{12}\right)^2\right] = 6.61$$

Then, by Eq. (4.181),

$$H = \frac{1}{2} \frac{4.01 \times 10^6}{6.61(12 + 12)} = 12{,}600 \text{ in-kip}$$

Flexural Rigidity. The other parameter, the flexural rigidity in the longitudinal direction, for the basic differential equation for the orthotropic plate [Eq. (4.178)] can be obtained from Eq. (4.180):

$$D_y = \frac{29{,}000 \times 376}{12 + 12} = 454{,}000 \text{ in-kips}$$

Plate Parameter. For use in determination of influence coefficients, the plate parameter is

$$\sqrt{\frac{H}{D_y}} = \sqrt{\frac{12{,}600}{454{,}000}} = 0.166$$

Relative Rigidity of Rib and Floorbeam. For calculating the effects of floorbeam deflections on rib moments, the flexibility coefficient is needed. It can be obtained from Eq. (4.192). For the purpose, the moment of inertia of the rib I_r can be taken as I_{NA} in Table 12.73*b*, and the moment of inertia of the floorbeam I_f can be taken as the average of the moment of inertia at midspan and that at the ends, from Table 12.80.

$$I_f = \tfrac{1}{2}(2{,}730 + 1{,}950) = 2{,}340 \text{ in}^4$$

Thus, by Eq. (4.192), with $l = 0.7 \times 360 = 252$ in because the floorbeams will be considered fixed at the supports under live loads,

$$\gamma = \frac{(252)^4 376}{\pi^4 (180)^3 (12 + 12) 2{,}340} = 0.0477$$

Live Load as Fourier Series. The roadway can be divided into two design lanes, centered in the 30-ft roadway. For maximum moments in the ribs, place a truck in each design lane close to the bridge centerline, as indicated in Fig 12.62. (Larger stresses result with the loads moved 1 ft off center, but the effect is small.) For use with the moment-influence coefficients to be computed, this loading has to be converted into a Fourier series. For the two axle loads in Fig. 12.62, Eq. (4.172) can be used for this purpose, with the wheel load $P = 16$ kips, spacing $c = 72$ in, and location of axle centers $x_1 = 120$ in and $x_2 = 240$ in. Because the loading is symmetrical, the integer n should be taken only as odd numbers. The load will be distributed, in this case, over the whole 30-ft roadway.

$$\begin{aligned} Q_n &= \frac{4 \times 16}{360} \cos \frac{72 n \pi}{2 \times 360} \left(\sin \frac{120\, n\pi}{360} + \sin \frac{240\, n\pi}{360} \right) \\ &= 0.178 \cos 18^\circ n (\sin 60^\circ n + \sin 120^\circ n) \qquad n = 1, 3, 5, \ldots \end{aligned}$$

Substitution of $n = 1$ yields

$$Q_1 = 0.178 \times 0.951(0.866 + 0.866) = 0.293$$

For $n = 3$, the sines become zero, and for $n = 5$, the cosine is zero. Hence,

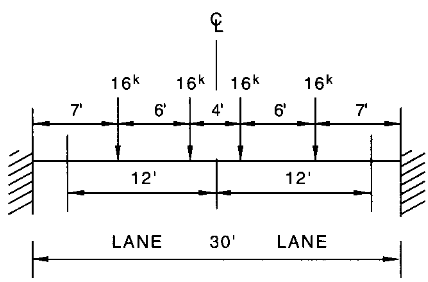


FIGURE 12.62 Positions of truck wheels for maximum moments in ribs and floorbeams.

$$Q_3 = Q_5 = 0$$

For $n = 7$,

$$Q_7 = 0.178(-0.588)(0.866 + 0.866) = -0.181$$

For this example, only two terms of the Fourier series will be used. In general, however, additional terms are required for accuracy, because the computations require subtraction of nearly equal numbers.

Parameters for Influence Coefficients. The ribs at this stage are considered continuous, with span $s = 180$ in, over rigid supports. For calculation of moment-influence coefficients, the parameter α_n given by Eq. (4.187) is needed.

$$\alpha_n = \frac{n\pi}{l}\sqrt{\frac{H}{D_y}}\sqrt{2} = \frac{n\pi}{360}0.166\sqrt{2} = 0.00205n$$

Values of functions of α_n for $n = 1$ and $n = 7$ needed for moment-influence coefficients are tabulated in Table 12.74. Also required is the carry-over factor κ_n given by Eq. (4.188). For this calculation, β_n and k_n are computed in Table 12.74*a*.

$$\kappa_1 = \sqrt{1.823^2 - 1} - 1.823 = -0.30 \qquad \kappa_7 = \sqrt{2.66^2 - 1} - 2.66 = -0.20$$

Moment-Influence Coefficients. Substitution of the computed values in Eq. (4.189) yields the influence coefficients for bending moment at an unyielding support:

$$m_{O1} = -2.500(3.61 \sinh 0.00205y - \cosh 0.00205y - 0.00722y + 1)$$

$$m_{O7} = -61.5(1.04 \sinh 0.01435y - \cosh 0.01435y - 0.00667y + 1)$$

Substitution of the computed values in Eq. (4.190) gives the influence coefficients for bending moment at midspan when supports are rigid:

$$m_{C1} = 240 \sinh 0.00205y - 858(0.1730 \sinh 0.00205y - \cosh 0.00205y + 1)$$

$$m_{C7} = 17.8 \sinh 0.01435y - 12.6(0.859 \sinh 0.01435y - \cosh 0.01435y + 1)$$

Rib Live-Load Moments, Rigid Floorbeams. For maximum moment in a rib at a support with H20 loading, place the 16-kip truck wheels 7 ft from the support on one span ($y = 84$) and the 4-kip wheels 7 ft from the same support on the adjoining span ($y = 84$). With values tabulated in Table 12.74*b*, the moment-influence coefficients become, for $y = 84$, $m_{O1} = -8.25$ and $m_{O7} = -12.35$.

The bending moments at the support will be computed at the centerline of the bridge, $x = 180$ in. Then, by Eq. (4.169),

$$Q_{nx} = Q_1 - Q_7 + \cdots = 0.293 + 0.181 + \cdots$$

By Eq. (4.191), the moment at the support due to the 16-kip wheel at $y = 84$ is

$$M_O = -24(8.25 \times 0.293 + 12.35 \times 0.181) = -112 \text{ in-kips}$$

Because of the 4-kip wheel in the adjoining span, also at $y = 84$,

$$M_O = -112 \times 4/16 = -28 \text{ in-kips}$$

Thus, the live-load moment at the support is $M_{LL} = -112 - 28 = -140$ in-kips.

TABLE 12.74 Values of Functions for Computing Influence Coefficients

(*a*) Functions of rib span *s*

$\alpha_1 = 0.00205$	$\alpha_1 s/2 = 0.185$	$\alpha_7 = 0.1435$	$\alpha_7 s/2 = 1.29$
$\alpha_1 s = 0.370$		$\alpha_7 s = 2.59$	
$\sinh \alpha_1 s = 0.379$	$\cosh \alpha_1 s/2 = 1.017$	$\sinh \alpha_7 s = 6.63$	$\cosh \alpha_7 s/2 = 1.954$
$\cosh \alpha_1 s = 1.069$	$\tanh \alpha_1 s/2 = 0.1730$	$\cosh \alpha_7 s = 6.70$	$\tanh \alpha_7 s/2 = 0.859$
$\tanh \alpha_1 s = 0.354$		$\tanh \alpha_7 s = 0.989$	
$\coth \alpha_1 s = 2.82$		$\coth \alpha_7 s = 1.011$	
$\beta_1 = \dfrac{0.379 - 0.370}{0.379} = 0.0238$		$\beta_7 = \dfrac{6.63 - 2.59}{6.63} = 0.608$	
$k_1 = \dfrac{0.370 \times 2.82 - 1}{0.0238} = 1.823$		$k_1 = \dfrac{2.59 \times 1.011 - 1}{0.608} = 2.66$	

(*b*) Functions for $y = 84$

$0.00205 \times 84 = 0.172$	$0.01435 \times 84 = 1.206$
$\sinh 0.172 = 0.1728$	$\sinh 1.206 = 1.520$
$\cosh 0.172 = 1.014$	$\cosh 1.206 = 1.820$

(*c*) Functions for $y = 90$

$0.00205 \times 90 = 0.185$	$0.01435 \times 90 = 1.292$
$\sinh 0.185 = 0.1861$	$\sinh 1.292 = 1.683$
$\cosh 0.185 = 1.017$	$\cosh 1.292 = 1.957$

(*d*) Functions for $y = 78$

$0.00205 \times 78 = 0.160$	$0.01435 \times 78 = 1.12$
$\sinh 0.160 = 0.1607$	$\sinh 1.12 = 1.369$
$\cosh 0.160 = 1.013$	$\cosh 1.12 = 1.696$

For maximum moment at midspan, place the 16-kip wheels there ($y = 90$). The 4-kip wheels will be on the adjoining span 6.5 ft from the support ($y = 78$). The midspan moments also will be computed for $x = 180$. The moment-influence coefficients become, with values from Table 12.74 for $y = 90$, $m_{C1} = 31.8$ and $m_{C7} = 23.9$. Then, for the 16-kip wheels, by Eq. (4.191),

$$M_C = 24(31.8 \times 0.293 + 23.9 \times 0.181) = 328 \text{ in-kips}$$

The effect of the 4-kip wheels on the midspan moment is found in several steps. First, the moment M_O at the support is computed. This requires determination of the moment-influence coefficients for $y = 78$. Then, the carry-over factors are used to calculate the midspan moment:

$$M_C = \frac{M_O(1 - \kappa_n)}{2} \tag{12.46}$$

With values from Table 12.74*d*, the moment-influence coefficients become $m_{O1} = -5.00$ and $m_{O7} = -12.6$. By Eq. (4.191), for the 4-kip wheels,

$$M_{O1} = -\tfrac{4}{16} \times 24 \times 5.00 \times 0.293 = -8.8 \text{ in-kips}$$

$$M_{O7} = -\tfrac{4}{16} \times 24 \times 12.6 \times 0.181 = -13.7 \text{ in-kips}$$

With Eq. (12.45), the midspan moment due to the 4-kip wheels is found to be

$$M_C = -8.8 \frac{1 + 0.300}{2} - 13.7 \frac{1 + 0.200}{2} = -14 \text{ in-kips}$$

Thus, the live-load moment at midspan with rigid supports is

$$M_{LL} = 328 - 14 = 314 \text{ in-kips}$$

Rib Live-Load Moments, Flexible Floorbeams. Because the floorbeams actually are not rigid and deflect under live loading, end moments in ribs are less than they would be with rigid supports, and midspan moments are larger. The decrease in support moments in this case will be ignored. The increase in midspan moment, however, will be computed from Eq. (4.176).

From Table 4.4, the influence coefficient for reaction due to a load at midspan is 0.601. The flexibility coefficient has previously been computed to be $\gamma = 0.0477$. From Table 4.7, the influence coefficient for midspan moment in a continuous beam on elastic supports, with load at support 0, is 0.027. Taking into account only the effects of the two supports on either side of midspan, the influence coefficient for change in midspan moment is $2(0.601 \times 0.027) = 0.0324$. By Eq. (4.176), the change in midspan moment is

$$\Delta M_C = 0.293 \times 180 \times 24 \times 0.0324 = 41 \text{ in-kips}$$

Therefore, the live-load moment at midspan with flexible supports is

$$M_{LL} = 314 + 41 = 355 \text{ in-kips}$$

Impact. For the 15-ft rib span, impact should be taken as 30% of live-load stresses.

At midspan, $M_I = 0.30 \times 355 = 106$ in-kips.
At supports, $M_I = 0.30(-140) = -42$ in-kips.

Maximum Rib Moments. The design moments previously calculated for member II are summarized in Table 12.75.

In a similar way, stress reversals can be computed for investigation of fatigue stresses.

Rib Stresses. Section properties for determination of member II stresses are given in Table 12.74*b*. Computations for maximum rib stresses at midspan and supports are given in Table 12.76. The compressive stress at the bottom of member II is augmented by the compressive stress induced when the rib acts as part of the top flange of member IV. This stress was

TABLE 12.75 Rib Moments, in-kips

	M_{DL}	M_{LL}	M_I	Total M
Midspan	14	355	106	475
Supports	−27	−140	−42	−209

TABLE 12.76 Bending Stresses in Ribs

At midspan:	
Top of deck plate	$f_b = 475/100 = 4.75$ ksi (compression)
Bottom of rib	$f_b = 475/43.6 = 10.9 < 27$ ksi (tension)
At supports:	
Bottom of rib	$f_b = 209/43.6 = 4.80$ ksi (compression)

previously computed to be 4.26 ksi. Hence, the total compressive stress at the bottom of the rib is

$$f_b = 4.80 + 4.26 = 9.06 < 1.25 \times 27 \text{ ksi}$$

Rib Stability. Closed ribs of the dimensions usually used have high resistance to buckling. In this case, therefore, there is no need to check the stability of the ribs, especially since compressive bending stresses are low.

12.15.3 Design of Floorbeam with Orthotropic-Plate Flange

Select Grade 50W steel for the floorbeams. This steel provides atmospheric corrosion resistance and has a yield strength $F_y = 50$ ksi in thicknesses up to 4 in.

The floorbeams are tapered. Web depth ranges from 21 in at midspan to 18 in at the box-girder supports (Fig. 12.63). The span is 30 ft. Spacing is 15 ft c to c. The floorbeams are considered simply supported for dead load, fixed end for live loads.

Floorbeam Dead Load. Each beam supports its own weight and the weights of framing details, a 15-ft-wide strip of deck, including 14 ribs, and a 2-in-thick wearing course (Table 12.77).

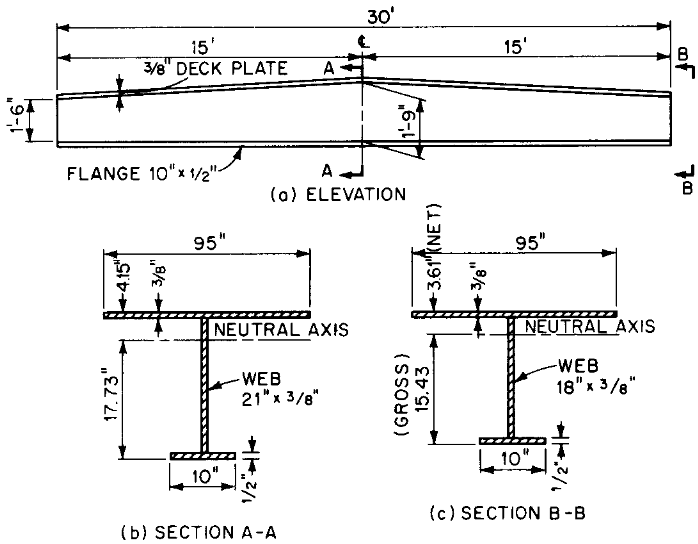


FIGURE 12.63 Floorbeam with orthotropic-plate top flange.

TABLE 12.77 Dead Load on Floorbeam, kips per ft, with Orthotropic-Plate Flange

Floorbeam	= 0.042
Deck plate: 0.153×15	= 0.230
Ribs: $14 \times 0.036 \times {}^{15}\!/_{30}$	= 0.252
Wearing course: $0.150 \times 15 \times {}^{2}\!/_{12}$	= 0.375
Details:	0.026
DL per beam:	0.925

Maximum dead-load moment occurs at midspan and equals

$$M_{DL} = \frac{0.925(30)^2}{8} = 104 \text{ ft-kips}$$

Maximum dead-load shear occurs at the supports and equals

$$V_{DL} = \frac{0.925 \times 30}{2} = 13.9 \text{ kips}$$

Floorbeam Live-Load Moments. Bending moments are computed in two stages. In the first, the floorbeams are assumed to act as rigid supports for the continuous ribs. In the second stage, the changes in moments due to flexibility of the floorbeams are calculated.

For maximum moments in a floorbeam in the first stage, the H20 truck live loads are placed in each of the two design lanes as indicated in Fig. 12.62. The 16-kip wheels are placed over the floorbeam. A 4-kip wheel is located 14 ft away on each of the adjoining rib spans. From Table 4.4, the reaction influence coefficient for this location is estimated to be 0.06. Hence, the wheel loads on the floorbeam equal

$$P = 16 \times 1.00 + 2 \times 4 \times 0.06 = 16.5 \text{ kips}$$

For this loading, the bending moment at the support is

$$M_{LL} = -\frac{16.5}{30^2}\,[7(23)^2 + 13(17)^2 + 17(13)^2 + 23(7)^2] = -210 \text{ ft-kips}$$

and the midspan moment is

$$M_{LL} = -210 + 2 \times 16.5 \times 13 - 16.5 \times 6 = 110 \text{ ft-kips}$$

The effects on these moments of floorbeam flexibility may be approximated in the following way, using the first term of the Fourier series for the loading in Fig. 12.62: For the 16-kip wheels, $Q_1 = 0.293$. Hence, $Q_1 = 0.293 \times 16.5/16 = 0.302$ for 16.5-kip wheels, and for 4-kip wheels, $Q_1 = 0.293 \times 4/16 = 0.073$. From Table 4.7, for $\gamma = 0.0477$, the reaction influence coefficient for unit load at the support is 0.75, and for unit load at an adjacent support, 0.14. From Table 4.4, the reaction influence coefficient for the 4-kip wheels 1 ft from the adjacent support is 0.99. Thus, for the first term of the Fourier series, the reaction of the loading on the floorbeam is

$$R = 0.302 \times 0.75 + 2 \times 0.073 \times 0.99 \times 0.14 = 0.247$$

Now, $Q_1 = 0.302$ corresponds to the loading that produced the live-load moments previously calculated on the assumption that the floorbeams were rigid. Also, $Q_1 = 0.247$ corresponds to loading with the same distribution on the floorbeam. Therefore, the bending moments can be found by proportion from those previously calculated. Thus, the moment at the support is

$$M_{LL} = -\frac{210 \times 0.247}{0.0302} = -172 \text{ ft-kips}$$

and the moment at midspan is

$$M_{LL} = \frac{110 \times 0.247}{0.302} = 90 \text{ ft-kips}$$

Impact. For a 30-ft span, impact is taken as 30%.

At midspan, $M_I = 0.30 \times 90 = 27$ ft-kips.
At supports, $M_I = 0.30(-172) = -52$ ft-kips.

Total Floorbeam Moments. The design moments previously calculated are summarized in Table 12.78.

Properties of Floorbeam Sections. For stress computations, an effective width s_o of the deck plate is assumed to act as the top flange of member III. For determination of s_o, the effective spacing of floorbeams s_f is taken equal to the actual spacing, 180 in. The effective span l_e, with the floorbeam ends considered fixed, is taken as $0.7 \times 30 \times 12 = 252$ in. Hence,

$$\frac{s_f}{l_e} = \frac{180}{252} = 0.715$$

From Table 4.6 for this ratio,

$$\frac{s_o}{s_f} = 0.53, \text{ and } s_o = 0.53 \times 180 = 95 \text{ in}$$

(Fig. 12.63*b* and *c*).

The neutral axis of the floorbeam sections at midspan and supports can be located by taking moments of component areas about middepth of the web. This computation and those for moments of inertia and section moduli are given in Table 12.79.

Floorbeam Stresses. These are determined for the total moments in Table 12.78 with the section properties given in Table 12.79. Calculations for the stresses at midspan and the supports are given in Table 12.80. Since the stresses are well within the allowable, the floorbeam sections are satisfactory.

TABLE 12.78 Moments, ft-kips, in Floorbeam with Orthotropic-Plate Flange

	M_{DL}	M_{LL}	M_I	Total M
Midspan	104	90	27	221
Supports	0	−712	−52	−224

TABLE 12.79 Floorbeam Moments of Inertia and Section Moduli

(a) At midspan

Material	A	d	Ad	Ad^2	I_o	I
Deck 95 × ⅜	35.6	10.69	381	4,070		4,070
Web 21 × ⅜	7.9				290	290
Bottom flange 10 × ½	5.0	−10.75	−54	580		580
	48.5		327			4,940
$d = 327/48.5 = 6.73$ in					$-6.73 \times 327 =$	−2,210
					$I_{NA} =$	2,730

Distance from neutral axis to:

Top of deck plate = 10.50 + 0.375 − 6.73 = 4.15 in

Bottom of rib = 10.50 + 0.50 + 6.73 = 17.73 in

Section moduli

Top of deck plate	Bottom of rib
$S_t = 2{,}730/4.15 = 658$ in³	$S_b = 2{,}730/17.73 = 154$ in³

(b) At supports, gross section

Material	A	d	Ad	Ad^2	I_o	I
Deck 95 × ⅜	35.6	9.19	327	3,010		3,010
Web 18 × ⅜	6.8				180	180
Bottom flange 10 × ½	5.0	−9.25	−46	430		430
	47.4		281			3,620
$d_g = 281/47.4 = 5.93$ in					$-5.93 \times 281 =$	−1,670
					Gross $I_{NA} =$	1,950

Distance from neutral axis to:

Bottom of rib = 9 + 0.50 + 5.93 = 15.43 in

Section modulus, bottom of rib

$S_b = 1{,}950/15.43 = 126$ in³

(c) At supports, net section

Material	A	d	Ad	Ad^2	I_o	I
Gross section	47.4		281			3,620
Top-flange holes	−10.2	9.19	−94	−860		−860
Bottom-flange holes	−1.0	−9.25	9	−90		−90
Web holes	−2.3				−120	−120
	33.9		196			2,550
$d_{net} = 196/33.9 = 5.77$ in					$-5.77 \times 196 =$	−1,130
					Net $I_{NA} =$	1,420

TABLE 12.79 Floorbeam Moments of Inertia and Section Moduli (*Continued*)

Distance from neutral axis to: Top of deck plate $= 9 + 0.375 - 5.77 = 3.61$
Section modulus, top of deck plate
$S_t = 1{,}420/3.61 = 392 \text{ in}^3$

Floorbeam Shears. For maximum shear, the truck wheels are placed in each design lane as indicated in Fig. 12.64. The 16-kip wheels are placed over the floorbeam. A 4-kip wheel is located 14 ft away on each of the adjoining rib spans. Thus, with the floorbeams assumed acting as rigid supports for the ribs, the wheel load is 16.5 kips, as for maximum floorbeam moment. (The effects of floorbeam flexibility can be determined as for bending moments.)

This loading produces a simple-beam reaction of 41.8 kips. It also causes end moments of -202 and -86, which induce a reaction of $(-86 + 202)/30 = 3.9$ kips. Hence, the maximum live-load reaction and shear equal

$$V_{LL} = 41.8 + 3.9 = 45.7 \text{ kips}$$

Shear due to impact is

$$V_I = 0.30 \times 45.7 = 13.7 \text{ kips}$$

MAXIMUM FLOORBEAM SHEARS, KIPS

V_{DL}	V_{LL}	V_I	Total V
13.9	45.7	13.7	73.3

Allowable shear stress in the web for Grade 50W steel is 17 ksi. Average shear stress in the web is

$$f_v = \frac{73.3}{18 \times 3/8} = 10.9 < 17 \text{ ksi}$$

Transverse stiffeners are not required.

Flange-to-Web Welds. The web will be connected to the deck plate and the bottom flange by a fillet weld on opposite sides of the web. These welds must resist the horizontal shear between flange and web. For the weld to the $10 \times \frac{1}{2}$-in bottom flange, the minimum size

TABLE 12.80 Bending Stresses in Member III

At midspan:	
Top of deck plate	$f_b = 221 \times 12/658 = 4.03$ ksi (compression)
Bottom flange	$f_b = 221 \times 12/154 = 17.2 < 27$ ksi (tension)
At supports:	
Top of deck plate	$f_b = 224 \times 12/392 = 6.9$ ksi (tension)
Bottom flange	$f_b = 224 \times 12/126 = 21.4 < 27$ ksi

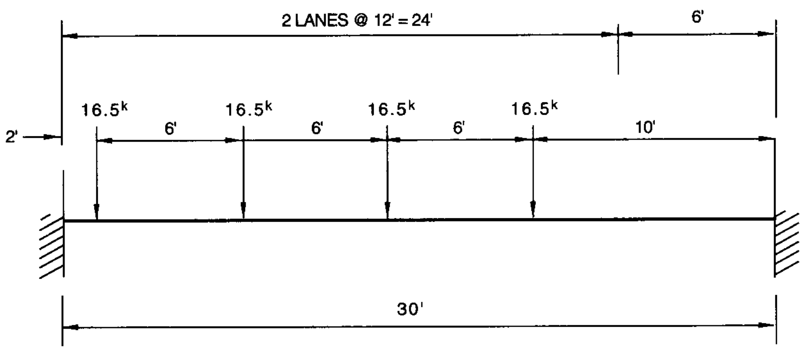


FIGURE 12.64 Positions of truck wheels for maximum shear in floorbeam.

fillet weld permissible with a ½-in plate, ¼ in, may be used. Shear, however, governs for the weld to the deck plate.

For computing the shear v, kips per in, between web and deck plate, the total maximum shear V is 73.3 kips and the moment of inertia of the floorbeam cross section I is 1,950 in.^{4} The static moment of the deck plate is

$$Q = 35.6(4.15 - 0.19) = 141 \text{ in}^3$$

Hence, the shear to be carried by the welds is

$$v = \frac{VQ}{I} = \frac{73.3 \times 141}{1{,}950} = 5.30 \text{ kips per in}$$

The allowable stress on the weld is 18.9 ksi. So the allowable load per weld is $18.9 \times 0.707 = 13.4$ kips per in, and for two welds, 26.7 kips per in. Therefore, the weld size required is $5.30/26.7 = 0.20$ in. Use ¼-in fillet welds.

Floorbeam Connections to Girders. Since the bottom flange of the floorbeam is in compression, it can be connected to the inner web of each box girder with a splice plate of the same area. Use a 10 × ½-in plate, shop-welded to the girder and field-bolted to the floorbeam. With A325 ⅞-in-dia. high-strength bolts in slip-critical connections with Class A surfaces, the allowable load per bolt is 9.3 kips. If the capacity of the 10 × ½-in flange is developed at the allowable stress of 27 ksi, the number of bolts required in the connection is $27 \times 5/9.3 = 15$. Use 16.

The deck plate is spliced to the girder with ⅞-in-dia. high-strength bolts. To meet girder requirements, the pitch may vary from 3 to 5½ in (Fig. 12.60*d*). But the bolts also must transmit the tensile forces from the deck plate to the girder when the plate acts as the top flange of member III. The shear in the bolts from the girder compression is perpendicular to the shear from the floorbeam tension. Hence, the allowable load per bolt decreases from 9.3 to $9.3 \times 0.707 = 6.6$ kips. With an average tensile stress in the deck plate of 6.2 ksi, and a net area after deduction of holes of $35.6 - 10.2 = 25.4$ in^2, the plate carries a tensile force of $25.4 \times 6.2 = 158$ kips. Thus, to transmit this force, $158/6.6 = 24$ bolts are needed. If a pitch of 3 in is used in the 95-in effective width of the plate, 31 bolts are provided. Use a 3-in pitch for 4 ft on each side of every floorbeam.

The web connection to the girder must transmit both vertical shear, $V = 73.3$ kips, and bending moment. The latter can be computed from the stress diagram for the cross section (Fig. 12.65*a*).

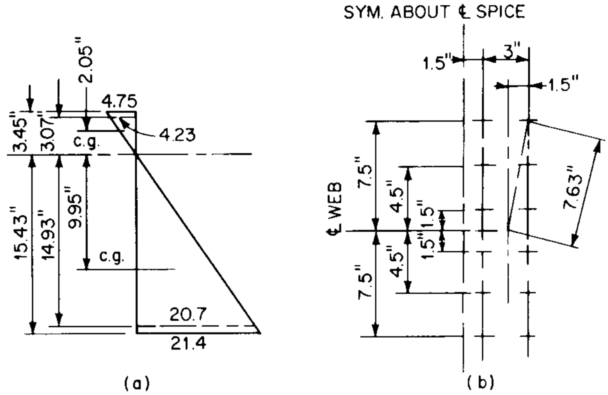


FIGURE 12.65 (*a*) Bending stresses in floorbeam at supports. (*b*) Bolted web connection of floorbeam to girder.

$$M = \tfrac{1}{2} \times \tfrac{3}{8}(4.23 \times 3.07 \times 2.05 + 20.7 \times 14.93 \times 9.95) = 581 \text{ in-kips}$$

Assume that the connection will be made with two rows of six bolts each, on each side of the connection centerline (Fig. 12.65*b*). The polar moment of inertia of these bolts can be computed as the sum of the moments of inertia about the *x* (horizontal) and *y* (vertical) axes.

$$\begin{aligned} I_x &= 4(1.5^2 + 4.5^2 + 7.5^2) && = 315 \\ I_y &= 12(1.5)^2 && = \underline{\ 27} \\ &= & J &= 342 \end{aligned}$$

Load per bolt due to shear is

$$P_v = \frac{73.3}{12} = 6.1 \text{ kips}$$

Load on the outermost bolt due to moment is

$$P_m = \frac{581 \times 7.63}{342} = 12.95 \text{ kips}$$

The vertical component of this load is

$$P_v = \frac{12.95 \times 1.5}{7.63} = 2.5 \text{ kips}$$

and the horizontal component is

$$P_h = \frac{12.95 \times 7.5}{7.63} = 12.7 \text{ kips}$$

The total load on the outermost bolt is the resultant

$$P = \sqrt{(6.1 + 2.5)^2 + 12.7^2} = 15.3 < 2 \times 9.3$$

For the web connection plates, try two plates $17\tfrac{1}{2} \times \tfrac{5}{16}$ in. They have a net moment of inertia

$$I = 2\,\frac{(5/16)17.5^3}{12} - 50 = 228 \text{ in}^4$$

To transmit the 581-in-kip moment in the web, they carry a bending stress of

$$f_b = \frac{581 \times 8.75}{228} = 22.3 < 27 \text{ ksi}$$

The assumed plates are therefore satisfactory if Grade 50W steel is used.

12.15.4 Design of Deck Plate

The deck plate is to be made of Grade 50W steel. This steel has a yield strength F_y = 50 ksi for the 3⁄8-in deck thickness.

Stresses. Bending stresses in the deck plate as the top flange of ribs (member II), floorbeams (member III), and girders (member IV) are relatively low. Combining the stresses of members II and IV yields $4.75 + 9.73 = 14.48 < 1.25 \times 27$ ksi.

Deflection. The thickness of deck plate to limit deflection to 1⁄300 of the rib spacing can be computed from Eq. 11.72. For a 16-kip wheel, assumed distributed over an area of $26 \times 12 = 312$ in^2, the pressure, including 30% impact, is

$$p = 1.3 \times 16/312 = 0.0667 \text{ ksi}$$

Required thickness with rib spacing $a = e = 12$ in is

$$t = 0.07 \times 12(0.0667)^{1/3} = 0.341 < 0.375 \text{ in}$$

The 3⁄8-in deckplate is satisfactory.

12.16 CONTINUOUS-BEAM BRIDGES

Articles 12.1 and 12.3 recommended use of continuity for multispan bridges. Advantages over simply supported spans include less weight, greater stiffness, smaller deflections, and fewer bearings and expansion joints. Disadvantages include more complex fabrication and erection and often the costs of additional field splices.

Continuous structures also offer greater overload capacity. Failure does not necessarily occur if overloads cause yielding at one point in a span or at supports. Bending moments are redistributed to parts of the span that are not overstressed. This usually can take place in bridges because maximum positive moments and maximum negative moments occur with loads in different positions on the spans. Also, because of moment redistribution due to yielding, small settlements of supports have no significant effects on the ultimate strength of continuous spans. If, however, foundation conditions are such that large settlements could occur, simple-span construction is advisable.

While analysis of continuous structures is more complicated than that for simple spans, design differs in only a few respects. In simple spans, maximum dead-load moment occurs at midspan and is positive. In continuous spans, however, maximum dead-load moment occurs at the supports and is negative. Decreasing rapidly with distance from the support, the negative moment becomes zero at an inflection point near a quarter point of the span. Between the two dead-load inflection points in each interior span, the dead-load moment is positive, with a maximum about half the negative moment at the supports.

As for simple spans, live loads are placed on continuous spans to create maximum stresses at each section. Whereas in simple spans maximum moments at each section are always positive, maximum live-load moments at a section in continuous spans may be positive or negative. Because of the stress reversal, fatigue stresses should be investigated, especially in the region of dead-load inflection points. At interior supports, however, design usually is governed by the maximum negative moment, and in the midspan region, by maximum positive moment. The sum of the dead-load and live-load moments usually is greater at supports than at midspan. Usually also, this maximum is considerably less than the maximum moment in a simple beam with the same span. Furthermore, the maximum negative moment decreases rapidly with distance from the support.

The impact fraction for continuous spans depends on the length L, ft, of the portion of the span loaded to produce maximum stress. For positive moment, use the actual loaded length. For negative moment, use the average of two adjacent loaded spans.

Ends of continuous beams usually are simply supported. Consequently, moments in three-span and four-span continuous beams are significantly affected by the relative lengths of interior and exterior spans. Selection of a suitable span ratio can nearly equalize maximum positive moments in those spans and thus permit duplication of sections. The most advantageous ratio, however, depends on the ratio of dead load to live load, which, in turn, is a function of span length. Approximately, the most advantageous ratio for length of interior to exterior span is 1.33 for interior spans less than about 60 ft, 1.30 for interior spans between about 60 to 110 ft, and about 1.25 for longer spans.

When composite construction is advantageous (see Art. 12.1), it may be used either in the positive-moment regions or throughout a continuous span. Design of a section in the positive-moment region in either case is similar to that for a simple beam. Design of a section in the negative-moment regions differs in that the concrete slab, as part of the top flange, cannot resist tension. Consequently, steel reinforcement must be added to the slab to resist the tensile stresses imposed by composite action.

Additionally, for continuous spans with a cast-in-place concrete deck, the sequence of concrete pavement is an important design consideration. Bending moments, bracing requirements, and uplift forces must be carefully evaluated.

12.17 ALLOWABLE-STRESS DESIGN OF BRIDGE WITH CONTINUOUS, COMPOSITE STRINGERS

The structure is a two-lane highway bridge with overall length of 298 ft. Site conditions require a central span of 125 ft. End spans, therefore, are each 86.5 ft (Fig. 12.66*a*). The typical cross section in Fig. 12.66*b* shows a 30-ft roadway, flanked on one side by a 21-in-wide barrier curb and on the other by a 6-ft-wide sidewalk. The deck is supported by six rolled-beam, continuous stringers of Grade 36 steel. Concrete to be used for the deck is Class A, with 28-day strength f'_c = 4,000 psi and allowable compressive stress f_c = 1,600 psi. Loading is HS20-44. Appropriate design criteria given in Sec. 11 will be used for this structure.

Concrete Slab. The slab is designed to span transversely between stringers, as in Art. 12.2. A 9-in-thick, two-course slab will be used. No provision will be made for a future 2-in wearing course.

Stringer Loads. Assume that the stringers will not be shored during casting of the concrete slab. Then, the dead load on each stringer includes the weight of a strip of concrete slab plus the weights of steel shape, cover plates, and framing details. This dead load will be referred to as *DL* and is summarized in Table 12.81.

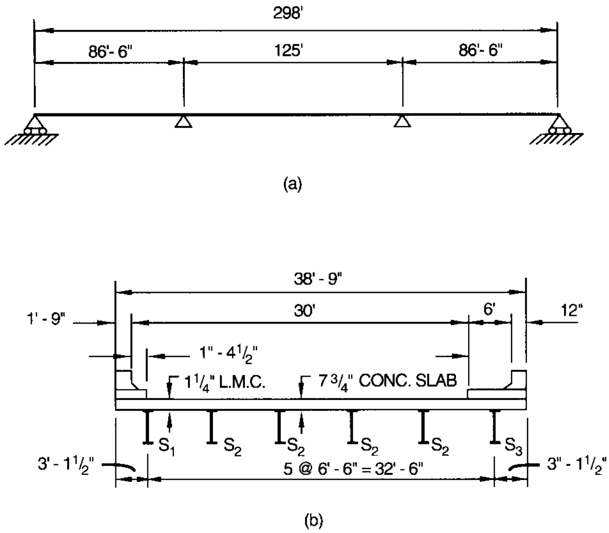


FIGURE 12.66 (*a*) Spans of a continuous highway bridge. (*b*) Typical cross section of bridge.

Sidewalks, parapet, and barrier curbs will be placed after the concrete slab has cured. Their weights may be equally distributed to all stringers. Some designers, however, prefer to calculate the heavier load imposed on outer stringers by the cantilevers by taking moments of the cantilever loads about the edge of curb, as shown in Table 12.82. In addition, the six composite beams must carry the weight, 0.016 ksf, of the 30-ft-wide latex-modified concrete wearing course. The total superimposed dead load will be designated *SDL*.

The HS20-44 live load imposed may be a truck load or lane load. For these spans, truck loading governs. With stringer spacing $S = 6.5$ ft, the live load taken by outer stringers S_1 and S_3 is

$$\frac{S}{4 + 0.25S} = \frac{6.5}{4 + 0.25 \times 6.5} = 1.155 \text{ wheels} = 0.578 \text{ axle}$$

The live load taken by S_2 is

TABLE 12.81 Dead Load, kips per ft, on Continuous Steel Beams

	Stringers S_1 and S_3	Stringers S_2
Slab	0.618	0.630
Haunch and SIP forms:	0.102	0.047
Rolled beam and details—assume:	0.320	0.320
DL per stringer	1.040	0.997

TABLE 12.82 Dead Load, kips per ft, on Composite Stringers

	SDL	*x*	Moment
Barrier curb: 0.530/6 =	0.088	1.33	0.117
Sidewalk: 0.510/6	0.085	3.50	0.298
Parapet: 0.338/6	0.056	6.50	0.364
Railing: 0.015/6	0.002	6.50	0.013
	0.231		0.675
1¼-in LMC course	0.078		
SDL for S_2:	0.309		

Eccentricity for $S_1 = 0.117/0.088 + 6.5 + 1.38 = 9.21$ ft

Eccentricity for $S_3 = 0.675/0.143 + 6.5 - 3.88 = 7.34$ ft

SDL for $S_1 = 0.309 \times 9.21/6.5 = 0.438$

SDL for $S_3 = 0.309 \times 7.34/6.5 = 0.349$

$$\frac{S}{5.5} = \frac{6.5}{5.5} = 1.182 \text{ wheels} = 0.591 \text{ axle}$$

Sidewalk live load (*SLL*) on each stringer is

$$w_{SLL} = \frac{0.060 \times 6}{6} = 0.060 \text{ kip per ft}$$

The impact factor for positive moment in the 86.5-ft end spans is

$$I = \frac{50}{L + 125} = \frac{50}{86.5 + 125} = 0.237$$

For positive moment in the 125-ft center span,

$$I = \frac{50}{125 + 125} = 0.200$$

And for negative moments at the interior supports, with an average loaded span $L = (86.5 + 125)/2 = 105.8$ ft,

$$I = \frac{50}{105.8 + 125} = 0.217$$

Stringer Moments. The steel stringers will each consist of a single rolled beam of Grade 36 steel, composite with the concrete slab only in regions of positive moment. To resist negative moments, top and bottom cover plates will be attached in the region of the interior supports. To resist maximum positive moments in the center span, a cover plate will be added to the bottom flange of the composite section. In the end spans, the composite section with the rolled beam alone must carry the positive moments.

For a precise determination of bending moments and shears, these variations in moments of inertia of the stringer cross sections should be taken into account. But this requires that the cross sections be known in advance or assumed, and the analysis without a computer is tedious. Instead, for a preliminary analysis, to determine the cross sections at critical points the moment of inertia may be assumed constant and the same in each span. This assumption considerably simplifies the analysis and permits use of tables of influence coefficients. (See, for example, "Moments, Shears, and Reactions for Continuous Highway Bridges,"*American Institute of Steel Construction.*) The resulting design also often is sufficiently accurate to serve as the final design. In this example, dead-load negative moment at the supports. computed for constant moment of inertia, will be increased 10% to compensate for the variations in moment of inertia.

Curves of maximum moment (moment envelopes) are plotted in Figs. 12.67 and 12.68 for S_1 and S_2, respectively. Because total maximum moments at critical points are nearly equal for S_1, S_2, and S_3, the design selected for S_1 will be used for all stringers. (In some cases, there may be some cost savings in using shorter cover plates for the stringers with smaller moments.)

Properties of Negative-Moment Section. The largest bending moment occurs at the interior supports, where the section consists of a rolled beam and top and bottom cover plates. With the dead load at the supports as indicated in Fig. 12.67 increased 10% to compensate for the variable moment of inertia, the moments in stringer S_1 at the supports are as follows:

S_1 MOMENTS AT INTERIOR SUPPORTS, FT-KIPS

M_{DL}	M_{SDL}	$M_{LL} + M_I$	M_{SLL}	Total M
−1,331	−510	−821	−78	−2,740

For computing the minimum depth-span ratio, the distance between center-span inflection points can be taken approximately as $0.7 \times 125 = 87.5 > 86.5$ ft. In accordance with AASHTO specifications, the depth of the steel beam alone should be at least $87.5 \times 12/30 = 35$ in. Select a 36-in wide-flange beam. With an effective depth of 8.5 in for the concrete slab, allowing ½ in for wear, overall depth of the composite section is 44.5 in. Required depth is $87.5 \times 12/25 = 4 < 44.5$ in.

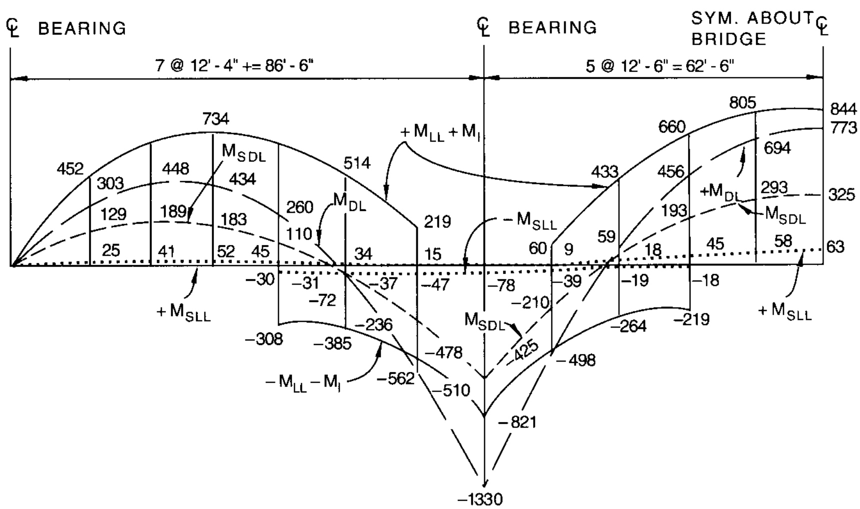


FIGURE 12.67 Maximum moments in outer stringer S_1.

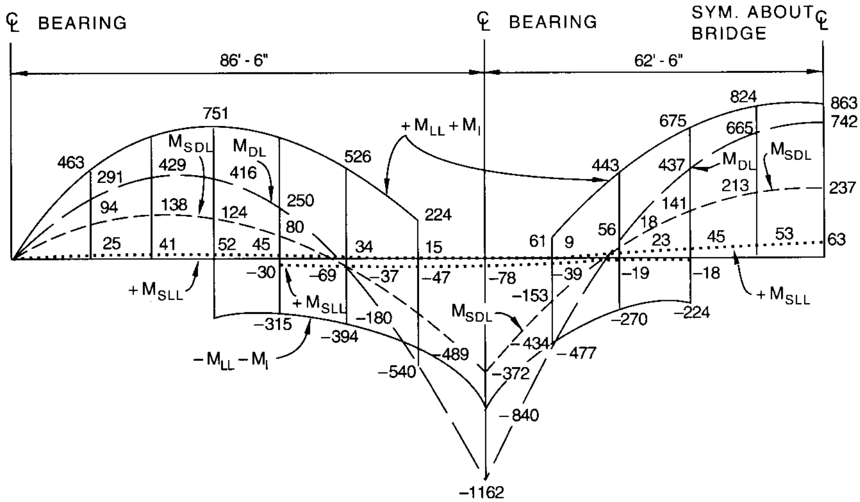


FIGURE 12.68 Maximum moments in interior stringer S_2.

With an allowable bending stress of 20 ksi, the cover-plated beam must provide a section modulus of at least

$$S = \frac{2{,}740 \times 12}{20} = 1{,}644 \text{ in}^3$$

Try a W36 × 280. It provides a moment of inertia of 18,900 in^4 and a section modulus of 1,030 in^3, with a depth of 36.50 in. The cover plates must increase this section modulus by at least $1{,}644 - 1{,}030 = 614$ in^3. Hence, for an assumed distance between plates of 37 in, area of each plate should be about $614/37 = 16.6$ in^2. Try top and bottom plates $14 \times 1\frac{3}{8}$ in (area = 19.25 in^2). The 16.6-in flange width provides at least 1 in on both sides of the cover plates for fillet welding the plates to the flange.

The assumed section provides a moment of inertia of

$$I = 18{,}900 + 2 \times 19.25(18.94)^2 = 32{,}700 \text{ in}^4$$

Hence, the section modulus provided is

$$S = \frac{32{,}700}{19.63} = 1{,}666 > 1{,}644 \text{ in}^3$$

Use a W36 × 280 with top and bottom cover plates $14 \times 1\frac{3}{8}$ in. Weld plates to flanges with $\frac{5}{16}$-in fillet welds, minimum size permitted for the flange thickness.

Allowable Compressive Stress near Supports. Because the bottom flange of the beam is in compression near the supports and is unbraced, the allowable compressive stress may have to be reduced to preclude buckling failure. AASHTO specifications, however, permit a 20% increase in the reduced stress for negative moments near interior supports. The unbraced length should be taken as the distance between diaphragms or the distance from interior support to the dead-load inflection point, whichever is smaller. In this example, if distance between diaphragms is assumed not to exceed about 22 ft, the allowable bending stress for a flange width of 16.6 in is computed as follows:

Allowable compressive stress F_b, ksi, on extreme fibers of rolled beams and built-up sections subject to bending, when the compression flange is partly supported, is determined from

$$F_b = \frac{50{,}000}{S_{xc}} C_b \left(\frac{I_{yc}}{l}\right) \sqrt{\frac{0.772J}{I_{yc}} + 9.87\left(\frac{d}{l}\right)^2} \leq 0.55F_y \tag{12.47}$$

where C_b = $1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2 \leq 2.3$
S_{xc} = section modulus with respect to the compression flange, in^3
= 1,666 in^3
I_{yc} = moment of inertia of compression flange about vertical axis in plane of web, in^4 = $1.57 \times 16.6^3/12 = 598$ in^4
l = length of unsupported flange between lateral connections, knee braces, or other points of support, in
= $22 \times 12 = 264$ in
J = torsional constant, in^4
= $\frac{1}{3}(bt_{fc}^3 + bt_{ft}^3 + dt_w^3)$
= $\frac{1}{3}[16.6(1.57)^3 + 16.6(1.57)^3 + 36.52(0.89)^3] = 51$ in^4
d = depth of girder, in = 36.52 in
M_1 = smaller end moment in the unbraced length of the stringer
= $-121 - 52 - 394 - 38 = -605$ ft-kip
M_2 = larger end moment in the unbraced length of the stringer
= $-1331 - 510 - 821 - 78 = -2{,}740$ ft-kip
C_b = $1.75 + 1.05(605/2{,}740) + 0.3(605/2{,}740)^2 = 2.00$

Substitution of the above values in Eq. (12.46) yields

$$F_b = \frac{50{,}000 \times 2.0}{1{,}666}\left(\frac{598}{264}\right)\sqrt{0.772(51/598) + 9.87(36.52/264)^2}$$
$$= 68.62 \text{ ksi} > (0.55 \times 36 = 19.8 \text{ ksi})$$

Use $F_b = 19.8$ ksi.

Cutoffs of Negative-Moment Cover Plates. Because of the decrease in moments with distance from an interior support, the top and bottom cover plates can be terminated where the rolled beam alone has sufficient capacity to carry the bending moment. The actual cutoff points, however, may be determined by allowable fatigue stresses for the base metal adjacent to the fillet welds between flanges and ends of the cover plates. The number of cycles of load to be resisted for HS20-44 loading is 500,000 for a major highway. For Grade 36 steel and these conditions, the allowable fatigue stress range for this redundant-load-path structure and the Stress Category E′ connection is $F_r = 9.2$ ksi.

Resisting moment of the W36 × 280 alone with $F_r = 9.2$ ksi is

$$M = \frac{9.2 \times 1{,}030}{12} = 790 \text{ ft-kips}$$

This equals the live-load bending-moment range in the end span about 12 ft from the interior support. Minimum terminal distance for the 14-in cover plate is $1.5 \times 14 = 21$ in. Try an actual cutoff point 12 ft 4 in from the support. At that point, the moment range is $219 - (-562) = 781$ ft-kips. Thus, the stress range is

$$F_r = \frac{781 \times 12}{1{,}030} = 9.1 \text{ ksi} < 9.2 \text{ ksi}$$

Fatigue does not govern. Use a cutoff 12 ft 4 in from the interior support in the end span.

In the center span, the resisting moment of the W36 equals the bending moment about 8 ft 4 in from the interior support. With allowance for the terminal distance, the plates may be cut off 10 ft 6 in from the support. Fatigue does not govern there.

Properties of End-Span Composite Section. The 9-in-thick roadway slab includes an allowance of 0.5 in for wear. Hence, the effective thickness of the concrete slab for composite action is 8.5 in.

The effective width of the slab as part of the top flange of the T beam is the smaller of the following:

$$\tfrac{1}{4}\text{ span} = \tfrac{1}{4} \times 86.5 \times 12 = 260\text{ in}$$

$$\text{Overhang} + \text{half the spacing of stringers} = 37.5 + 78/2 = 76.5\text{ in}$$

$$12 \times \text{slab thickness} = 12 \times 8.5 = 102\text{ in}$$

Hence the effective width is 76.5 in (Fig. 12.69).

To resist maximum positive moments in the end span, the W36 × 280 will be made composite with the concrete slab. As in Art. 12.2, the properties of the end-span composite section are computed with the concrete slab, ignoring the haunch area, transformed into an equivalent steel area. The computations for neutral-axis locations and section moduli for the composite section are tabulated in Table 12.83. To locate the neutral axes for $n = 24$ and $n = 8$ moments are taken about the neutral axis of the rolled beam.

Stresses in End-Span Composite Section. Since the stringers will not be shored when the concrete is cast and cured, the stresses in the steel section for load *DL* are determined with the section moduli of the steel section alone. Stresses for load *SDL* are computed with section moduli of the composite section when $n = 24$. And stresses in the steel for live loads and impact are calculated with section moduli of the composite section when $n = 8$. See Table 12.68. Maximum positive bending moments in the end span are estimated from Fig. 12.67:

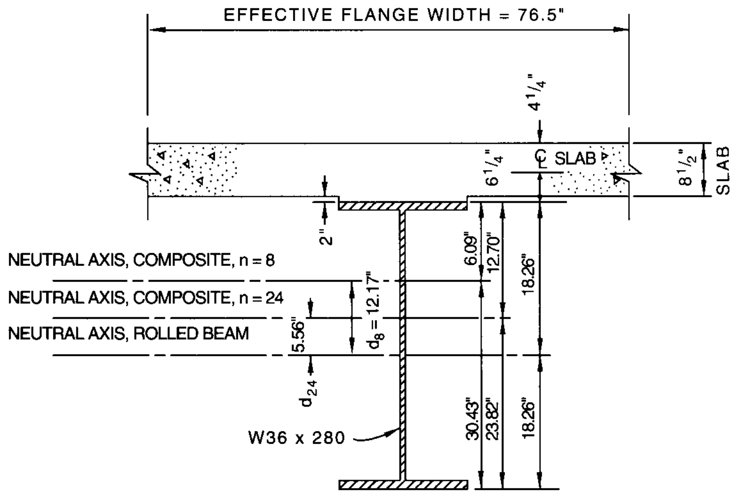


FIGURE 12.69 Composite section for end span of continuous girder.

TABLE 12.83 End-Span Composite Section

(*a*) For dead loads, n = 24

Material	A	d	Ad	Ad^2	I_o	I
Steel section	82.4				18,900	18,900
Concrete 76.5 × 7.75/24	24.7	24.14	596	14,400	120	14,520
	107.1		596			33,420
$d_{24} = 596/107.1 = 5.56$ in					$-5.56 \times 596 =$	$-3,320$
					$I_{NA} =$	30,100

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 18.26 - 5.56 = 12.70 \text{ in}$$
$$\text{Bottom of steel} = 18.26 + 5.56 = 23.82 \text{ in}$$
$$\text{Top of concrete} = 12.70 + 2 + 7.75 = 22.45 \text{ in}$$

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 30,100/12.70$ $= 2,370 \text{ in}^3$	$S_{sb} = 30,100/23.82$ $= 1,264 \text{ in}^3$	$S_c = 30,100/22.45$ $= 1,341 \text{ in}^3$

(*b*) For live loads, n = 8

Material	A	d	Ad	Ad^2	I_o	I
Steel section	82.4				18,900	18,900
Concrete 76.5 × 8.5/8	81.3	24.51	1,993	48,840	490	49,330
	163.7		1,993			68,230
$d_8 = 1993/163.7 = 12.17$ in					$-12.17 \times 1,993 =$	$-24,260$
					$I_{NA} =$	43,970

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 18.26 - 12.17 = 6.09 \text{ in}$$
$$\text{Bottom of steel} = 18.26 + 12.17 = 30.43 \text{ in}$$
$$\text{Top of concrete} = 6.09 + 2 + 8.5 = 16.59 \text{ in}$$

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 43,970/6.09$ $= 7,220 \text{ in}^3$	$S_{sb} = 43,970/30.43$ $= 1,445 \text{ in}^3$	$S_c = 43,970/16.59$ $= 2,650 \text{ in}^3$

MAXIMUM POSITIVE MOMENTS IN CENTER SPAN, FT-KIPS

M_{DL}	M_{SDL}	$M_{LL} + M_I$	M_{SLL}
434	183	734	52

Stresses in the concrete are determined with the section moduli of the composite section with $n = 24$ for *SDL* from Table 12.83*a* and $n = 8$ for $LL + I$ from Table 12.83*b* (Table 12.84).

Since the bending stresses in steel and concrete are less than the allowable, the assumed steel section is satisfactory for the end span.

Properties of Center-Span Section for Maximum Positive Moment. For maximum positive moment in the middle portion of the center span, the rolled beam will be made composite with the concrete slab and a cover plate will be added to the bottom flange. Area of cover plate required A_{sb} will be estimated from Eq. (12.1*a*) with $d_{cg} = 35$ in and $t = 8.5$ in.

MAXIMUM POSITIVE MOMENTS IN CENTER SPAN, FT-KIPS

M_{DL}	M_{SDL}	$M_{LL} + M_I$	M_{SLL}
773	325	844	63

$$A_{sb} = \frac{12}{20}\left(\frac{773}{35} + \frac{325 + 844 + 63}{35 + 8.5}\right) = 30.2 \text{ in}^2$$

The bottom flange of the W36 × 280 provides an area of 26.0 in^2. Hence, the cover plate should supply an area of about $30.2 - 26.0 = 4.2$ in^2. Try a 10 × ½-in plate, area = 5.0 in^2.

The trial section is shown in Fig. 12.70. Properties of the cover-plated steel section alone are computed in Table 12.85. In determination of the properties of the composite section, use is made of the computations for the end-span composite section in Table 12.84. Calculations for the center-span section are given in Table 12.86. In all cases, the neutral axes are located by taking moments about the neutral axis of the rolled beam.

Midspan Stresses in Center Span. Stresses caused by maximum positive moments in the center span are computed in the same way as for the end-span composite section (Table 12.87*a*). Stresses in the concrete are computed with the section moduli of the composite section with $n = 24$ for *SDL* and $n = 8$ for $LL + I$ (Table 12.87*b*).

Since the bending stresses in steel and concrete are less than the allowable, the assumed steel section is satisfactory. Use the W36 × 280 with 10 × ½-in cover plate on the bottom flange. Weld to flange with ⅜-in fillet welds, minimum size permitted for the flange thickness.

Cutoffs of Positive-Moment Cover Plate. Bending moments decrease almost parabolically with distance from midspan. At some point on either side of midspan, therefore, the bottom cover plate is not needed for carrying bending moment. After the plate is cut off, the remaining section of the stringer is the same as the composite section in the end span. Properties of this section can be obtained from Table 12.83. Try a theoretical cutoff point 12.5 ft on both sides of midspan.

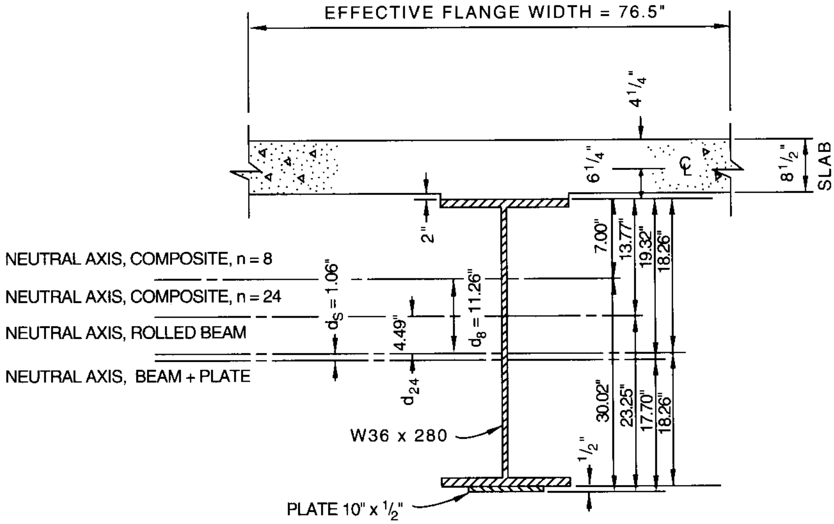


FIGURE 12.70 Composite section for center span of continuous girder.

Center-span Moments, ft-kips, 12.5 ft from Midspan

M_{DL}	M_{SDL}	$M_{LL} + M_I$	M_{SLL}
694	293	805	58

Calculations for the stresses at the theoretical cutoff point are given in Table 12.88. The composite section without cover plate is adequate at the theoretical cutoff point. With an

TABLE 12.84 Stresses, ksi, in End Span for Maximum Positive Moment

(*a*) Steel stresses	
Top of steel (compression)	Bottom of steel (tension)
DL: $f_b = 434 \times 12/1{,}030 = 5.06$	$f_b = 434 \times 12/1{,}030 = 5.06$
SDL: $f_b = 183 \times 12/2{,}370 = 0.93$	$f_b = 183 \times 12/1{,}264 = 1.74$
LL + I: $f_b = 786 \times 12/7{,}220 = \underline{1.31}$	$f_b = 786 \times 12/1{,}445 = \underline{6.53}$
Total: 7.30 < 20	13.33 < 20
(*b*) Stresses at top of concrete	
SDL: $f_c = 183 \times 12/(1{,}341 \times 24) = 0.07$	
LL + I: $f_c = 786 \times 12/(2{,}650 \times 8) = \underline{0.44}$	
Total: 0.51 < 1.6	

TABLE 12.85 Rolled Beam with Cover Plate

Material	A	d	Ad	Ad^2	I_o	I
W36 × 280	82.4				18,900	18,900
Cover plate 10 × ½	5.0	−18.51	−93	1,710		1,710
	87.4		−93			20,610
$d_s = -93/87.4 = -1.06$ in					$-1.06 \times 93 =$	−100
					$I_{NA} =$	20,510

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 18.26 + 1.06 = 19.32 \text{ in}$$

$$\text{Bottom of steel} = 18.26 + 0.50 - 1.06 = 17.70 \text{ in}$$

Section moduli	
Top of steel	Bottom of steel
$S_{st} = 20{,}510/19.32 = 1{,}062 \text{ in}^3$	$S_{sb} = 20{,}510/17.70 = 1{,}159 \text{ in}^3$

allowance of 1.5 × 10 = 15 in for the terminal distance, actual cutoff would be about 14 ft from midspan. Since there is no stress reversal, fatigue does not govern there. Use a cover plate 10 × ½ in by 28 ft long.

Stringer design as determined so far is illustrated in Fig. 12.71.

Bolted Field Splice. The 298-ft overall length of the stringer is too long for shipment in one piece. Hence, field splices are necessary. They should be made where bending stresses are small. Suitable locations are in the center span near the dead-load inflection points. Provide a bolted field splice in the center span 20 ft from each support. Use A325 ⅞-in-dia high-strength bolts in slip-critical connections with Class A surfaces.

Bending moments at each splice location are identical because of symmetry. They are obtained from Fig. 12.67.

MOMENTS AT FIELD SPLICE, FT-KIPS

	M_{DL}	M_{SDL}	$M_{LL} + M_I$	M_{SLL}	Total M
Positive	−80	−50	280	10	160
Negative	−80	−50	−330	−20	−480

Because of stress reversal, a slip-critical connection must be used. Also, fatigue stresses in the base metal adjacent to the bolts must be taken into account for 500,000 cycles of loading. The allowable fatigue stress range ksi, in the base metal for tension or stress reversal for the Stress Category B connection and the redundant-load-path structure is 29 ksi. The allowable shear stress for bolts in a slip-critical connection is 15.5 ksi.

The web splice is designed to carry the shear on the section. Since the stresses are small, the splice capacity is made 75% of the web strength. For web strength 0.885 × 36.5 = 32.3 and $F_v = 12$ ksi,

$$V = 0.75 \times 32.3 \times 12 = 291 \text{ kips}$$

Each bolt has a capacity in double shear of 2 × 0.601 × 15.5 = 18.6 kips. Hence, the

TABLE 12.86 Center-Span Composite Section for Maximum Positive Moment

(*a*) For dead loads, n = 24

Material	A	d	Ad	Ad^2	I
End-span composite section	107.1		596		33,420
Cover plate 10 × ½	5.0	−18.51	−93	1,710	1,710
	112.1		503		35,130
$d_{24} = 503/112.1 = 4.49$ in				$-4.49 \times 503 =$	−2,260
				$I_{NA} =$	32,870

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 18.26 - 4.49 = 13.77 \text{ in}$$
$$\text{Bottom of steel} = 18.26 + 0.50 + 4.49 = 23.25 \text{ in}$$
$$\text{Top of concrete} = 13.77 + 2 + 7.75 = 23.52 \text{ in}$$

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 32{,}870/13.77$ $= 2{,}387 \text{ in}^3$	$S_{sb} = 32{,}870/23.25$ $= 1{,}414 \text{ in}^3$	$S_c = 32{,}870/23.52$ $= 1{,}398 \text{ in}^3$

(*b*) For live loads, n = 8

Material	A	d	Ad	Ad^2	I
End-span composite section	163.7		1,993		68,230
Cover plate 10 × ½	5.0	−18.51	−93	1,710	1,710
	168.7		1,900		69,940
$d_s = 1900/168.7 = 11.26$ in				$-11.26 \times 1{,}900 =$	−21,390
				$I_{NA} =$	48,550

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 18.26 - 11.26 = 7.00 \text{ in}$$
$$\text{Bottom of steel} = 18.26 + 0.50 + 11.26 = 30.02 \text{ in}$$
$$\text{Top of concrete} = 7.00 + 2 + 8.5 = 17.50 \text{ in}$$

Section moduli

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 48{,}550/7.00$ $= 6{,}936 \text{ in}^3$	$S_{sb} = 48{,}550/30.02$ $= 1{,}617 \text{ in}^3$	$S_c = 48{,}550/17.50$ $= 2{,}774 \text{ in}^3$

TABLE 12.87 Stresses, ksi, in Center Span for Maximum Positive Moment

(*a*) Steel stresses

Top of steel (compression)		Bottom of steel (tension)	
DL: $f_b = 773 \times 12/1{,}062 =$	8.73	$f_b = 773 \times 12/1{,}159 =$	8.00
SDL: $f_b = 325 \times 12/2{,}387 =$	1.63	$f_b = 325 \times 12/1{,}414 =$	2.76
LL + *I*: $f_b = 907 \times 12/6{,}936 =$	1.56	$f_b = 907 \times 12/1{,}617 =$	6.70
Total:	11.92 < 20		17.46 < 20

(*b*) Stresses at top of concrete

SDL: $f_c = 325 \times 12/(1{,}398 \times 24) =$	0.12
LL + *I*: $f_c = 907 \times 12/(2{,}774 \times 8) =$	0.49
Total:	0.61 < 1.6

number of bolts required is 291/18.6 = 16. Use two rows of bolts on each side of the splice, each row with 10 bolts and 3-in pitch. Also, use on each side of the web a 30 × 9⁄16-in splice plate, total area = 33.7 > 32.3 in^2.

The flange splice is designed to carry the moment on the section. With the allowable bending stress of 20 ksi, the W36 × 280 has a resisting moment of

$$M = \frac{1{,}030 \times 20}{12} = 1{,}720 > 480 \text{ ft-kips}$$

The average of the resisting and calculated moment is 1,100 ft-kips, which is less than 0.75 × 1,720 = 1,290 ft-kips. Therefore, the splice should be designed for a moment of 1,290 ft-kips. With a moment arm of 35 in, force in each flange is

$$P = \frac{1{,}290 \times 12}{35} = 442 \text{ kips}$$

Then, the number of bolts in double shear required is 442/18.6 = 24. Use on each side of the splice four rows of bolts, each row with six bolts. But to increase the net section of the flange splice plates, the bolts in inner and outer rows should be staggered 1½ in.

The flange splice plates should provide a net area of 442/20 = 22.1 in^2. Try a 16 × 1-in plate on the outer face of each flange and a 6½ × 1-in plate on the inner face on both sides of the web. Table 12.89 presents the calculations for the net area of the splice plates. The plates can be considered satisfactory. See Fig. 12.71.

TABLE 12.88 Tensile Stresses, ksi, 12.5 ft from Midspan

DL: $f_b = 694 \times 12/1{,}030 =$	8.09
SDL: $f_b = 293 \times 12/1{,}264 =$	2.78
LL + *I*: $f_b = 863 \times 12/1{,}445 =$	7.17
Total:	18.04 < 20

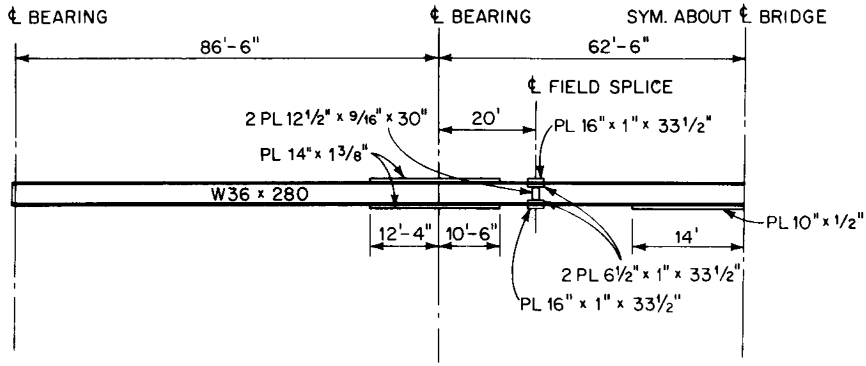


FIGURE 12.71 Cover plates and field splice for typical girder.

Shear in Web. Maximum shear in the stringer totals 270 kips. Average shear stress in the web, which has an area of 32.3 in^2 is

$$f_v = \frac{270}{32.3} = 8.35 \text{ ksi}$$

With an allowable stress in shear of 12 ksi, the web has ample capacity. Furthermore, since the shear stress is less than $0.75 \times 12 = 9$ ksi, bearing stiffeners are not required.

Shear Connectors. To ensure composite action of concrete slab and steel stringer, shear connectors welded to the top flange of the stringer must be embedded in the concrete (Art. 12.5.10). For this structure, ¾-in-dia. welded studs are selected. They are to be installed at specified locations in the positive-moment regions of the stringer in groups of three (Fig. 12.72*b*) to resist the horizontal shear at the top of the steel stringer. With height $H = 6$ in, they satisfy the requirement $H/d \geq 4$, where d = stud diameter, in.

Computation of number of welded studs required and pitch is similar to that for the simply supported stringer designed in Art. 11.2. With $f'_c = 4{,}000$ psi for the concrete, the ultimate strength of each stud is $S_u = 27$ kips. By Eq. (12.4), the allowable load range, kips per stud, for fatigue resistance is, for 500,000 cycles of load, $Z_r = 5.97$.

In the end-span positive-moment region, the strength of the rolled beam is

$$P_1 = A_s F_y = 82.4 \times 36 = 2{,}970 \text{ kips}$$

The compressive strength of the concrete slab is

$$P_2 = 0.85 f'_c bt = 0.85 \times 4.0 \times 76.5 \times 8.5 = 2{,}210 < 2{,}970 \text{ kips}$$

Concrete strength governs. Therefore, the number of studs to be provided between the point

TABLE 12.89 Net Area of Splice Plates, in^2

Plate	Gross area	Hole area	$S^2/4g$	Net area
16 × 1	16	−4	2(2.25/12 + 2.25/26.4)	12.55
2—6½ × 1	13	−4	2(2.25/12)	9.38
Total	29			21.93

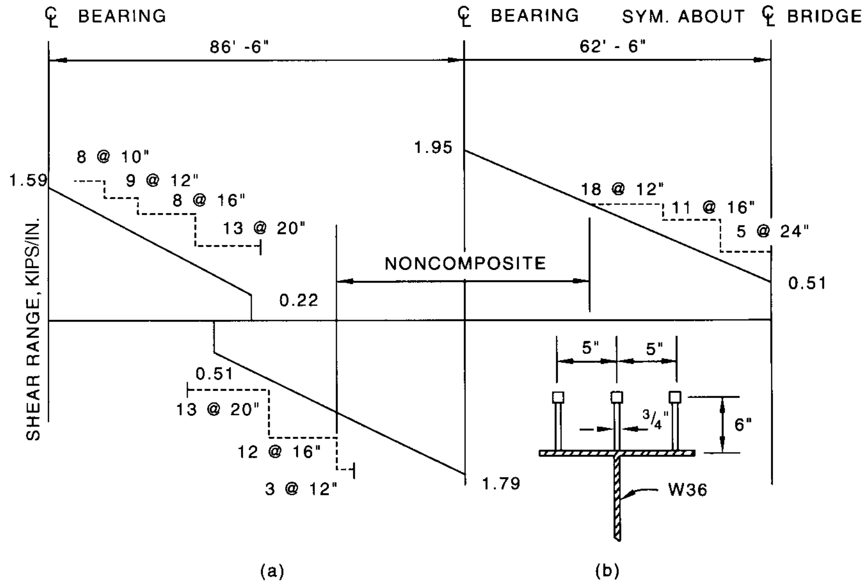


FIGURE 12.72 Variation of shear range (solid lines) and pitch selected for shear connectors (dash lines) for continuous girder.

of maximum moment and both the support and the dead-load inflection point must be at least

$$N = \frac{P_2}{0.85S_u} = \frac{2{,}210}{0.85 \times 27} = 96$$

Since the point of maximum moment is about 37 ft from the support, and the studs are placed in groups of three, there should be at least 32 groups within that distance. Similarly, there should be at least 32 groups in the 23 ft from the point of maximum moment and the dead-load inflection point.

Pitch is determined by fatigue requirements. The sloping lines in Fig. 12.72*a* represent the range of horizontal shear stress, kips per in, $S_r = V_rQ/I$, where V_r is the shear range, or change in shear caused by live loads, Q is the moment about the neutral axis of the transformed concrete area ($n = 8$), and I is the moment of inertia of the composition section. Shear resistance provided, kips per in, equals $3Z_r/p = 17.91/p$, where p is the pitch.

Spacing, in	10	12	14	16	20	24
Shear resistance, kips per in	1.79	1.49	1.28	1.12	0.90	0.75

The shear-connector spacings selected to meet the preceding requirements are indicated in Fig. 12.72*a*.

Additional connectors are required at the dead-load inflection point in the end span over a distance of one-third the effective slab width. The number depends on A_r, the total area, in^2, of longitudinal slab reinforcement for the stringer over the interior support. AASHTO specifications require that in the negative moment regions of continuous spans, the minimum longitudinal reinforcement including the longitudinal distribution reinforcement must equal or exceed 1% of the cross sectional area of concrete section. Therefore,

$$A_r = 0.01 \times 76.5 \times 8.5 = 6.50 \text{ in}^2$$

The range of stress in the reinforcement may be taken as $f_r = 10$ ksi. Then, the additional connectors needed total

$$N_c = \frac{A_r f_r}{Z_r} = \frac{6.50 \times 10}{5.97} = 10.9, \text{ say } 12$$

These are indicated for the noncomposite region at the inflection point in Fig. 12.72*a*.

In the center span also, concrete strength also determines the number of shear connectors required between mid-span and each dead-load inflection point. As in the end span, at least 18 groups of connectors should be provided. In addition, at least three groups are required at the inflection points within a distance of $^{75}\!/_{3} = 25$ in. The pitch is determined by the shear range as for the end span. Figure 12.72*a* indicates the pitch selected.

Bearings. Fixed and expansion bearings for the continuous stringers are the same as for simply supported stringers. However, some bearings may require uplift restraint.

Camber. Dead-load deflections can be computed by a method described in Sec. 3 for the actual moments of inertia along the stringer. The camber to offset these deflections is indicated in Fig. 12.73.

Live-Load Deflection. Maximum live-load deflection occurs at the middle of the center span and equals 1.39 in. The deflection-span ratio is 1.39/(125 × 12) = 1/1,080. This is less than 1/1,000, the maximum for bridges in urban areas and is satisfactory.

12.18 EXAMPLE-LOAD AND RESISTANCE FACTOR DESIGN (LRFD) OF COMPOSITE PLATE-GIRDER BRIDGE

As discussed in Section 11, the AASHTO LRFD Bridge Design Specifications represent a major step forward in improved highway bridge design and are intended to replace the AASHTO Standard Specifications. It is anticipated that bridges designed using LRFD should exhibit superior serviceability, enhanced long-term maintainability, and a more uniform level of safety.

To illustrate LRFD design, calculations are presented for simply supported, composite, plate-girder stringers for a two-lane highway bridge (Fig. 12.13) similar to that considered in Art. 12.4 and 12.5 by ASD and LFD methods. The span length is 100 ft and the girder spacing is 8 ft 4 in. The HL-93 live load for LRFD will be used (Art. 11.4).

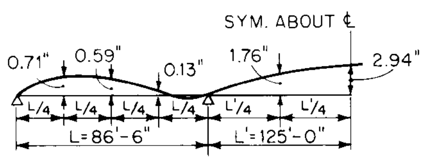


FIGURE 12.73 Camber of girder to offset dead-load deflections.

12.18.1 Stringer Design Procedure

The design procedure for LRFD in most cases resembles that discussed in Art. 12.5.1 for LFD design, but the detailed design criteria differ as discussed in the following articles.

12.18.2 Concrete Slab

The slab is designed to span transversely between stringers using a procedure similar to that for LFD (Art. 12.5.2). The same 9-in-thick, one-course concrete slab is used for this example. AASHTO LRFD Bridge Design Specifications allow use of approximate elastic methods or refined analysis methods for design of decks. Also allowed, for monolithic concrete bridge decks satisfying specific conditions, is the use of an empirical design method that does not require analysis. The AASHTO LRFD Bridge Design Specifications provide deck slab design tables that can be used to determine design live load moments for different girder spacings. Consideration should be given to the assumptions and limitations used in developing those tables: (1) concrete slabs are supported on parallel girders continuous over at least three girders; (2) distance between the centerlines of fascia girders is 14 ft or more; (3) equivalent strip method is used to calculate the moments; (4) tabulated values are for live load HL-93; (5) effects of multiple presence factors and dynamic load allowance are included. For other limitations, refer to AASHTO LRFD specifications.

A table in the AASHTO LRFD Bridge Design Specifications provides maximum live load moments per unit width for different girder spacings at various transverse locations from the girder centerline. This feature allows the user to determine the negative moments at a section that corresponds to the effective slab span length between girders.

The negative live load moments for a center-to-center distance between girders of 8 ft 4 in can be obtained by interpolation from the AASHTO table values listed below as follows.

$$\text{8 ft 3 in c/c spacing, } M_{LL} = -5.74 \text{ ft-kips per ft, 3 in from CL}$$

$$M_{LL} = -4.90 \text{ ft-kips per ft, 6 in from CL}$$

$$\text{8 ft 6 in c/c spacing, } M_{LL} = -5.82 \text{ ft-kips per ft, 3 in from CL}$$

$$M_{LL} = -4.98 \text{ ft-kips per ft, 6 in from CL}$$

For a concrete slab on steel beams, the effective span length will be the distance between quarter points of the top flange plate, which is 16 in/4 = 4 in. By interpolation, 4 in from centerline of girder and for 8 ft 4 in c/c spacing, the negative slab moment is $M_{LL} = -5.49$ ft-kips per ft, which includes the effects of multiple presence factors and dynamic load allowance.

Maximum positive live load moment is located midway between the girders. AASHTO table values are as follows:

$$\text{8 ft 3 in c/c beam spacing, } M_{LL} = 5.83 \text{ ft-kips per ft}$$

$$\text{8 ft 6 in c/c beam spacing, } M_{LL} = 5.99 \text{ ft-kips per ft}$$

By interpolation, for 8 ft 4 in spacing, the positive slab moment is $M_{LL} = 5.88$ ft-kips per ft.

For computation of permanent dead loads,

$$\text{Weight of concrete slab: } 0.150 \times 9/12 = 0.113$$

$$3/8\text{-in extra concrete in stay-in-place forms: } 0.150(3/8)/12 = \underline{0.005}$$

$$\text{Total component dead loads, } DC = 0.12 \text{ kips per ft}$$

$$\text{For dead load of wearing surfaces and utilities, } DW = 0.025 \text{ kips per ft}$$

Assuming a factor of 0.8 is applied to account for continuity of slab over more than three stringers, the maximum dead-load bending moments are

$$+M_{DC} = -M_{DC} = w_{DC}S^2/10 = 0.12(8.33)^2/10 = 0.83 \text{ ft-kips per ft}$$

$$+M_{DW} = -M_{DW} = w_{DW}S^2/10 = 0.025(8.33)^2/10 = 0.17 \text{ ft-kips per ft}$$

To determine the negative dead load moments at the same section as the live load moments,

$$+R_{DC} = w_{DC}S/2 = 0.12(8.33)/2 = 0.50 \text{ kips per ft}$$

$$+R_{DW} = w_{DW}S/2 = 0.025(8.33)/2 = 0.10 \text{ kips per ft}$$

and 4 in from centerline of girder

$$-M_{DC} = 0.83 - 0.50(0.33) + 0.12(0.33)^2/2 = 0.68 \text{ ft-kips per ft}$$

$$-M_{DW} = 0.17 - 0.10(0.33) + 0.025(0.33)^2/2 = 0.15 \text{ ft-kips per ft}$$

Load Combinations. Fatigue need not be investigated for concrete deck slabs in multigirder applications. By inspection, Strength I Limit State will govern the deck slab design. The effect of factored loads is expressed as

$$Q = \eta \Sigma \gamma_i q_i \tag{12.48}$$

where

$$\eta = \eta_D \eta_R \eta_I \geq 0.95 \tag{12.49}$$

and

η_D = 1.0 for conventional designs
η_R = 1.0 for conventional levels of redundancy
η_I = 1.0 for typical bridges

Thus, for this design

$$\eta = 1.0 \geq 0.95$$

From Eq. 12.48, since $\gamma_p = 1.25$ for load DC and $\gamma_p = 1.50$ for load DW, the positive moment is

$$Q = 1.0(1.25 \times 0.83 + 1.50 \times 0.17 + 1.75 \times 5.88) = 11.58 \text{ ft-kips per ft}$$

And the negative moment is

$$Q = 1.0(1.25 \times 0.68 + 1.50 \times 0.15 + 1.75 \times 5.49) = -10.68 \text{ ft-kips per ft}$$

The required factored resistance, M_r, must be no greater than the nominal resistance, M_n, times the factor ϕ. Thus, $M_r \leq \phi M_n$. For rectangular sections, the nominal resistance reduces to

$$M_n = A_s f_y(d_s - a/2) \tag{12.50}$$

where A_s = area, in^2, of steel reinforcement
f_y = yield point = 60.0 ksi for Grade 60 rebars
d_s = effective depth of steel reinforcement
= 9 in − 2½ in (concrete cover) − ½(⅝) (assuming No. 6 rebar) = 6.13 in

From Eq. 12.9, for f'_c = 4.0 ksi and b = 12 in strip, depth of compressive stress block is

$$a = A_s f_y/(0.85\ f'_c b) = 60A_s/(0.85 \times 4 \times 12) = 1.47A_s$$

For flexure of reinforced concrete, $\phi = 0.90$ and negative moment governs the design. Equate the negative moment ϕM_n, where M_n is given by Eq. 12.50, and solve for the required steel area as follows:

$$10.68(12) = 0.90 \times 60A_s(6.13 - 1.47A_s/2)$$

which leads to

$$A_s^2 - 8.34A_s + 3.23 = 0$$

Thus, A_s = 0.41 in^2/ft. Try No. 5 bars @ 8 in: A_s = 0.47 in^2 per ft > 0.41 in^2 per ft—OK

Check minimum reinforcement as follows. AASHTO requires that the amount of tensile reinforcement at any section of a flexural component be adequate to develop a factored flexural resistance, M_r, at least equal to the lesser of 1.2M_{cr} or 1.33 M_{fT}, where M_{cr} is the critical moment as defined below and M_{fT} is the factored transverse slab moment.

$$M_{cr} = f_r S \tag{12.51}$$

where f_r is modulus of rupture and S is section modulus. For normal weight concrete,

$$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{4.0} = 0.48 \text{ ksi. } S = bh^2/6 = 12(8.5)^2/6 = 144.5 \text{ in}^3.$$

Thus,

$$M_r \geq 1.2[144.5 \times 0.48/12] = 6.94 \text{ ft-kips per ft (Governs)}$$

$$M_r \geq 1.33(11.58) = 15.40 \text{ ft-kips per ft}$$

Since the reinforcement provided for flexure in deck slab is adequate for the factored moment of M = 11.58 ft-kips per ft is greater than 6.94 ft-kips per ft, the minimum reinforcement requirement of AASHTO is met.

Alternatively, for non-prestressed components, the minimum reinforcement provision of AASHTO will be considered satisfied if the following minimum reinforcement ratio is provided:

$$\rho_{min} \geq 0.03 f'_c/f_y \tag{12.52}$$

In this case, the criterion becomes $\rho_{min} \geq 0.03(4.0/60.0) = 0.002$. For the provided steel area,

$$\rho = A_s/12d_s = 0.47/(6.13 \times 12) = 0.0064 > 0.002.$$

Therefore minimum reinforcement requirement is met.

Check maximum reinforcement as follows. AASHTO requires that

$$(c/d_e) \leq 0.42 \tag{12.53}$$

where $d_e = d_s$ in a non-prestressed concrete section, and c is the distance from the extreme compression fiber to the neutral axis defined as

$$c = a/\beta_1 \tag{12.54}$$

where the factor $\beta_1 = 0.85$ for $f'_c = 4.0$ ksi concrete. Thus, for the provided steel area,

$$c = 1.47 \times 0.47/0.85 = 0.81, \text{ and } c/d_e = 0.81/6.13 = 0.13 < 0.42\text{—OK.}$$

Maximum reinforcement requirement is also met. Section is termed "under-reinforced" and sufficient ductility is provided.

Check distribution of reinforcement for control of cracking. AASHTO imposes the following stress limitation on the tensile stress in the reinforcement at the service limit state:

$$f_{sa} \leq Z/(d_c A)^{1/3} \leq 0.6f_y \tag{12.55}$$

where Z = 170 ksi for moderate exposure conditions
d_c = depth of reinforcement bars
= 2.0 in (max. conc. cover) + ½(⅝ in) = 2.31 in for top bars
d_c = 1.0 in (concrete cover) + ½(⅝ in) = 1.31 in for bottom bars
A = area of concrete around rebar
A_{top} = 2 × 2.31 × 8 in (bar spacing) = 36.96 in^2
A_{bot} = 2 × 1.31 × 8 in (bar spacing) = 20.96 in^2

Substitution in Eq 12.55 yields $f_{sa} = 38.61$ ksi for top bars, but it is limited to $f_{sa} = 0.6 \times 60 = 36.0$ ksi for Grade 60 rebars. There is no need to calculate f_{sa} for bottom rebars because it will not be critical.

Next determine the tensile stress in the reinforcement at the service limit state, because AASHTO requires Service I Limit State to be used for crack control. The negative moment is calculated as previously but with $\gamma_p = 1.00$:

$$Q = 1.0(1.00 \times 0.68 + 1.00 \times 0.15 + 1.00 \times 5.49) = 6.32 \text{ ft-kips per ft}$$

$$a = 1.47A_s = 1.47 \times 0.47 = 0.69 \text{ in}$$

The stress in the reinforcement is determined by substituting in $M_r = \phi M_n$:

$$6.32 \times 12 = 0.9 \times 0.47 f_s (6.13 - 0.69/2)$$

$$f_s = 30.99 < 0.6f_y = 36.0 \text{ ksi—OK}$$

Therefore, the AASHTO requirement for distribution of reinforcement for control of cracking is met.

For main reinforcement at top and bottom of the deck slab, use No. 5 bars @ 8 in.

12.18.3 Loads, Moments and Shears

There are fewer load combinations specified in LRFD than LFD and some of the load combinations apply only to concrete superstructures. The following Strength I Limit State load combination will govern in this example problem:

$$\text{Strength I Limit State: } \gamma_p \text{ (Dead Load)} + 1.75(LL + IM)$$

Different load factors are applied to different types of dead loads. In addition, AASHTO specifies minimum and maximum values for these loads factors, and the most unfavorable load factor must be used in the design.

$$\gamma_p = 1.25 \text{ for component and attachments, } DC$$

$$\gamma_p = 1.50 \text{ for wearing surfaces and utilities, } DW$$

Then, Strength I Limit State load combination becomes:

$$1.25DC + 1.50DW + 1.75(LL + IM)$$

and loads are calculated as follows.

Permanent Load of Member Components, DC:

Slab—0.150 × 8.33 × 9⁄12	= 0.938
Haunch—16 × 2 in: 0.150 × 1.33 × 0.167	= 0.034
Steel stringer and framing details—assume:	= 0.323
Stay-in-place forms and additional concrete in forms:	= 0.090
Two barrier curbs 2 × 0.530/4 stringers	= 0.265
DC per stringer	= 1.650 kips per ft

Permanent Load of Wearing Surfaces and Utilities, DW:

Future wearing surface 0.025 × 8.33	= 0.210
DW per stringer	= 0.210 kips per ft

AASHTO states that vehicular live loading on bridges, designated HL-93, shall consist of a combination of the design truck or tandem, and design lane load. The configuration of the design truck is similar to an HS-20 truck as specified in AASHTO Standard Specifications, design tandem to Alternate Military load, and design lane load to lane load (without the concentrated load). (See Art. 11.4).

The Multiple Presence Factor, *m*, will be taken as 1.0 for 2 lanes for a 26-foot wide bridge. Dynamic Load Allowance, *IM*, to be applied to the static load, to account for wheel load impact from moving vehicles, will be taken as $(1 + IM/100)$, or 1.33, since *IM* is given in AASHTO LRFD Specifications as 33% for all limit states except for Fatigue and Fracture Limit States.

Distribution of live loads per lane for moment in interior steel beams with concrete decks when two or more design lanes are loaded may be obtained from Eq. 11.11 as

$$DF = 0.075 + (S/9.5)^{0.6}(S/L)^{0.2}[K_g/(12.0Lt_s^3)]^{0.1}$$

where S = beam spacing, 8.33 ft
L = span of beam, 100 ft
t_s = deck slab thickness, 8.5 in (½ in allowance for wearing surface)
K_g = longitudinal stiffness parameter defined as

$$K_g = n(I + Ae_g^2) = 1{,}457{,}000 \text{ in}^4$$

where $n = E_B/E_D$ ($n = 8$ for $f'_c = 4.0$ ksi concrete
I = moment of inertia of steel beam only, 42,780 in^4
e_g = distance between c.g. of basic beam and c.g. of deck, 38.92 in + 2 in + 4.25 in = 45.17 in
A = area of beam, 68.3 in^2

Substituting the above values,

$$DF = 0.075 + 0.924 \times 0.608 \times 1.070 = 0.676$$

Distribution of live loads per lane for shear for this condition may be obtained from

$$DF = 0.2 + S/12 - (S/35)^2 \tag{12.55}$$

where S is the spacing of girders, ft. Substitution of $S = 8.33$ ft gives $DF = 0.838$.

AASHTO LRFD Specifications have provisions for reduction of live load distribution factors for moment in longitudinal beams on skewed supports and correction factors for live load distribution factors for end shear at the obtuse corner, but since the bridge in this example has no skew, no adjustments to the distribution factors will be made.

Maximum dead load moments occur at mid-span:

$$M_{DC} = \tfrac{1}{8}(1.65)(100)^2 = 2063 \text{ ft-kips}$$

$$M_{DW} = \tfrac{1}{8}(0.21)(100)^2 = 263 \text{ ft-kips}$$

Maximum dead load shears at supports are

$$V_{DC} = \tfrac{1}{2}(1.65)(100) = 82.5 \text{ kips}$$

$$V_{DW} = \tfrac{1}{2}(0.21)(100) = 10.5 \text{ kips}$$

Maximum moment due to the design truck occurs when the center axle is at 47.67 ft from a support (see Fig. 12.14a). Then, the maximum live load moment per lane is

$$M(\text{Truck}) = 72(100/2 + 2.33)^2/100 - 32 \times 14 = 1{,}524 \text{ ft-kips}$$

Similarly, maximum live load moment due to design tandem occurs when the front (or rear) axle is 49 ft from a support.

$$M(\text{Tandem}) = 50(100/2 + 1.00)^2/100 - 25 \times 4 = 1{,}201 \text{ ft-kips}$$

$$M(\text{Lane}) = \tfrac{1}{8}(0.64)(100)^2 = 800 \text{ ft-kips}$$

Maximum live load shears due to truck, tandem and lane loads are as follows:

$$V(\text{Truck}) = 72(100 - 14 + 4.66)/100 = 65.3 \text{ kips}$$

$$V(\text{Tandem}) = 50(100 - 4 + 2.00)/100 = 49.0 \text{ kips}$$

$$V(\text{Lane}) = \tfrac{1}{2}(0.64)(100) = 32.0 \text{ kips}$$

The governing live load combination is Design Truck and Design Lane load.

$$M_{LL} = (1{,}524 + 800)0.676 = 1{,}571 \text{ ft-kips}$$

$$M_{IM} = 1{,}524 \times 0.676 \times 0.33 = 340 \text{ ft-kips}$$

since dynamic load allowance is not applied to Design Lane Load according to AASHTO.

$$M_{LL+IM} = 1{,}911 \text{ ft-kips}$$

Similarly,

$$V_{LL} = (65.3 + 32.0)0.838 = 81.5 \text{ kips}$$

$$V_{IM} = 65.3 \times 0.838 \times 0.33 = 18.1 \text{ kips}$$

$$V_{LL+IM} = 99.6 \text{ kips.}$$

The total factored moment and shear values for Strength I Limit State are:

$$M_{fT} = 1.25 \times 2{,}063 + 1.50 \times 263 + 1.75 \times 1{,}911 = 6{,}318 \text{ ft-kips}$$

$$V_{fT} = 1.25 \times 82.5 + 1.50 \times 10.5 + 1.75 \times 99.6 = 293.2 \text{ kips}$$

The midspan factored bending moment, ft-kips, due to the various loads and the total are as follows:

M_{fDC}	M_{fDW}	$M_{fLL} + M_{fIM}$	M_{fT}
2,579	395	3,344	6,318

The corresponding factored end shears, kips, are as follows:

V_{fDC}	V_{fDW}	$V_{fLL} + V_{fIM}$	V_{fT}
103.1	15.8	174.3	293.2

The Strength I Limit State shear diagram due to factored loads is shown in Fig. 12.74.

Fatigue Limit State. The Fatigue Limit State is defined as a fatigue and fracture load combination relating to repetitive gravitational vehicular live load and dynamic responses under a single design truck having the weights and spacing of axles as shown in Fig. 12.75. For this limit state the rear axle spacing remains constant. A dynamic load allowance of 15% will be applied to the fatigue load.

Distribution of live loads per lane for moment in interior steel beams with concrete decks when one design lane is loaded may be obtained from Eq. 11.10 as

$$D = 0.06 + (S/14)^{0.4}(S/L)^{0.3}[K_g/(12.0Lt_s^3)]^{0.1}$$
$$= 0.06 + 0.812 \times 0.474 \times 1.070 = 0.472$$

Distribution of live loads per lane for shear for this condition may be obtained from

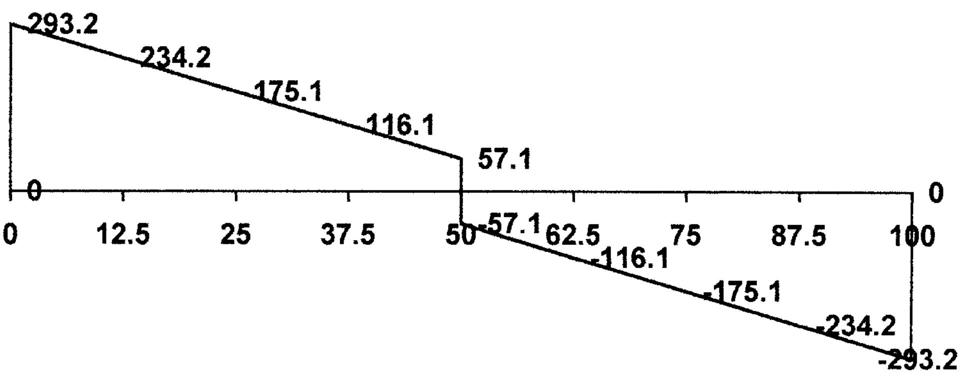


FIGURE 12.74 Plot of factored shear, kips, versus length along span, ft.

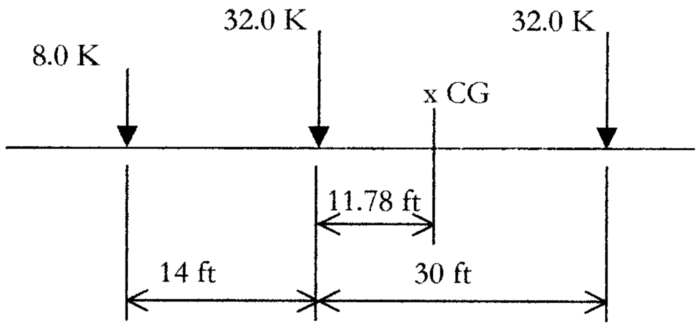


FIGURE 12.75 Axle load configuration of fatigue design truck.

$$DF = 0.36 + S/25 \tag{12.59}$$

where S is the spacing of girders, ft. Substitution of $S = 8.33$ ft gives $DF = 0.693$.

The Multiple Presence Factor for live load is not to be applied to the Fatigue Limit State for which a single design truck is used without regard to the number of design lanes on the bridge. Since the distribution factor is obtained using the single-lane approximate method, but not by the statistical method or level rule, the Multiple Presence Factor, $m = 1.20$ will be removed from the distribution factors for fatigue investigation.

Total load for the three axles of the fatigue truck is 72 kips, and maximum moment occurs when the center axle is ½(100 − 11.78) = 44.11 ft from one support, 55.89 ft from the other. Finding the end reaction and taking moments gives

$$M_{Fatigue} = \frac{72(55.89^2)}{100} - 32 \times 30 = 1{,}289 \text{ ft-kips}$$

Similarly, the maximum shear due to the fatigue truck is

$$V_{Fatigue} = 72(100 - 30 + 11.78)/100 = 58.9 \text{ kips.}$$

The factored fatigue bending moments at 5.89 ft from midspan are as follows:

$$M_{fLL} = 0.75 \times 1{,}289 \times 0.472/1.20 = 380 \text{ ft-kips}$$

$$M_{fIM} = 0.15 \times 380 = 57 \text{ ft-kips}$$

$$M_{f(LL+IM)} = 437 \text{ ft-kips}$$

The factored fatigue end shears are:

$$V_{fLL} = 0.75 \times 58.9 \times 0.693/1.20 = 25.5 \text{ kips}$$

$$V_{fIM} = 0.15 \times 25.5 = 3.8 \text{ kips}$$

$$V_{f(LL+IM)} = 29.3 \text{ kips}$$

12.18.4 Trial Girder Section

Flexural components will be proportioned such that:

$$0.1 \le I_{yc}/I_y \le 0.9 \tag{12.60}$$

where I_y = moment of inertia of the steel section about the vertical axis, in^4
I_{yc} = moment of inertia of the compression flange about the vertical axis, in^4

A trial section with a web plate 60 × 7⁄16 in, top flange plate 16 × ¾ in, and bottom flange plate 20 × 1½ in will be assumed in this example. Assumed cross section of the plate girder for the maximum factored moment is illustrated in Fig. 12.76. Neglecting the moment of inertia of the web about vertical axis,

$$I_{yc} = (3/4)16^3/12 = 256 \text{ in}^4$$

$$I_y = 256 + (1\tfrac{1}{2})20^3/12 = 1{,}256 \text{ in}^4$$

$$0.1 < (256/1{,}256 = 0.20) < 0.9\text{—OK}$$

Webs without longitudinal stiffeners should be proportioned such that:

$$2D_c/t_w \leq 6.77\sqrt{(E/f_c)} \tag{12.61}$$

where D_c is depth of the web in compression in the elastic range, in, f_c is stress in the compression flange due to the factored loading under investigation, ksi. For the steel-only section and for permanent dead loads on steel section, $D_c = 38.92 - 0.75 = 38.17$ in, and $f_c = 2{,}579 \times 12/1{,}100 = 28.13$ ksi. Substitute in Eq. 12.61 to find

$$2(38.17)/(7/16) < 6.77\sqrt{(29{,}000/28.13)}$$

$$174.5 < 217.4\text{—OK}$$

The concrete section for the interior stringer, not including the concrete haunch, is 8 ft 4 in wide (c to c stringers) and 8½ in deep (½ in of slab is deducted as the wearing course). Elastic section properties are tabulated for the trial steel section and composite section in Tables 12.90 and 12.91.

The plastic moment capacity of the composite section will be determined by force equilibrium. Concrete haunch and deck reinforcement will be neglected. Assume plastic neutral axis (PNA) is at top of top flange:

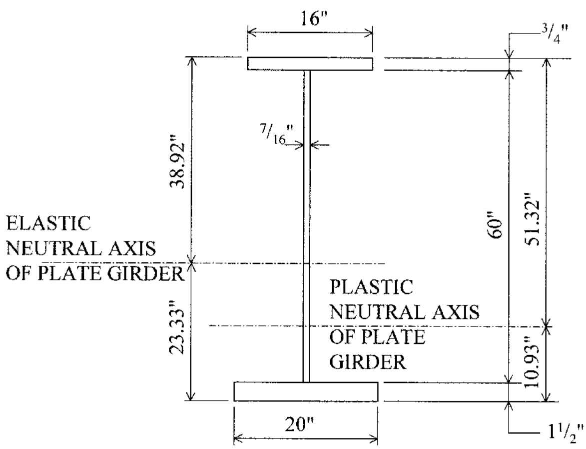


FIGURE 12.76 Cross section assumed for plate girder for LRFD example.

TABLE 12.90 Properties of Steel Section for Maximum Factored Moment

Material	A	d	Ad	Ad^2	I_o	I
Top Flange 16 × ¾	12.0	30.38	365	11,090		11,090
Web 60 × 7⁄16	26.3				7,880	7,880
Bottom Flange 20 × 1½	30.0	−30.75	−923	28,370		28,370
	68.3		−558			47,340
$d_s = 558/68.3 = -8.17$ in					$-8.17 \times 558 =$	−4,560
					$I_{NA} =$	42,780

Distance from neutral axis of steel section to:

$$\text{Top of steel} = 30 + 0.75 + 8.17 = 38.92 \text{ in}$$

$$\text{Bottom of steel} = 30 + 1.50 - 8.17 = 23.33 \text{ in}$$

Section modulus, top of steel	Section modulus, bottom of steel
$S_{st} = 42{,}780/38.92 = 1{,}100 \text{ in}^3$	$S_{sb} = 42{,}780/23.33 = 1{,}830 \text{ in}^3$

$$P_t + P_w + P_e \geq P_s$$

$$A_{bf}F_y + A_wF_y + F_{tf}F_y \geq 0.85f'_cb_{eff}t_s$$

$$30(36) + 26.25(36) + 12(36) \geq 0.85(4.0)(100)(8\tfrac{1}{2})$$

$$1{,}080 + 947 + 432 = 2{,}459 < 2{,}890 \text{ kips}$$

Therefore, PNA is in deck slab.

$$y_{bar} = (2{,}890 - 2{,}459)/(0.85 \times 4.0 \times 100) = 1.27 \text{ in from bottom of slab.}$$

Compute the plastic moment capacity, M_p, by summing moments about the PNA.

$$M_p = P_sd_s + P_cd_c + P_wd_w + P_td_t$$

$$= 2{,}459(8.50 - 1.27)/2 + 1{,}080(61.50 + 2 + 1.27) + 947(30.75 + 2 + 1.27) + 432(0.38 + 2 + 1.27)$$

$$= 8{,}890 + 69{,}950 + 32{,}220 + 1{,}580$$

$$M_p = 9{,}380 \text{ ft-kips}$$

Flexural members should be designed for the strength limit state flexural resistance, the service limit state control of permanent deflection, the fatigue and fracture limit state for details and the fatigue requirements of webs, the strength limit state shear resistance, and for constructibility. The following design process will be adopted in this example using simplified methods given in AASHTO LRFD specifications:

- Check D_c/t_w for fatigue induced by web flexure or shear
- Flexural design
    - Pouring/Loading sequence

TABLE 12.91 Properties of Composite Section for Maximum Factored Moment

(*a*) For Superimposed dead loads, $n = 24$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	68.3		−558			47,340
Concrete 100 × 8.5/24	35.4	37.00	1,310	48,470	210	48,680
	103.7		752			96,020
$d_{24} = 752/103.7 = 7.25$ in					$-7.25 \times 752 =$	−5,450
					$I_{NA} =$	90,570

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 30.75 - 7.25 = 23.50 \text{ in}$$

$$\text{Bottom of steel} = 31.50 + 7.25 = 38.75 \text{ in}$$

$$\text{Top of concrete} = 23.50 + 2 + 8.50 = 34.00 \text{ in}$$

Section Modulus:

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 90{,}570/23.50 = 3{,}850 \text{ in}^3$	$S_{sb} = 90{,}570/38.75 = 2{,}340 \text{ in}^3$	$S_c = 90{,}570/34.00 = 2{,}660 \text{ in}^3$

(*b*) For live loads, $n = 8$

Material	A	d	Ad	Ad^2	I_o	I
Steel section	68.3		−558			47,340
Concrete 100 × 8.5/8	106.3	37.00	3,933	145,520	640	146,160
	174.6		3,375			193,500
$d_8 = 3375/174{,}6 = 19.33$ in					$-19.33 \times 3{,}375 =$	−65,240
					$I_{NA} =$	128,260

Distance from neutral axis of composite section to:

$$\text{Top of steel} = 30.75 - 19.33 = 11.42 \text{ in}$$

$$\text{Bottom of steel} = 31.50 + 19.33 = 50.83 \text{ in}$$

$$\text{Top of concrete} = 11.42 + 2 + 8.50 = 21.92 \text{ in}$$

Section Modulus:

Top of steel	Bottom of steel	Top of concrete
$S_{st} = 128{,}260/11.42 = 11{,}230 \text{ in}^3$	$S_{sb} = 128{,}260/50.83 = 2{,}520 \text{ in}^3$	$S_c = 128{,}260/21.92 = 5{,}850 \text{ in}^3$

- Determine if section is compact
- Check web slenderness
- Check flange slenderness
- Check flange bracing
- Calculate flexural resistance
- Check positive flexure ductility for compact sections

- Flexural stress limits for lateral torsional buckling
- Shear design
  - Stiffened or unstiffened web
  - Transverse stiffener design
  - Longitudinal stiffener design
  - Bearing stiffener design
  - Shear connector design
- Constructibility check for
  - General proportions
  - Flexure, and
  - Shear

12.18.5 Check D_c / t_w for Fatigue Induced by Web Flexure or Shear

AASHTO LRFD Specifications require webs without longitudinal stiffeners to satisfy the following:

$$\text{If } 2D_c/t_w \leq 5.70\sqrt{(E/F_{yw})} \qquad \text{then } f_{cf} = F_{yw} \tag{12.62}$$

otherwise

$$f_{cf} \leq 32.5E(t_w/2D_c)^2 \tag{12.63}$$

where f_{cf} = maximum compressive elastic flexural stress in compression flange due to unfactored permanent and fatigue loading, ksi
F_{yw} = specified minimum yield strength of web, ksi
D_c = depth of the web in compression in elastic range, in

For composite sections in positive flexure, D_c is a function of the algebraic sum of the stresses caused by loads acting on the steel and composite sections. D_c may be computed from

$$D_c = \frac{f_{DC1} + f_{DC2} + f_{DW} + f_{LL+IM}}{\dfrac{f_{DC1}}{c_s} + \dfrac{f_{DC2} + f_{DW}}{c_{3n}} + \dfrac{f_{LL+IM}}{c_n}} - t_f \tag{12.64}$$

Calculate values as follows:

$$f_{DC1} = 1{,}731 \times 12/1{,}100 = 18.88 \text{ ksi}$$

$$f_{DC2} = 332 \times 12/3{,}850 = 1.04 \text{ ksi} \quad \text{(due to loads from barrier curbs)}$$

$$f_{DW} = 263 \times 12/3{,}850 = 0.82 \text{ ksi}$$

$$f_{LL+IM} = 437 \times 12/11{,}230 = 0.47 \text{ ksi}$$

$$D_c = \frac{18.88 + 1.04 + 0.82 + 0.47}{\dfrac{18.88}{39.92} + \dfrac{1.04 + 0.82}{23.50} + \dfrac{0.47}{11.42}} - 0.75 \text{ in} = 35 \text{ in}$$

Then, $2D_c/t_w = 2(35)/(7/16) = 160.0 \leq 5.70\sqrt{(29{,}000/36)} = 161.8$ and $f_{cf} = F_{yf} = 36.0$ ksi.

AASHTO requires that the live load flexural stress and shear stress resulting from the fatigue load be taken as twice the fatigue load combination. Therefore, the fatigue live load stress is

$$f_{(LL+IM)f} = 2(437 \times 12)/11{,}230 = 0.94 \text{ ksi}$$

Then, $f_{cf} = 36 \text{ ksi} > 18.88 + 1.04 + 0.82 + 0.94 = 21.68$ ksi—OK

Next, the web section will be checked for shear to see if it satisfies

$$v_{cf} \leq 0.58CF_{yw} \tag{12.65}$$

where v_{cf} = maximum elastic shear stress in the web due to unfactored permanent load and fatigue loading, ksi

C = ratio of shear buckling stress to the shear yield strength

$$v_{cf} = \frac{V_{DC} + V_{DW} + V_{LL+IM}}{Dt_w} = \frac{82.5 + 10.5 + 29.3}{60(7/16)} = 4.66 \text{ ksi}$$

Determine the ratio, C, from

$$C = [1.52/(D/t_w)^2](Ek/F_{yw}) \text{ when } D/t_w > 1.38\sqrt{(Ek/F_{yw})} \tag{12.66}$$

where $k = 5 + 5/(d_0/D)^2$

Assuming, as in Example 12.5, $d_0 = 150$ in, then $k = 5.8$.

$$D/t_w = 60(7/16) = 137.1 > 1.38\sqrt{(29{,}000 \times 5.8/36)} = 1.38(68.35) = 94.3$$

then $C = [1.52/(137.1)^2](68.35)^2 = 0.378$ and $0.58 \times 0.378 \times 36 = 7.88 \text{ ksi} > 4.66$ ksi—OK

Since both the flexure and shear requirements are satisfied, significant elastic flexing of the web is not expected to occur, and the web is assumed to be able to sustain an infinite number of smaller loadings without fatigue cracking.

12.18.6 Flexural Design

Pouring/Loading Sequence. Composite sections in unshored construction should be investigated as non-composite sections for strength and stability during the deck placement sequence. The steel-only section will be checked under the factored non-composite dead loads. If the entire deck is not cast in one stage, parts of the girder may become composite causing changes to loading, stiffness and bracing. Temporary stresses during deck staging can be higher than the final composite stresses. Deck staging can cause significant tensile strains in the previously hardened deck sections in adjacent spans of continuous superstructures. Therefore changes to loads and stiffnesses during deck construction stages should be considered. In this simply supported girder example, it is conservatively assumed that the entire deck is cast at once.

Compact-Section Web Slenderness. Previously it was determined that the plastic neutral axis was in the deck slab. As a result, D_{cp} will be taken as zero, and the compact-section web slenderness requirement of AASHTO will be considered to be satisfied.

Compact-Section Compression Flange Slenderness. It is also stated in the AASHTO LRFD specifications that the compression flange slenderness and bracing should not be investigated for the strength limit state for the composite sections in positive flexure because the hardened deck slab prevents local and lateral compression flange buckling.

Flexural Resistance. The nominal flexural resistance, M_n, of the composite section in the positive flexural region of a simple span will be taken as

$$M_n = M_p \qquad \text{when } D_p \leq D' \tag{12.67}$$

where D_p is the distance from the top of the slab to the neutral axis at the plastic moment, in. The depth D' is defined as

$$D' = \beta(d + t_s + t_h)/7.5 \tag{12.68}$$

where β = 0.9 for F_y = 36.0 ksi
d = depth of steel section, in
t_h = thickness of concrete haunch above top flange, in
t_s = thickness of the concrete slab, in.

Thus,

$$D' = 0.9(62.25 + 8.5 + 2.0)/7.5 = 8.73 \text{ in}$$

$$D_p = 8.5 - 1.27 = 7.23 \text{ in} < 8.73 \text{ in}$$

Recall that $M_n = M_p$ and $M_r = \phi_f M_n$ where ϕ is the resistance factor for flexure, 1.00. Then,

$$M_r = M_n = M_p = 9{,}380 \text{ ft-kips} > M_{fT} = 6{,}318 \text{ ft-kips}$$

Therefore the section satisfies the strength limit state for flexure.

Ductility Requirement. The section must also be checked to see if it satisfies the ductility requirement to ensure that the concrete slab is protected from premature crushing and spalling when the composite section approaches the plastic moment. The following ratio is limited to a value of 5 to ensure that the steel tension flange reaches strain hardening before the concrete slab crushes:

$$D_p/D' \leq 5 \tag{12.69}$$

In this design, $7.23/8.73 = 0.83 < 5$—OK

12.18.7 Flexural Stress Limits for Lateral Torsional Buckling

Compression Flanges. The nominal flexural resistance of the compression flange will be determined as:

$$F_n = C_b R_b R_h F_{yc}[1.33 - 0.187(L_b/r_t)\sqrt{(F_{yc}/E)}] \leq R_b R_h F_{yc} \tag{12.70}$$

$$\text{if } L_p \leq L_r = 4.44 r_t \sqrt{(E/F_{yc})}$$

where C_b = moment gradient correction factor
= 1.0 for members where the moment within a significant portion of the unbraced segment exceeds the larger of the segment end moments
R_b = load shedding factor
R_h = hybrid factor = 1.0 for a homogeneous girder
L_b = unbraced length, in
r_t = radius of gyration of a notional section comprised of the compression flange plus one-third of depth of web in compression taken about the vertical axis, in
F_{yc} = yield point of compression flange, ksi
E = modulus of elasticity of steel, ksi

Calculate R_b from the following:

$$R_b = 1 - \left[\frac{a_r}{1200 + 300a_r}\right]\left[\frac{2D_c}{t_w} - \lambda_b\sqrt{(E/f_c)}\right] \tag{12.71}$$

for which

$$a_r = 2D_c t_w / A_c \tag{12.72}$$

where A_c = area of compression flange, 12 in^2
λ_b = 4.64 for members with compression flange area less than tension flange area
f_c = stress in compression flange due to the factored loading under investigation, ksi

For this design, L_b = 25 ft × 12 = 300 in, f_c = 18.88 × 1.25 = 23.60 ksi, A_c = 12 in^2, and a_r = 2(38.17)(7/16)/12.0 = 2.78. Therefore,

$$R_b = 1 - \left[\frac{2.78}{1200 + 300(2.78)}\right]\left[\frac{2(38.17)}{(7/16)} - 4.64\sqrt{(29{,}000/23.60)}\right]$$
$$= 0.98$$

Calculate r_t for the notional section comprised of the 16 × ¾ in flange plus a 13 × 7/16 in portion of the web (⅓ × 38.92 = 13 in):

$$I_{yf} = \tfrac{3}{4}(16)^3/12 = 256 \text{ in}^4$$

$$A_{yf} = \tfrac{3}{4}(16) + \tfrac{7}{16}(13) = 17.69 \text{ in}^2$$

$$r_t = \sqrt{(256/17.69)} = 3.80 \text{ in}$$

Also, $L_r = 4.44(3.80)\sqrt{(29{,}000)/36)} = 479 \text{ in} > L_b = 300$ in. Therefore, from Eq. 12.70,

$$F_n = 1.0(0.98)(1.0)(36.0)[1.33 - 0.187(300/3.80)\sqrt{(36/29{,}000)}]$$
$$= 28.57 \text{ ksi} < 1.0(0.98)(36) = 35.28 \text{ ksi}$$

Since f_c = 23.60 ksi < F_n = 28.57 ksi, the lateral torsional buckling provision of AASHTO for the compression flange is met.

12.18.8 Shear Design

Next, the nominal shear resistance, V_n, of an unstiffened web will be determined.

Where $D/t_w > 3.07\sqrt{(E/F_{yw})}$

$$V_n = \frac{4.55 t_w^3 E}{D} \tag{12.73}$$

In this design, $D/t_w = 60/(7/16) = 137$, which is greater than $3.07\sqrt{(E/F_{yw})} = 87$, and therefore

$$V_n = \frac{4.55(7/16)^3\ 29{,}000}{60} = 184 \text{ kips}$$

The factored shear resistance, V_r, is equal to:

$$V_r = \phi_v V_n \tag{12.74}$$

where the resistance factor for shear, $\phi_v = 1.0$. Thus, $V_r = 1.0(184) = 184$ kips $< V_{fT} = 293.2$ kips (the factored shear load for the Strength I Limit State). Therefore, intermediate transverse stiffeners are required for the range where $V_{fT} > 184$ kips.

Transverse Stiffener Design. Transverse stiffeners may consist of plates or angles welded or bolted to the web on one side or both sides. The width, b_t, of each projecting stiffener element should satisfy both of the following conditions to prevent local buckling of the transverse stiffener:

$$2 + d/30 \le b_t \le 0.48 t_p \sqrt{(E/F_{yw})} \tag{12.75}$$

$$0.25 b_f \le b_t \le 16 t_p \tag{12.76}$$

where d = depth of the steel section, in
t_p = thickness of projecting element, in
b_f = full width of wider flange at a section, in.

For Eq. 12.75

$$2 + 62.25/30 \le b_t \le 0.48 t_p(28.38) \quad \text{or} \quad 4.08 \le b_t \le 13.62 t_p.$$

For Eq. 12.76

$$0.25(20) \le b_t \le 16 t_p \qquad \text{or} \qquad 5.0 \le b_t \le 16 t_p$$

For $b_t = 6$ in, these requirements become $t_p \ge 6/13.62 = 0.44$ in, and $t_p \ge 6/16 = 0.375$ in.

Try a pair of transverse stiffener plates of 6 × ½ in for the end panels. Check to see if the moment of inertia of the transverse stiffeners satisfies the following condition:

$$I_t \ge d_0 t_w^3 J \tag{12.77}$$

where

$$J = 2.5(D_p/d_0)^2 - 2.0 \ge 0.5 \tag{12.78}$$

In the above, I_t = moment of inertia of the transverse stiffener taken about the edge in contact with the web for single stiffeners, or taken about the mid-thickness of the web for stiffener pairs, in^4; D_p = web depth for webs without longitudinal stiffeners or maximum subpanel depth for webs with longitudinal stiffeners, in; and d_0 = stiffener spacing, in. The maximum stiffener spacing in end panels is limited to $1.5D$, which in this case is $1.5 \times 60 = 90$ in. From Eq. 12.78, for $d_0 = 90$ and $D_p = 60$, find $J = 0.5$. Then, from Eq. 12.77, $I_t = 90(7/16)^3(0.5) = 3.77$ in^4.

Next, the nominal shear resistance of interior web panels of compact sections, V_n, can be determined from Eq. 12.23 if $M_u \leq 0.5\phi_f M_p$ where M_u is the maximum moment in the panel under consideration, ϕ_f is the resistance factor for flexure, 1.00, and M_p is the plastic moment resistance, 9,380 ft-kips in this case.

The first intermediate diaphragm is located 25 ft or 300 in from centerline of end bearing and stiffeners are usually equally spaced between the diaphragms. For a first trial, check stiffener spacing of $d_0 = 300/2 = 150$ in. At 24 ft from end bearing, $M_u = 1.25 \times 1{,}504.8 + 1.50 \times 191.3 + 1.75 \times 1{,}430.3 = 4{,}671$ ft-kips. This does not exceed $0.5\phi_f M_p = 0.5 \times 1.0 \times 9{,}380$ ft-kips, so Eq. 12.23 applies. From the LFD example of Art. 12.5, $V_n = 322$ kips, which is greater than $V_{ft} = 293.2$ kips and is OK. Therefore, continue with the design of transverse stiffeners for the 150 in spacing. Note that this does not exceed the maximum spacing permitted, $3D = 180$ in.

Assuming a pair of $6 \times 1/2$ in stiffeners, substitute in Eqs. 12.77 and 12.78:

$$J = 2.5(60/150)^2 - 2.0 = -1.6 < 0.5 \qquad \text{Use } J = 0.5.$$

$$I_t \geq 150(7/16)^3 \times 0.5 = 6.3 \text{ in}^4$$

Then, $I_t = A\bar{y}^2 + I_0 = 2[6 \times 1/2 \times (3 + 7/32)^2 + 1/2(6)^3/12] = 80.2 \text{ in}^4 > 6.3 \text{ in}^4$—OK

Transverse stiffeners are required to have sufficient area to carry the vertical component of forces imposed by tension field action of web:

$$A_s \geq [0.15BDt_w(1.0 - C)(V_u/V_r) - 18.0t_w^2](F_{yw}/F_{ys}) \tag{12.79}$$

where B = 1.0 for stiffeners on both sides of web
C = ratio of shear buckling stress to shear yield strength
V_u = shear due to factored loads at the Strength Limit State, kips

For this design, $C = 0.39$ (from Art. 12.5), $V_u = (293.2 + 234.2)/2 = 263.7$ kips, and $A_s = 2 \times 6 \times 1/2 = 6.0$ in^2. From Eq. 12.79,

$$6.0 \geq [0.15 \times 1.0 \times 60 \times 7/16 \times (1.0 - 0.39)(263.7/322) - 18.0 \times (7/16)^2] \times 1$$

$$6.0 \geq -1.48$$

The negative result indicates that the web alone is sufficient to carry the vertical component of the tension field. Therefore, the assumed transverse stiffeners satisfy all requirements. Use 6 in × 1/2 in transverse stiffener plates on both sides of the web of the interior girders.

Flange-to-Web Welds. Each flange will be connected to the web by a fillet web on each side of the web. The horizontal shear between the web and the flange must be resisted by the weld. Fillet welded connections subjected to shear on the effective area will be designed for the lesser of either the factored resistance of the connected material or the factored resistance of the weld metal.

The factored resistance of the weld metal, R_r, is

$$R_r = 0.6\phi_{e2}F_{exx} \tag{12.80}$$

where ϕ_{e2} = resistance factor for shear in throat of weld metal = 0.80
F_{exx} = classification strength of weld metal = 70.0 ksi (for E70XX weld metal)

Thus, $R_r = 0.6 \times 0.80 \times 70.0 = 33.60$ ksi.

The gross moment of inertia of the steel section, $I = 42{,}780$ in^4, will be used in computing the shear, v, kips per inch, between flange and web. The static moment of the flange area is $Q = 1.5 \times 20.0 \times (23.33 - 1.5/2) = 677$ in^3, and the factored maximum shear is 293.2 kips. Thus, $v = VQ/I = 293.2 \times 677/42{,}780 = 4.64$ kips/in. The weld size required to resist the shear is $v/(R_r \times 0.707 \times 2 \text{ sides of web}) = 4.64/(33.60 \times 0.707 \times 2) = 0.10$ in. The minimum size fillet weld required for a base metal thicker than ¾ in, is 5⁄16 in. Therefore, use two 5⁄16-in continuous fillet welds to connect the web to the bottom flange.

Bearing Stiffener Design. Bearing stiffeners should be welded or bolted on both sides of the web of rolled beams and plate girders at all bearing locations and at other points of concentrated load when:

$$V_u > 0.75\phi_b V_n \tag{12.81}$$

where V_u = shear due to factored loads, kips
ϕ_b = resistance factor for bearing = 1.0 for milled surfaces
V_n = nominal shear resistance, kips

The stiffeners should extend the full depth of the web and, as closely as practical, to the outer edges of flanges. The width, b_t, of each projecting element should satisfy

$$b_t \le 0.48 t_p \sqrt{(E/F_{ys})} \tag{12.82}$$

where t_p = thickness of projecting element, in
F_{ys} = yield stress of stiffener, ksi

For the design of the stiffeners at the end bearings, try 2 stiffener plates, 9-in wide, welded to each side of the web. Calculate minimum required thickness from the following form of Eq. 12.82:

$$t_p \ge \frac{b_t}{0.48\sqrt{E/F_{ys}}}$$

$$t_p \ge \frac{9}{0.48(28.38)} = 0.66 \text{ in}$$

Try t_p = ¾ in. The factored bearing resistance, B_r, of the stiffeners is calculated from

$$B_r = \phi_b A_{pn} F_{ys} \tag{12.83}$$

Assuming a 1-in long clip at the stiffener base to clear the web-to-flange fillet weld,

$$A_{pn} = 2(9.0 - 1.0) \times 3/4 = 12.0 \text{ in}^2$$

$$B_r = 1.0 \times 12.0 \times 36.0 = 432 \text{ kips} > 293.2 \text{ kips—OK.}$$

The factored axial resistance, P_r, is determined from

$$P_r = \phi_c P_n \tag{12.84}$$

where ϕ_c is the resistance factor for compression, 0.90, and P_n, is the nominal compressive resistance, equal to the axial resistance of an equivalent column section that consists of the stiffener pair plus a centrally located strip of web extending $9t_w$ on each side of the stiffeners as shown in Fig. 12.77. The gross cross sectional area of the equivalent column is $A_s = 2[(7/16)3.94 + (3/4)9] = 16.94\ \text{in}^2$. The moment of inertia of the equivalent column about centerline of web is $I_s = (3/4)(2 \times 9 + 7/16)^3/12 = 392\ \text{in}^4$ and the radius of gyration is $r_s = \sqrt{(392/16.94)} = 4.8$ in.

The slenderness ratio must satisfy $KL/r < 120$ where the effective length factor, K, is 0.75 and $L = D$. Thus, $KL/r_s = 0.75(60)/4.8 = 9.4 < 120$—OK

The nominal compressive resistance is as follows:

when $\lambda \leq 2.25$

$$P_n = 0.66^{\lambda} F_y A_s \tag{12.85}$$

when $\lambda > 2.25$

$$P_n = 0.88 F_y A_s / \lambda \tag{12.86}$$

where

$$\lambda = \left(\frac{KL}{r_s \pi}\right)^2 \frac{F_y}{E} \tag{12.87}$$

Thus, $\lambda = (9.4/\pi)^2(36/29{,}000) = 0.011 < 2.25$. From Eq. 12.85, $P_n = 0.66^{0.011}\,(36)(16.94) = 607$ kips, and from Eq. 12.84, $P_r = 0.90(607) = 546\ \text{kips} > 293.2$ kips—OK. Use two $3/4 \times 9$ in plates for bearing stiffeners.

The welds to the web must be capable of developing the entire reaction. Minimum size fillet weld for the 3/4-in bearing stiffener is 1/4 in as specified in AASHTO for base metal thickness of thicker part joined $\geq$ 3/4-in. Subtracting for top and bottom clips, the length available for each weld is $60 - 2 \times 2\frac{1}{2} = 55$ in. For two stiffeners, there are $2 \times 2 = 4$ lines of weld. Total shear resistance that can be developed in welds is, $R = 4 \times 1/4 \times 0.707 \times 55 \times 33.60 = 1306\ \text{kips} > 293.2$ kips—OK. Use 1/4-in full height fillet welds for connect bearing stiffeners to web.

Shear Connector Design. For shear connectors, as in the example of Art. 12.5, 7/8-in diameter by 6-in-long welded studs will be used to satisfy the AASHTO requirement that the ratio of the height to the diameter of a stud connector not to be less than 4.0.

The fatigue resistance of an individual shear connector, Z_r, is

$$Z_r = \alpha d^2 \geq 5.5 d^2/2 \tag{12.88}$$

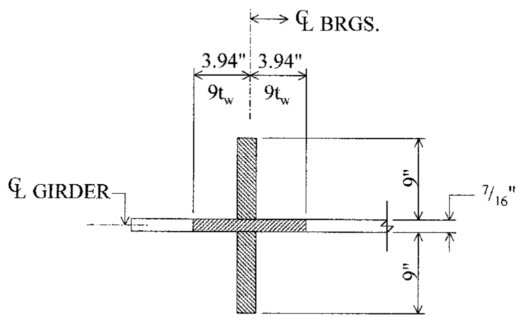


FIGURE 12.77 Equivalent column for bearing stiffener design.

where $\alpha = 34.5 - 4.28 \log N$
d = diameter of stud, in
N = number of cycles

The number of cycles is determined as follows (see Art. 11.10). The bridge is located on a major highway with an average daily truck traffic (ADTT) in one direction less than 2,500. ADTT will be adjusted for single lane truck traffic as specified in AASHTO, since no specific site information is available, as follows:

$$\mathrm{ADTT}_{SL} = p \times \mathrm{ADTT} \tag{12.89}$$

where p is the fraction of truck traffic in a single lane, = 0.85 for a bridge with 2 lanes available to trucks. Thus, $\mathrm{ADTT}_{SL} = 0.85 \times 2{,}500 = 2{,}125$. Calculate N from

$$N = (365 \text{ days/year})(75 \text{ years})(n)\mathrm{ADTT}_{SL} \tag{12.90}$$

where n is number of stress range cycles per truck passage, = 1.0 for simple span girders with $L > 40$ ft. In this case, $N = 365 \times 75 \times 1.0 \times 2{,}125 = 5.8 \times 10^7$ cycles. Then, calculate α as, $\alpha = 34.5 - 4.28 \log (5.8 \times 10^7) = 1.27$. From Eq. 12.88, since $1.27 < 5.5/2$, $Z_r = 5.5d^2/2 = 5.5(7/8)^2/2 = 2.1$ kips/stud.

Determine the pitch, p (in), of shear connectors from

$$p \le \frac{nZ_r I}{V_{sr} Q} \tag{12.91}$$

where n = number of shear connectors in a cross section
I = moment of inertia of composite section for short-term loads, in^4,
Q = first moment of the transformed area of the slab about the neutral axis of the short-term composite section, in^3
V_{sr} = shear force range under $LL + IM$ determined for the Fatigue Limit state, kips

The pitch should also satisfy

$$6d < p < 24 \text{ in} \tag{12.92}$$

which, for $d = 7/8$ in, becomes 5.25 in $< p <$ 24 in.

Assume 3 studs per row and substitute the following values in Eq. 12.91: $V_{sr} = 35.2$ kips as calculated before, $I = 128{,}260$ in^4, and $Q = 106.3(21.92 - 8.5/2) = 1{,}878$ in^3.

$$p \le \frac{3 \times 2.1 \times 128{,}260}{35.2 \times 1{,}878}$$

$$p \le 12.2 \text{ in}$$

Thus, to satisfy the Fatigue and Fracture Limit State, the longitudinal stud spacing must not exceed 12.2 in.

Check the transverse spacing of studs. The minimum transverse spacing of stud shear connectors is 4 stud diameters center-to-center and minimum clear distance between the edge of the top flange and the nearest shear stud is 1.0 in as required by AASHTO. Assuming 3 studs per row and setting the edge distance as 1.0-inch, transverse stud spacing is calculated as $[16.0 - 2(1.0) - 7/8]/2 = 6\,9/16$ in $> 4.0(7/8) = 3\frac{1}{2}$ in—OK.

Next check the Strength Limit State. The factored resistance of shear connectors, Q_r, is

$$Q_r = \phi_{sc} Q_n \tag{12.93}$$

where ϕ_{sc} = resistance factor for shear connectors, 0.85, and Q_n is the nominal shear resistance of a single shear connector embedded in a concrete slab.

$$Q_n = 0.5A_{sc}\sqrt{(f'_c E_c)} \leq A_{sc}F_u \tag{12.94}$$

where A_{sc} = cross sectional area of a stud shear connector, in^2
f'_c = minimum specified compressive strength of concrete, ksi
F_u = specified minimum tensile strength of a stud shear connector, 60 ksi
E_c = modulus of elasticity of concrete, ksi

Substitution of values gives $Q_n = 0.5 \times 0.60\sqrt{4.0 \times 3{,}600} = 36.0 \leq 0.60 \times 60 = 36.0$ kips and $Q_r = 0.85 \times 36.0 = 30.6$ kips.

The number of shear connectors provided between the maximum positive moment section and zero moment section (in a simple span) must not be less than

$$n = V_h/Q_r \tag{12.95}$$

The nominal horizontal shear force, V_h, is the lesser of the following:

$$V_h = 0.85f'_c bt_s \qquad \text{(concrete deck limit)} \tag{12.96}$$

$$V_h = F_{yw}Dt_w + F_{yt}b_t t_t + F_{yc}b_f t_f \qquad \text{(steel girder limit)} \tag{12.97}$$

From Eq. 12.96, $V_h = 0.85 \times 4.0 \times 100 \times 8.5 = 2{,}890$ kips, and from Eq. 12.97, $V_h = 36$ $(60 \times 7/16 + 16 \times 3/4 + 20 \times 1\tfrac{1}{2}) = 2{,}457$ kips. Thus, $V_h = 2{,}457$ kips. From Eq. 12.95, $n = 2{,}457/30.6 = 81$ studs. With the studs placed in groups of three, there should be at least 27 rows of studs in each half of the girder. Then the average pitch is $p = 50$ ft$/27 =$ 1.85 ft or 22.2 in. This is greater than the value of 12.2 in previously determined from the fatigue requirements and does not control.

Consequently, use three 7/8-in-dia by 6-in-long shear studs per row, spaced at 12 in.

12.18.19 Constructibility Check

The constructibility check of the girder will be based on a non-composite section. During construction, such sections must satisfy the traditional non-compact section compression flange slenderness and web slenderness requirements, because the concrete deck has not hardened and they are not yet composite.

General Proportions. Flexural components must be proportioned such that $0.1 \leq I_{yc}/I_y \leq 0.9$ and webs without longitudinal stiffeners such that $2D_c/t_w \leq 6.77\sqrt{(E/f_c)}$. The trial section in this example satisfies these requirements as shown in Art. 12.18.4.

Flexure Check. First, check to see if the following non-compact section compression flange requirement of AASHTO for a girder without longitudinal stiffeners is satisfied:

$$b/2t_f \leq 1.38\sqrt{E/(f_c\sqrt{2D_c/t_w})} \tag{12.98}$$

For the constructibility investigation, f_c is determined for the compression flange under factored construction loads for steel section only as 23.61 ksi. D_c, the depth of the web in compression, is 38.17 in as calculated in Art. 12.18.4. Thus,

$$16/(2 \times 0.75) \leq 1.38\sqrt{29{,}000/(23.61/\sqrt{2 \times 38.17/0.4375})}$$

$$10.7 \leq 13.3\text{—OK}$$

The compression flange slenderness requirement for non-compact sections is met.

Shear Check. For investigation of the deck construction sequence of homogeneous sections with transverse stiffeners (with or without longitudinal stiffeners), the nominal shear resistance should meet the following:

$$V_n = CV_p \tag{12.99}$$

where C = ratio of shear buckling stress to shear yield stress
V_p = plastic shear capacity, kips,
= $0.58F_{yw}Dt_w$.

In this case, $C = 0.378$ (see Art. 12.18.5) and $V_p = 0.58(36)(60)(7/16) = 548$ kips. Thus, $V_n = 0.378(548) = 207.2$ kips.

The factored shear due to construction loads is

$$V_{fDC1} = 1.25(1/2)(1.365)(100) = 85.3 \text{ kips} < V_p = 207.2 \text{ kips—OK}$$

The trial section has satisfied flexure, shear, fatigue and fracture, and the constructibility requirements of the AASHTO LRFD Specifications. For completeness, the need for temporary wind bracing during construction should also be investigated.