2 General Electrotechnical Formulae

2.1 Electrotechnical symbols as per DIN 1304 Part 1

Table 2-1

Mathematical symbols for electrical quantities (general)

Symbol	Quantity	SI unit
Q	quantity of electricity, electric charge	C
E	electric field strength	V/m
D	electric flux density, electric displacement	C/m^2
U	electric potential difference	V
φ	electric potential	V
ε	permittivity, dielectric constant	F/m
ε_o	electric field constant, $\varepsilon_o = 0.885419 \cdot 10^{-11}$ F/m	F/m
ε_r	relative permittivity	1
C	electric capacitance	F
I	electric current	A
J	electric current density	A/m^2
x, γ, σ	specific electric conductivity	S/m
ρ	specific electric resistance	Ωm
G	electric conductance	S
R	electric resistance	Ω
θ	electromotive force	A

Table 2-2

Mathematical symbols for magnetic quantities (general)

Symbol	Quantity -	SI unit
Φ	magnetic flux	Wb
B	magnetic induction	T
H	magnetic field strength	A/m
V	magnetomotive force	A
φ	magnetic potential	A
μ	permeability	H/m
μ_o	absolute permeability, $\mu_o = 4\,\pi \cdot 10^{-7} \cdot$ H/m	H/m
μ_r	relative permeability	1
L	inductance	H
L_{mn}	mutual inductance	H

Table 2-3

Mathematical symbols for alternating-current quantities and network quantities

Symbol	Quantity	SI unit
S	apparent power	W, VA
P	active power	W
Q	reactive power	W, Var
D	distortion power	W
φ	phase displacement	rad
ϑ	load angle	rad
λ	power factor, $\lambda = P/S$, λ cos φ [1]	1
δ	loss angle	rad
d	loss factor, $d = \tan \delta$	1
Z	impedance	Ω
Y	admittance	S
R	resistance	Ω
G	conductance	S
X	reactance	Ω
B	susceptance	S
γ	impedance angle, $\gamma = \arctan X/R$	rad

Table 2-4

Numerical and proportional relationships

Symbol	Quantity	SI unit
η	efficiency	1
s	slip	1
p	number of pole-pairs	1
w, N	number of turns	1
$ü$	transformation ratio	1
m	number of phases and conductors	1
γ	amplitude factor	1
k	overvoltage factor	1
ν	ordinal number of a periodic component	1
s	wave content	1
g	fundamental wave content	1
k	harmonic content, distortion factor	1
ζ	increase in resistance due to skin effect, $\zeta = R_{\sim} / R_{-}$	1

[1] Valid only for sinusoidal voltage and current.

2.2 Alternating-current quantities

With an alternating current, the instantaneous value of the current changes its direction as a function of time $i = f(t)$. If this process takes place periodically with a period of duration T, this is a periodic alternating current. If the variation of the current with respect to time is then sinusoidal, one speaks of a sinusoidal alternating current.

The frequency f and the angular frequency ω are calculated from the periodic time T with

$$f = \frac{1}{T} \text{ and } \omega = 2\,\pi\,f = \frac{2\pi}{T}.$$

The *equivalent d. c. value* of an alternating current is the average, taken over one period, of the value:

$$|\bar{i}| = \frac{1}{T}\int_0^T |i|\,\mathrm{d}t = \frac{1}{2\pi}\int_0^{2\pi} |i|\,\mathrm{d}\omega t.$$

This occurs in rectifier circuits and is indicated by a moving-coil instrument, for example.

The root-mean-square value (rms value) of an alternating current is the square root of the average of the square of the value of the function with respect to time.

$$I = \sqrt{\frac{1}{T}\cdot\int_0^T i^2\mathrm{d}t} = \sqrt{\frac{1}{2\pi}\cdot\int_0^{2\pi} i^2\mathrm{d}\omega t}.$$

As regards the generation of heat, the root-mean-square value of the current in a resistance achieves the same effect as a direct current of the same magnitude.

The root-mean-square value can be measured not only with moving-coil instruments, but also with hot-wire instruments, thermal converters and electrostatic voltmeters.

A non-sinusoidal current can be resolved into the fundamental oscillation with the fundamental frequency f and into harmonics having whole-numbered multiples of the fundamental frequency. If I_1 is the rms value of the fundamental oscillation of an alternating current, and I_2, I_3 etc. are the rms values of the harmonics having frequencies $2\,f$, $3\,f$, etc., the rms value of the alternating current is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots}$$

If the alternating current also includes a direct-current component i_{-}, this is termed an undulatory current. The rms value of the undulatory current is

$$I = \sqrt{I_{-}^2 + I_1^2 + I_2^2 + I_3^2 + \ldots}$$

The fundamental oscillation content g is the ratio of the rms value of the fundamental oscillation to the rms value of the alternating current

$$g = \frac{I_1}{I}.$$

The harmonic content k (distortion factor) is the ratio of the rms value of the harmonics to the rms value of the alternating current.

$$k = \frac{\sqrt{I_2^2 + I_3^2 + \ldots}}{I} = \sqrt{1 - g^2}$$

The fundamental oscillation content and the harmonic content cannot exceed 1.

In the case of a sinusoidal oscillation

the fundamental oscillation content $g = 1$,
the harmonic content $k = 0$.

Forms of power in an alternating-current circuit

The following terms and definitions are in accordance with DIN 40110 for the sinusoidal wave-forms of voltage and current in an alternating-current circuit.

apparent power	$S = UI = \sqrt{P^2 + Q^2}$,
active power	$P = UI \cdot \cos\varphi = S \cdot \cos\varphi$,
reactive power	$Q = UI \cdot \sin\varphi = S \cdot \sin\varphi$,
power factor	$\cos\varphi = \dfrac{P}{S}$,
reactive factor	$\sin\varphi = \dfrac{Q}{S}$.

When a three-phase system is loaded symmetrically, the apparent power is

$$S = 3\,U_1 I_1 = \sqrt{3} \cdot U \cdot I_1,$$

where I_1 is the rms phase current, U_1 the rms value of the phase to neutral voltage and U the rms value of the phase to phase voltage. Also

active power	$P = 3\,U_1 I_1 \cos\varphi = \sqrt{3} \cdot U \cdot I_1 \cdot \cos\varphi$,
reactive power	$Q = 3\,U_1 I_1 \sin\varphi = \sqrt{3} \cdot U \cdot I_1 \cdot \sin\varphi$.

The unit for all forms of power is the watt (W). The unit watt is also termed volt-ampere (symbol VA) when stating electric apparent power, and Var (symbol var) when stating electric reactive power.

Resistances and conductances in an alternating-current circuit

impedance	$Z = \dfrac{U}{I} = \dfrac{S}{I^2} = \sqrt{R^2 + X^2}$
resistance	$R = \dfrac{U\cos\varphi}{I} = \dfrac{P}{I^2} = Z\cos\varphi = \sqrt{Z^2 - X^2}$
reactance	$X = \dfrac{U\sin\varphi}{I} = \dfrac{Q}{I^2} = Z\sin\varphi = \sqrt{Z^2 - R^2}$
inductive reactance	$X_i = \omega L$
capacitive reactance	$X_c = \dfrac{1}{\omega C}$
admittance	$Y = \dfrac{I}{U} = \dfrac{S}{U^2} = \sqrt{G^2 + B^2} = \dfrac{1}{Z}$
conductance	$G = \dfrac{I\cos\varphi}{U} = \dfrac{P}{U^2} = Y\cos\varphi = \sqrt{Y^2 - B^2} = \dfrac{R}{Z^2}$
conductance	$B = \dfrac{I\sin\varphi}{U} = \dfrac{Q}{U^2} = Y\sin\varphi = \sqrt{Y^2 - G^2} = \dfrac{X}{Z^2}$
inductive susceptance	$B_i = \dfrac{1}{\omega L}$
capacitive susceptance	$B_c = \omega C$

$\omega = 2\,\pi\,f$ is the angular frequency and φ the phase displacement angle of the voltage with respect to the current. U, I and Z are the numerical values of the alternating-current quantities $\underline{U}$, $\underline{I}$ and $\underline{Z}$.

Complex presentation of sinusoidal time-dependent a. c. quantities

Expressed in terms of the load vector system:

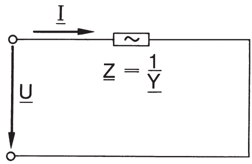


Fig. 2- 1
Equivalent circuit diagram

$\underline{U} = \underline{I} \cdot \underline{Z}$, $\underline{I} = \underline{U} \cdot \underline{Y}$
The symbols are underlined to denote that they are complex quantities (DIN 1304).

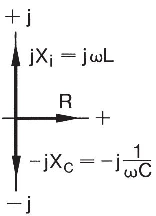


Fig. 2-2
Vector diagram of resistances

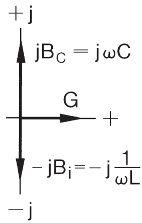


Fig. 2-3
Vector diagram of conductances

If the voltage vector $\underline{U}$ is laid on the real reference axis of the plane of complex numbers, for the equivalent circuit in Fig. 2-1 with $\underline{\underline{Z}} = R + \mathrm{j}\,X_i$: we have

$$\underline{U} = U,$$

$$\underline{I} = I_w - \mathrm{j}\,I_b = I\,(\cos\varphi - \mathrm{j}\sin\varphi),$$

$$I_w = \frac{P}{U};\ I_b = \frac{Q}{U};$$

$$\underline{S}^{1)} = U\,\underline{I}^* = U\,I\,(\cos\varphi + \mathrm{j}\sin\varphi) = P + \mathrm{j}\,Q,$$

$$S = |\underline{S}| = U\,I = \sqrt{P^2 + Q^2},$$

$$\underline{\underline{Z}} = R + \mathrm{j}\,X_i = \frac{U}{\underline{I}} = \frac{U}{I\,(\cos\varphi - \mathrm{j}\sin\varphi)} = \frac{U}{I}\,(\cos\varphi + \mathrm{j}\sin\varphi),$$

$$\text{where } R = \frac{U}{I}\cos\varphi \text{ and } X_i = \frac{U}{I}\sin\varphi,$$

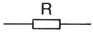
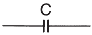
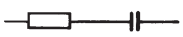
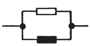
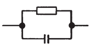
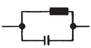
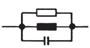
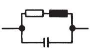
$$\underline{Y} = \mathrm{G} - \mathrm{j}B = \frac{\underline{I}}{U} = \frac{I}{U}\,(\cos\varphi - \mathrm{j}\sin\varphi)$$

$$\text{where } \mathrm{G} = \frac{I}{U}\cos\varphi \text{ and } B_i = \frac{I}{U}\sin\varphi.$$

[1)] $\underline{S}$: See DIN 40110
$\underline{I}^*$ = conjugated complex current vector

Table 2-5

Alternating-current quantities of basic circuits

	Circuit	$\underline{Z}$	$\lvert\underline{Z}\rvert$
1.		R	R
2.		$j\,\omega L$	ωL
3.		$-j/(\omega C)$	$1/\omega C$
4.		$R + j\,\omega L$ [1]	$\sqrt{R^2 + (\omega L)^2}$
5.		$R - j/(\omega C)$	$\sqrt{R^2 + 1/(\omega C)^2}$
6.		$j\,(\omega L - 1/(\omega C))$ [2]	$\sqrt{(\omega L - 1/(\omega C))^2}$
7.		$R + j(\omega L - 1/(\omega C))$ [2]	$\sqrt{R^2 + (\omega L - 1/(\omega C))^2}$
8.		$\dfrac{R\,\omega L}{\omega L - jR}$	$\dfrac{R\,\omega L}{\sqrt{R^2 + (\omega L)^2}}$
9.		$\dfrac{R - j\,\omega C R^2}{1 + (\omega C)^2 R^2}$ [3]	$\dfrac{R}{\sqrt{1 + (\omega C)^2 R^2}}$
10.		$\dfrac{j}{1/(\omega L) - \omega C}$	$\dfrac{1}{\sqrt{(1/\omega L)^2 - (\omega C)^2}}$
11.	 [4]	$\dfrac{1}{1/R + j\,(\omega C - 1/(\omega L))}$ $[\underline{Y} = 1/R^2 + j\,(\omega C - 1/(\omega L))]$	$\dfrac{1}{\sqrt{1/R^2 + (\omega C - 1/(\omega L))^2}}$
12.	 [5]	$\dfrac{R + j\,(L\,(1 - \omega^2 LC) - R^2 C)}{(1 - \omega^2 LC)^2 + (R\,\omega C)^2}$	$\dfrac{\sqrt{R^2 + [L\,(1 - \omega^2 LC) - R^2 C]^2}}{(1 - \omega^2 LC)^2 + (R\,\omega C)^2}$

[1] With small loss angle δ (= $1/\varphi$) ≈ tan δ (error at 4° about 1 ‰): $\underline{Z} \approx \omega L\,(\delta + j)$.

[2] Series resonance (voltage resonance) for $\omega L = 1/(\omega C)$:

$X_{res} = |X_L| = |X_c| = \sqrt{L/C}$ $\quad f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad \underline{Z}_{res} = R.$

Close to resonance ($|\Delta f| < 0.1\,f_{res}$) is $\underline{Z} \approx R + j\,X_{res} \cdot 2\,\Delta f / f_{res}$ with $\Delta f = f - f_{res}$

[3] With small loss angle δ (= $1/\varphi$) ≈ tan $\delta = -1/(\omega C R)$:

$\underline{Z} = \dfrac{\delta + j}{\omega C}$ $\quad B_{res} = \sqrt{C/L}$: $f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad \underline{Y}_{res} = G.$

[4] Close to resonance ($|\Delta f| < 0.1\,f_{res}$):

$\underline{Y} = G + j\,B_{res} \cdot 2\,\Delta f$ with $\Delta f = f - f_{res}$

[5] e. g. coil with winding capacitance.

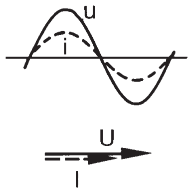
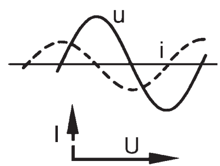
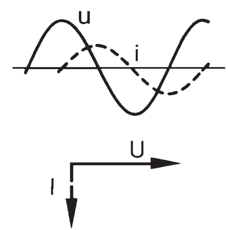
Table 2-6

Current / voltage relationships

		Ohmic resistance R	Capacitance (capacitor) C	Inductance (choke coil) L
General law	$u =$	$i\,R$	$\frac{1}{C}\int i\,\mathrm{d}t$	$L \cdot \frac{\mathrm{d}i}{\mathrm{d}t}$
	$i =$	$\frac{u}{R}$	$C \cdot \frac{\mathrm{d}u}{\mathrm{d}t}$	$\frac{1}{L}\int u\,\mathrm{d}t$
Time law	$u =$	$\hat{u} \sin \omega t$	$\hat{u} \sin \omega t$	$\hat{u} \sin \omega t$
hence	$u =$	$\hat{\imath} R \sin \omega t = \hat{u} \sin \omega t$	$-\frac{1}{\omega C}\,\hat{\imath} \cos \omega t = -\,\hat{u} \cos \omega t$	$\omega L\,\hat{\imath} \cos \omega t = \hat{u} \cos \omega t$
	$i =$	$\frac{\hat{u}}{R} \sin \omega t = \hat{\imath} \sin \omega t$	$\omega C\,\hat{u} \cos \omega t = \hat{\imath} \cos \omega t$	$-\frac{1}{\omega L}\,\hat{u} \cos \omega t = -\,\hat{\imath} \cos \omega t$
Elements of calculation	$\hat{\imath} =$	$\hat{u} / R$	$\omega C\,\hat{u}$	$\hat{u} / (\omega L)$
	$\hat{u} =$	$\hat{\imath} R$	$\hat{\imath} / (\omega C)$	$\hat{\imath}\,\omega L$
	$\varphi =$	0 u and i in phase	$\arctan \frac{1}{\omega C \cdot 0} = -\frac{\pi}{2}$ i leads u by 90 °	$\arctan \frac{\omega L}{0} = \frac{\pi}{2}$ i lags u by 90 °
	$f =$	$\frac{\omega}{2\pi}$	$\frac{\omega}{2\pi}$	$\frac{\omega}{2\pi}$

(continued)

Table 2-6 (continued)

		Ohmic resistance R	Capacitance (capacitor) C	Inductance (choke coil) L
Alternating current impedance	$\underline{Z} =$	R	$\frac{-\,j}{\omega\, C}$	$j\,\omega\, L$
	$\lvert\underline{Z}\rvert =$	R	$\frac{1}{\omega\, C}$	$\omega\, L$
Diagrams				

2.3 Electrical resistances

2.3.1 Definitions and specific values

An ohmic resistance is present if the instantaneous values of the voltage are proportional to the instantaneous values of the current, even in the event of time-dependent variation of the voltage or current. Any conductor exhibiting this proportionality within a defined range (e. g. of temperature, frequency or current) behaves within this range as an ohmic resistance. Active power is converted in an ohmic resistance. For a resistance of this kind is

$$R = \frac{P}{I^2}.$$

The resistance measured with direct current is termed the *d. c. resistance* R_{-}. If the resistance of a conductor differs from the d. c. resistance only as a result of skin effect, we then speak of the a. c. resistance $R_{\sim}$ of the conductor. The ratio expressing the increase in resistance is

$$\zeta = \frac{R_{\sim}}{R_{-}} = \frac{\textit{a. c.}\ \text{resistance}}{\text{d. c. resistance}}.$$

Specific values for major materials are shown in Table 2-7.

Table 2-7

Numerical values for major materials

Conductor	Specific electric resistance ρ (mm² Ω/m)	Electric conductivity $x = 1/\rho$ (m/mm² Ω)	Temperature coefficient α (K^{-1})	Density (kg/dm³)
Aluminium, 99.5 % Al, soft	0.0278	36	$4 \cdot 20^{-3}$	2.7
Al-Mg-Si	0.03…0.033	33…30	$3.6 \cdot 10^{-3}$	2.7
Al-Mg	0.06…0.07	17…14	$2.0 \cdot 10^{-3}$	2.7
Al bronze, 90 % Cu, 10 % Al	0.13	7.7	$3.2 \cdot 10^{-3}$	8.5
Bismuth	1.2	0.83	$4.5 \cdot 10^{-3}$	9.8
Brass	0.07	14.3	$1.3…1.9 \cdot 10^{-3}$	8.5
Bronze, 88 % Cu, 12 % Sn	0.18	5.56	$0.5 \cdot 10^{-3}$	8.6…9
Cast iron	0.60…1.60	1.67…0.625	$1.9 \cdot 10^{-3}$	7.86…7.2
Conductor copper, soft	0.01754	57	$4.0 \cdot 10^{-3}$	8.92
Conductor copper, hard	0.01786	56	$3.92 \cdot 10^{-3}$	8.92
Constantan	0.49…0.51	2.04…1.96	$-0.05 \cdot 10^{-3}$	8.8
CrAl 20 5	1.37	0.73	$0.05 \cdot 10^{-3}$	—
CrAl 30 5	1.44	0.69	$0.01 \cdot 10^{-3}$	—
Dynamo sheet	0.13	7.7	$4.5 \cdot 10^{-3}$	7.8
Dynamo sheet alloy (1 to 5 % Si)	0.27…0.67	3.7…1.5	—	7.8
Graphite and retort carbon	13…100	0.077…0.01	$-0.8…-0.2 \cdot 10^{-3}$	2.5…1.5
Lead	0.208	4.8	$4.0 \cdot 10^{-3}$	11.35
Magnesium	0.046	21.6	$3.8 \cdot 10^{-3}$	1.74
Manganin	0.43	2.33	$0.01 \cdot 10^{-3}$	8.4
Mercury	0.958	1.04	$0.90 \cdot 10^{-3}$	13.55
Molybdenum	0.054	18.5	$4.3 \cdot 10^{-3}$	10.2
Monel metal	0.42	2.8	$0.19 \cdot 10^{-3}$	—
Nickel silver	0.33	3.03	$0.4 \cdot 10^{-3}$	8.5

(continued)

Table 2-7 (continued)

Numerical values for major materials

Conductor	Specific electric resistance ρ (mm² Ω/m)	Electric conductivity $x = 1/\rho$ (m/mm² Ω)	Temperature coefficient α (K⁻¹)	Density (kg/dm³)
Ni Cr 30 20	1.04	0.96	$0.24 \cdot 10^{-3}$	8.3
Ni Cr 6015	1.11	0.90	$0.13 \cdot 10^{-3}$	8.3
Ni Cr 80 20	1.09	0.92	$0.04 \cdot 10^{-3}$	8.3
Nickel	0.09	11.1	$6.0 \cdot 10^{-3}$	8.9
Nickeline	0.4	2.5	$0.18 \ldots 0.21 \cdot 10^{-3}$	8.3
Platinum	0.1	10	$3.8 \ldots 3.9 \cdot 10^{-3}$	21.45
Red brass	0.05	20	—	8.65
Silver	0.0165	60.5	$41 \cdot 10^{-3}$	10.5
Steel, 0.1% C, 0.5 % Mn	0.13…0.15	7.7…6.7	$4 \ldots 5 \cdot 10^{-3}$	7.86
Steel, 0.25 % C, 0.3 % Si	0.18	5.5	$4 \ldots 5 \cdot 10^{-3}$	7.86
Steel, spring, 0.8 % C	0.20	5	$4 \ldots 5 \cdot 10^{-3}$	7.86
Tantalum	0.16	6.25	$3.5 \ldots 10^{-3}$	16.6
Tin	0.12	8.33	$4.4 \cdot 10^{-3}$	7.14
Tungsten	0.055	18.2	$4.6 \cdot 10^{-3}$	19.3
Zinc	0.063	15.9	$3.7 \cdot 10^{-3}$	7.23

Resistance varies with temperature, cf. Section 2.3.3

2.3.2 Resistances in different circuit configurations

Connected in series (Fig. 2-4)

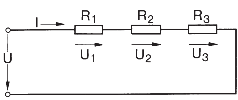


Fig. 2-4

Total resistance = Sum of individual resistances

$$R = R_1 + R_2 + R_3 + \ldots$$

The component voltages behave in accordance with the resistances $U_1 = I\,R_1$ etc.

The current at all resistances is of equal magnitude $I = \frac{U}{R}$.

Connected in parallel (Fig. 2-5)

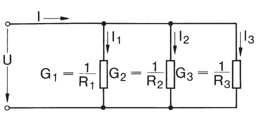


Fig. 2-5

Total conductance = Sum of the individual conductances

$$\frac{1}{R} = G = G_1 + G_2 + G_3 + \ldots \qquad R = \frac{1}{G}.$$

In the case of n equal resistances the total resistance is the nth part of the individual resistances. The voltage at all the resistances is the same. Total current

$$I = \frac{U}{R} = \text{Sum of components } I_1 = \frac{U}{R_1} \text{ etc.}$$

The currents behave inversely to the resistances

$$I_1 = I\frac{R}{R_1};\ I_2 = I\frac{R}{R_2};\ I_3 = I\frac{R}{R_3}.$$

Transformation delta-star and star-delta (Fig. 2-6)

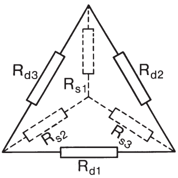


Fig. 2-6

Conversion from delta to star connection with the same total resistance:

$$R_{S1} = \frac{R_{d2}\,R_{d3}}{R_{d1} + R_{d2} + R_{d3}}$$

$$R_{S2} = \frac{R_{d3}\,R_{d1}}{R_{d1} + R_{d2} + R_{d3}}$$

$$R_{S3} = \frac{R_{d1}\,R_{d2}}{R_{d1} + R_{d2} + R_{d3}}$$

Conversion from star to delta connection with the same total resistance:

$$R_{d1} = \frac{R_{S1}\,R_{S2} + R_{S2}\,R_{S3} + R_{S3}\,R_{S1}}{R_{S1}}$$

$$R_{d2} = \frac{R_{S1}\,R_{S2} + R_{S2}\,R_{S3} + R_{S3}\,R_{S1}}{R_{S2}}$$

$$R_{d3} = \frac{R_{S1}\,R_{S2} + R_{S2}\,R_{S3} + R_{S3}\,R_{S1}}{R_{S3}}$$

Calculation of a bridge between points A and B (Fig. 2-7)

To be found:

1. the total resistance R_{tot} between points A and B,
2. the total current I_{tot} between points A and B,
3. the component currents in R_1 to R_5.

Given:

voltage $U = 220$ V.
resistance $R_1 = 10\ \Omega$,
$R_2 = 20\ \Omega$,
$R_3 = 30\ \Omega$,
$R_4 = 40\ \Omega$,
$R_5 = 50\ \Omega$.

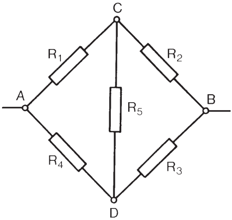


Fig. 2-7

First delta connection CDB is converted to star connection CSDB (Fig. 2-8):

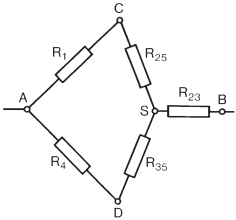


Fig. 2-8

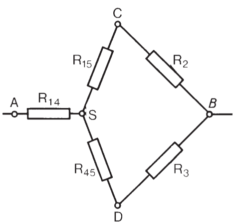
$$R_{25} = \frac{R_2\,R_5}{R_2 + R_3 + R_5} = \frac{20 \cdot 50}{20 + 30 + 50} = 10\ \Omega,$$

$$R_{35} = \frac{R_3\,R_5}{R_2 + R_3 + R_5} = \frac{30 \cdot 50}{20 + 30 + 50} = 15\ \Omega,$$

$$R_{23} = \frac{R_2\,R_3}{R_2 + R_3 + R_5} = \frac{20 \cdot 30}{20 + 30 + 50} = 6\ \Omega,$$

$$R_{tot} = \frac{(R_1 + R_{25})\,(R_4 + R_{35})}{R_1 + R_{25} + R_4 + R_{35}} + R_{23} =$$

$$= \frac{(10 + 10)\,(40 + 15)}{10 + 10 + 40 + 15} + 6 = 20.67\ \Omega.$$



$$I_{tot} = \frac{U}{R_{tot}} = \frac{220}{20.67} = 10.65\ \text{A}.$$

$$I_{R1} = I_{tot}\,\frac{R_{tot} - R_{23}}{R_1 + R_{25}} = 10.65 \cdot \frac{20.67 - 6}{10 + 10} = 7.82\ \text{A},$$

$$I_{R4} = I_{tot}\,\frac{R_{tot} - R_{23}}{R_4 + R_{35}} = 10.65 \cdot \frac{20.67 - 6}{40 + 15} = 2.83\ \text{A},$$

By converting the delta connection CDA to star connection CSDA, we obtain the following values (Fig. 2-9): $R_{15} = 5\,\Omega$; $R_{45} = 20\,\Omega$; $R_{14} = 4\,\Omega$; $I_{R2} = 7.1$ A; $I_{R3} = 3.55$ A.

Fig. 2-9

With alternating current the calculations are somewhat more complicated and are carried out with the aid of resistance operators. Using the symbolic method of calculation, however, it is basically the same as above.

2.3.3 The influence of temperature on resistance

The resistance of a conductor is

$$R = \frac{l \cdot \rho}{A} = \frac{l}{x \cdot A}$$

where

l = Total length of conductor

A = Cross-sectional area of conductor

ρ = Specific resistance (at 20 °C)

$x = \frac{1}{\rho}$ Conductance

α = Temperature coefficient.

Values for ρ, x and α are given in Table 2-7 for a temperature of 20 °C.

For other temperatures ϑ[1] (ϑ in °C)

$$\rho_\vartheta = \rho_{20}\,[1 + \alpha\,(\vartheta - 20)]$$

[1] Valid for temperatures from – 50 to + 200 °C.

and hence for the conductor resistance

$$R_\vartheta = \frac{l}{A} \cdot \rho_{20} [1 + \alpha (\vartheta - 20)].$$

Similarly for the conductivity

$$x_\vartheta = x_{20} [1 + \alpha (\vartheta - 20)]^{-1}$$

The temperature rise of a conductor or a resistance is calculated as

$$\Delta \vartheta = \frac{R_w / R_k - 1}{\alpha}.$$

The values R_k and R_w are found by measuring the resistance of the conductor or resistance in the cold and hot conditions, respectively.

Example:

The resistance of a copper conductor of $l = 100$ m and $A = 10$ mm² at 20 °C is

$$R_{20} = \frac{100 \cdot 0.0175}{10} = 0.175\ \Omega.$$

If the temperature of the conductor rises to $\vartheta = 50$ °C, the resistance becomes

$$R_{50} = \frac{100}{10} \cdot 0.0175 [1 + 0.004 (50 - 20)] \approx 0.196\ \Omega.$$

2.4 Relationships between voltage drop, power loss and conductor cross section

Especially in low-voltage networks is it necessary to check that the conductor cross-section, chosen with respect to the current-carrying capacity, is adequate as regards the voltage drop. It is also advisable to carry out this check in the case of very long connections in medium-voltage networks. (See also Sections 6.1.6 and 13.2.3).

Direct current

voltage drop $$\Delta U = R'_L \cdot 2 \cdot l \cdot I = \frac{2 \cdot l \cdot I}{x \cdot A} = \frac{2 \cdot l \cdot P}{x \cdot A \cdot U}$$

percentage voltage drop $$\Delta u = \frac{\Delta U}{U_n} 100\ \% = \frac{R'_L \cdot 2 \cdot l \cdot I}{U_n} 100\ \%$$

power loss $$\Delta P = I^2 R'_L\, 2 \cdot l = \frac{2 \cdot l \cdot P^2}{x \cdot A \cdot U^2}$$

percentage power loss $$\Delta p = \frac{\Delta P}{P_n} 100\ \% = \frac{I^2 R'_L \cdot 2 \cdot l}{P_n} 100\ \%$$

conductor cross section $$A = \frac{2 \cdot l \cdot I}{x \cdot \Delta U} = \frac{2 \cdot l \cdot I}{x \cdot \Delta u \cdot U} 100\ \% = \frac{2 \cdot l \cdot P}{\Delta p \cdot U^2 \cdot x} 100\ \%$$

Single-phase alternating current

voltage drop[2)] $\Delta U = I \cdot 2 \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)$

percentage voltage drop[2)] $\Delta u = \dfrac{\Delta U}{U_n} 100\,\% = \dfrac{I \cdot 2 \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)}{U_n}$

power loss $\Delta P = I^2 R'_L \cdot 2 \cdot l = \dfrac{2 \cdot l \cdot P^2}{x \cdot A \cdot U^2 \cdot \cos^2\varphi}$

percentage power loss $\Delta p = \dfrac{\Delta P}{P_n} 100\,\% = \dfrac{I^2 \cdot R'_L \cdot 2 \cdot l}{P_n} 100\,\%$

conductor cross-section[1)]

$$A = \frac{2 \cdot l \cos\varphi}{x\left(\dfrac{\Delta U}{I} - X'_L \cdot 2 \cdot l \cdot \sin\varphi\right)} = \frac{2 \cdot l \cos\varphi}{x\left(\dfrac{\Delta u \cdot U_n}{I \cdot 100\,\%} - X'_L \cdot 2 \cdot l \cdot \sin\varphi\right)}$$

Three-phase current

voltage drop[2)] $\Delta U = \sqrt{3} \cdot I \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)$

percentage voltage drop[2)] $\Delta u = \dfrac{\Delta U}{U_n} 100\,\% = \dfrac{\sqrt{3} \cdot I \cdot l\,(R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi)}{U_n} 100\,\%$

power loss $\Delta P = 3 \cdot I^2 R'_L \cdot l = \dfrac{l \cdot P^2}{x \cdot A \cdot U^2 \cdot \cos^2\varphi}$

percentage power loss $\Delta p = \dfrac{\Delta P}{P_n} 100\,\% = \dfrac{3\,I^2 \cdot R'_L \cdot l}{P_n} 100\,\%$

conductor cross-section[1)]

$$A = \frac{l \cdot \cos\varphi}{x\left(\dfrac{\Delta U}{\sqrt{3} \cdot I} - X'_L \cdot l \cdot \sin\varphi\right)} = \frac{l \cdot \cos\varphi}{x\left(\dfrac{\Delta u \cdot U}{\sqrt{3} \cdot I \cdot 100\,\%} - X'_L \cdot l \cdot \sin\varphi\right)}$$

l = one-way length of conductor
R'_L = Resistance per km
P = Active power to be transmitted ($P = P_n$)
U = phase-to-phase voltage
X'_L = Reactance per km
I = phase-to-phase current

In single-phase and three-phase a.c. systems with cables and lines of less than 16 mm^2 the inductive reactance can usually be disregarded. It is sufficient in such cases to calculate only with the d.c. resistance.

1) Reactance is slightly dependent on conductor cross section.
2) Longitudinal voltage drop becomes effectively apparent.

Table 2-8

Effective resistances per unit length of PVC-insulated cables with copper conductors as per DIN VDE 0271 for 0.6/1 kV

Number of conductors and cross-section	D. C. resistance at 70 °C	Ohmic resistance at 70 °C	Inductive reactance	Effective resistance per unit length $R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi$ at $\cos\varphi$ 0.95	0.9	0.8	0.7	0.6
	R'_{L-}	$R'_{L\sim}$	X'_L					
mm²	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km
4 × 1.5	14.47	14.47	0.115	13.8	13.1	11.65	10.2	8.77
4 × 2.5	8.71	8.71	0.110	8.31	7.89	7.03	6.18	5.31
4 × 4	5.45	5.45	0.107	5.21	4.95	4.42	3.89	3.36
4 × 6	3.62	3.62	0.100	3.47	3.30	2.96	2.61	2.25
4 × 10	2.16	2.16	0.094	2.08	1.99	1.78	1.58	1.37
4 × 16	1.36	1.36	0.090	1.32	1.26	1.14	1.020	0.888
4 × 25	0.863	0.863	0.086	0.847	0.814	0.742	0.666	0.587
4 × 35	0.627	0.627	0.083	0.622	0.60	0.55	0.498	0.443
4 × 50	0.463	0.463	0.083	0.466	0.453	0.42	0.38	0.344
4 × 70	0.321	0.321	0.082	0.331	0.326	0.306	0.283	0.258
4 × 95	0.231	0.232	0.082	0.246	0.245	0.235	0.221	0.205
4 × 120	0.183	0.184	0.080	0.2	0.2	0.195	0.186	0.174
4 × 150	0.149	0.150	0.080	0.168	0.17	0.168	0.162	0.154
4 × 185	0.118	0.1202	0.080	0.139	0.143	0.144	0.141	0.136
4 × 240	0.0901	0.0922	0.079	0.112	0.117	0.121	0.121	0.119
4 × 300	0.0718	0.0745	0.079	0.0954	0.101	0.107	0.109	0.108

Example:

A three-phase power of 50 kW with $\cos\varphi = 0.8$ is to be transmitted at 400 V over a line 100 m long. The voltage drop must not exceed 2 %. What is the required cross section of the line?

The percentage voltage drop of 2 % is equivalent to

$$\Delta U = \frac{\Delta u}{100\,\%} U_n = \frac{2\,\%}{100\,\%} 400\text{ V} = 8.0\text{ V}.$$

The current is

$$I = \frac{P}{\sqrt{3} \cdot U \cdot \cos\varphi} = \frac{50\text{ kW}}{\sqrt{3} \cdot 400\text{ V} \cdot 0.8} = 90\text{ A}.$$

Calculation is made easier by Table 2-8, which lists the effective resistance per unit length $R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi$ for the most common cables and conductors. Rearranging the formula for the voltage drop yields

$$R'_L \cdot \cos\varphi + X'_L \cdot \sin\varphi = \frac{\Delta U}{\sqrt{3} \cdot I \cdot l} = \frac{8.0}{\sqrt{3} \cdot 90\text{ A} \cdot 0.1\text{ km}} = 0.513\ \Omega/\text{km}.$$

According to Table 2-8 a cable of 50 mm² with an effective resistance per unit length of 0.42 Ω/km should be used. The actual voltage drop will then be

$$\Delta U = \sqrt{3} \cdot I \cdot l \,(R'_{\mathrm{L}} \cdot \cos\varphi + X'_{\mathrm{L}} \cdot \sin\varphi)$$
$$= \sqrt{3} \cdot 90\ \mathrm{A} \cdot 0.1\ \mathrm{km} \cdot 0.42\ \Omega/\mathrm{km} = 6.55\ \mathrm{V}.$$

This is equivalent to $\Delta u = \frac{\Delta U}{U_{\mathrm{n}}} 100\ \% = \frac{6.55\ \mathrm{V}}{400\ \mathrm{V}} 100\ \% = 1.6\ \%.$

2.5 Current input of electrical machines and transformers

Direct current

Motors:

$$I = \frac{P_{mech}}{U \cdot \eta}$$

Generators:

$$I = \frac{P}{U}$$

Single-phase alternating current

Motors:

$$I = \frac{P_{mech}}{U \cdot \eta \cdot \cos\varphi}$$

Transformers and synchronous generators:

$$I = \frac{S}{U}$$

Three-phase current

Induction motors:

$$I = \frac{P_{mech}}{\sqrt{3} \cdot U \cdot \eta \cdot \cos\varphi}$$

Transformers and synchronous generators:

$$I = \frac{S}{\sqrt{3} \cdot U}$$

Synchronous motors:

$$I \approx \frac{P_{mech}}{\sqrt{3} \cdot U \cdot \eta \cdot \cos\varphi} \cdot \sqrt{1 + \tan^2\varphi}$$

In the formulae for three-phase current, U is the phase voltage.

Table 2-9

Motor current ratings for three-phase motors (typical values for squirrel-cage type)

Smallest possible short-circuit fuse (Service category gG[1)]) for three-phase motors. The maximum value is governed by the switching device or motor relay.

Motor output data			Rated currents at							
			230 V		400 V		500 V		600 V	
			Motor	Fuse	Motor	Fuse	Motor	Fuse	Motor	Fuse
kW	$\cos\varphi$	η %	A	A	A	A	A	A	A	A
0.25	0.7	62	1.4	4	0.8	2	0.6	2	—	—
0.37	0.72	64	2.0	4	1.2	4	0.9	2	0.7	2
0.55	0.75	69	2.7	4	1.5	4	1.2	4	0.9	2
0.75	0.8	74	3.2	6	1.8	4	1.5	4	1.1	2
1.1	0.83	77	4.3	6	2.5	4	2	4	1.5	2
1.5	0.83	78	5.8	16	3.3	6	2.6	4	2	4
2.2	0.83	81	8.2	20	4.7	10	3.7	10	2.9	6
3	0.84	81	11.1	20	6.4	16	5	10	3.5	6

(continued)

Table 2-9 (continued)

Motor current ratings for three-phase motors (typical values for squirrel-cage type)

Smallest possible short-circuit fuse (Service category gG[1]) for three-phase motors. The maximum value is governed by the switching device or motor relay.

Motor output data			Rated currents at							
			230 V		400 V		500 V		660 V	
			Motor	Fuse	Motor	Fuse	Motor	Fuse	Motor	Fuse
kW	cos φ	η %	A	A	A	A	A	A	A	A
4	0.84	82	14.6	25	8.4	20	6.4	16	4.9	10
5.5	0.85	83	19.6	35	11.3	25	8.6	20	6.7	16
7.5	0.86	85	25.8	50	14.8	35	11.5	25	9	16
11	0.86	87	36.9	63	21.2	35	17	35	13	25
15	0.86	87	50	80	29	50	22.5	35	17.5	25
18.5	0.86	88	61	100	35	63	27	50	21	35
22	0.87	89	71	100	41	63	32	63	25	35
30	0.87	90	96	125	55	80	43	63	33	50
37	0.87	90	119	200	68	100	54	80	42	63
45	0.88	91	141	225	81	125	64	100	49	63
55	0.88	91	172	250	99	160	78	125	60	100
75	0.88	91	235	350	135	200	106	160	82	125
90	0.88	92	279	355	160	225	127	200	98	125
110	0.88	92	341	425	196	250	154	225	118	160
132	0.88	92	409	600	235	300	182	250	140	200
160	0.88	93	491	600	282	355	220	300	170	224
200	0.88	93	613	800	353	425	283	355	214	300
250	0.88	93	—	—	441	500	355	425	270	355
315	0.88	93	—	—	556	630	444	500	337	400
400	0.89	96	—	—	—	—	534	630	410	500
500	0.89	96	—	—	—	—	—	—	515	630

[1] see 7.1.2 for definitions

The motor current ratings relate to normal internally cooled and surface-cooled threephase motors with synchronous speeds of 1500 min^{-1}.

The fuses relate to the stated motor current ratings and to direct starting:
starting current max. 6 × rated motor current,
starting time max. 5 s.

In the case of slipring motors and also squirrel-cage motors with star-delta starting ($t_{start} \leqq 15$ s, $I_{start} = 2 \cdot I_n$) it is sufficient to size the fuses for the rated current of the motor concerned.
Motor relay in phase current: set to 0.58 × motor rated current.

With higher rated current, starting current and/or longer starting time, use larger fuses. Note comments on protection of lines and cables against overcurrents (Section 13.2.3).

2.6 Attenuation constant *a* of transmission systems

The transmission properties of transmission systems, e. g. of lines and two-terminal pair networks, are denoted in logarithmic terms for the ratio of the output quantity to the input quantity of the same dimension. When several transmission elements are arranged in series the total attenuation or gain is then obtained, again in logarithmic terms, by simply adding together the individual partial quantities.

The natural logarithm for the ratio of two quantities, e. g. two voltages, yields the voltage gain in Neper (Np):

$$\frac{a}{\mathrm{Np}} = \ln U_2/U_1.$$

If $P = U^2/R$, the power gain, provided $R_1 = R_2$ is

$$\frac{a}{\mathrm{Np}} = \frac{1}{2} \ln P_2/P_1.$$

The conversion between logarithmic ratios of voltage, current and power when $R_1 \neq R_2$ is

$$\ln U_2/U_1 = \ln I_2/I_1 + \ln R_2/R_1 = \frac{1}{2} \ln P_2/P_1 + \frac{1}{2} \ln R_2/R_1.$$

The common logarithm of the power ratio is the power gain in Bel. It is customary to calculate with the decibel (dB), one tenth of a Bel:

$$\frac{a}{\mathrm{dB}} = 10 \lg P_2/P_1.$$

If $R_1 = R_2$, for the conversion we have

$$\frac{a}{\mathrm{dB}} = 20 \lg U_2/U_1 \text{ respectively } \frac{a}{\mathrm{dB}} = 20 \lg I_2/I_1.$$

If $R_1 \neq R_2$, then

$$10 \lg P_2/P_1 = 20 \lg U_2/U_1, - 10 \lg R_2/R_1, = 20 \lg I_2/I_1, + 10 \lg R_2/R_1.$$

Relationship between Neper and decibel:

1 dB = 0.1151 Np
1 Np = 8.6881 dB

In the case of absolute levels one refers to the internationally specified values $P_0 = 1$ mW at 600 Ω, equivalent to $U_0 \cdot 0.775$ V, $I_0 \cdot 1.29$ mA (0 Np or 0 dB).

For example, 0.36 Np signifies a voltage ratio of $U/U_0 = e^{0.35} = 1.42$.

This corresponds to an absolute voltage level of $U = 0.776 \text{ V} \cdot 1.42 = 1.1$ V. Also 0.35 Np = 0.35 · 8.6881 = 3.04 dB.